Design and Analysis of Spatially Coupled LDPC Convolutional Codes

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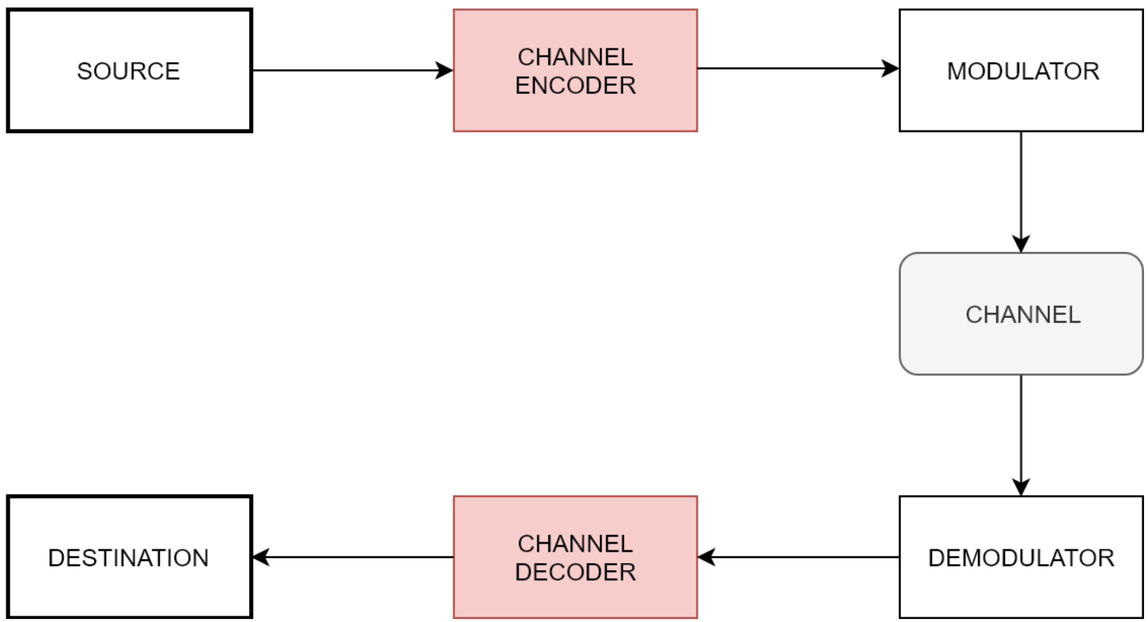
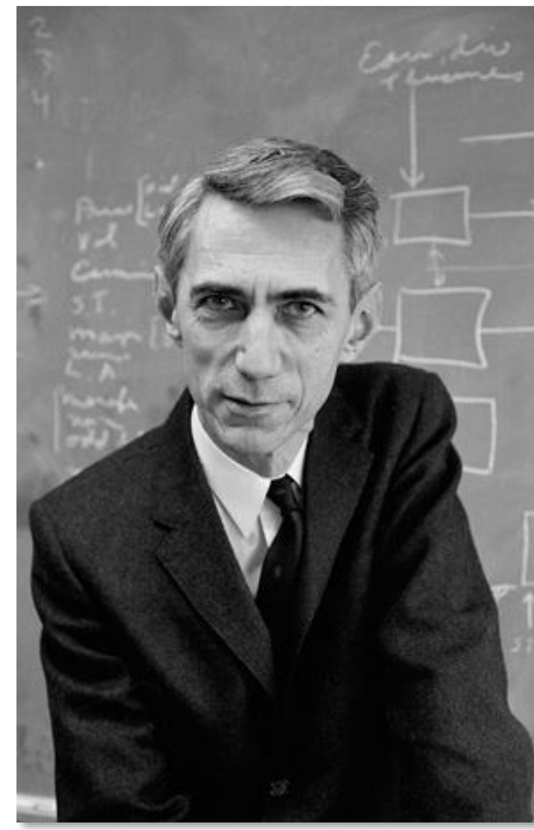
A Digital Communication Scheme





[1] C. E. Shannon, “A mathematical theory of communication”, Bell system technical journal, vol. 27, 1948.

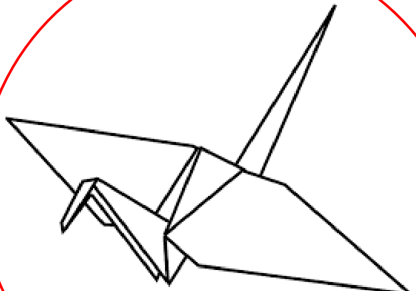
LDPC Codes: Error Correction and Beyond

- LDPC codes allow a convenient trade-off between error rate performance and decoding complexity

WiMAX

DVB S2

ERROR CORRECTION



LEDAcrypt

CODE-BASED CRYPTO

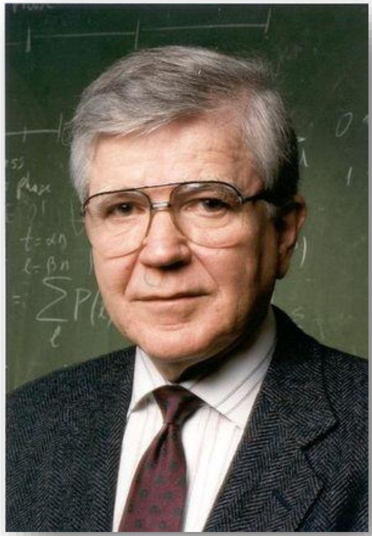
Random LDPC Codes vs. Structured LDPC Codes

- In most scenarios, **protograph-based** codes are considered

- **Structured** LDPC codes often guarantee significant savings in terms of encoding and decoding complexity

- **Quasi-cyclic** LDPC codes allow a convenient theoretical analysis and they can be seen as protograph-based structured codes

- Random LDPC codes have to be analyzed in probabilistic terms and leave less space to the code design

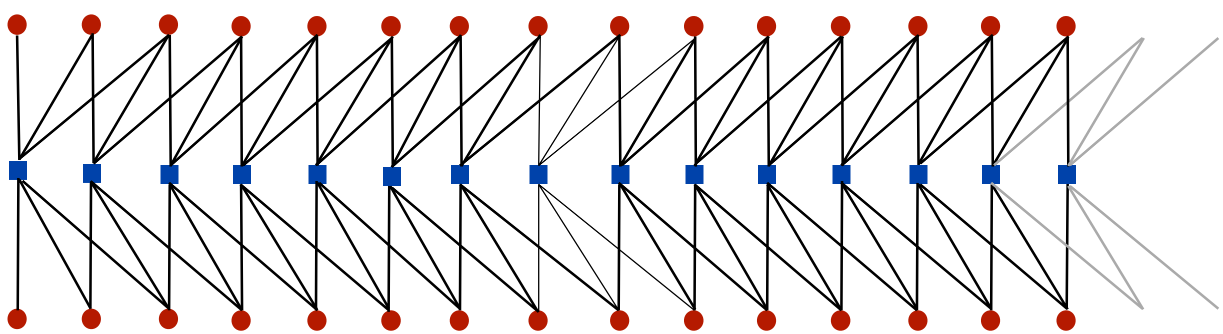


SC Codes and Optical Communications: an Ideal Match?

- Modern optical communication systems require high-performing error correcting codes that support throughputs of 100 Gbit/s (or multiples)

- Coding gains close to the theoretical limits at a target $\mathrm{BER} = 10^{-15}$ are recommended

- **Protograph-based spatially coupled codes** are recognized as a suitable solution to these challenges[2]



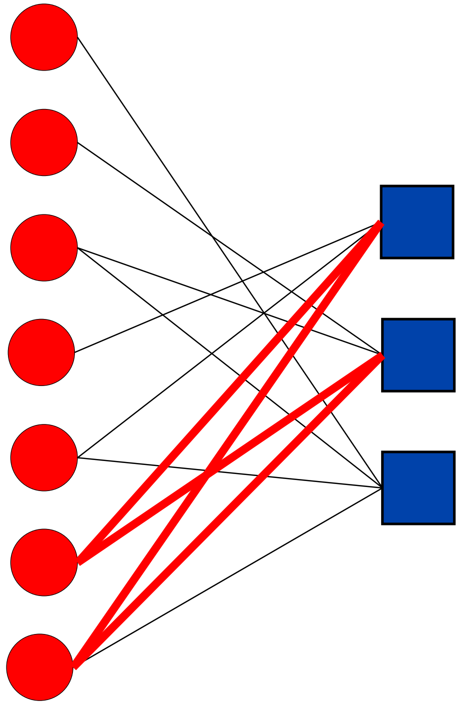
[2] A. Graell I Amat, C. Häger, F. Brännström, E. Agrell, "Spatially-coupled codes for optical communications: state-of-the-art and open problems", 2015 Opto-Electronics and Communications Conference (OECC), Shangai, China, Jun-Jul 2015.

LDPC codes

- Low-Density Parity-Check codes[3]

- Graph-based codes → Tanner graph[4]

$$\boldsymbol{H} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

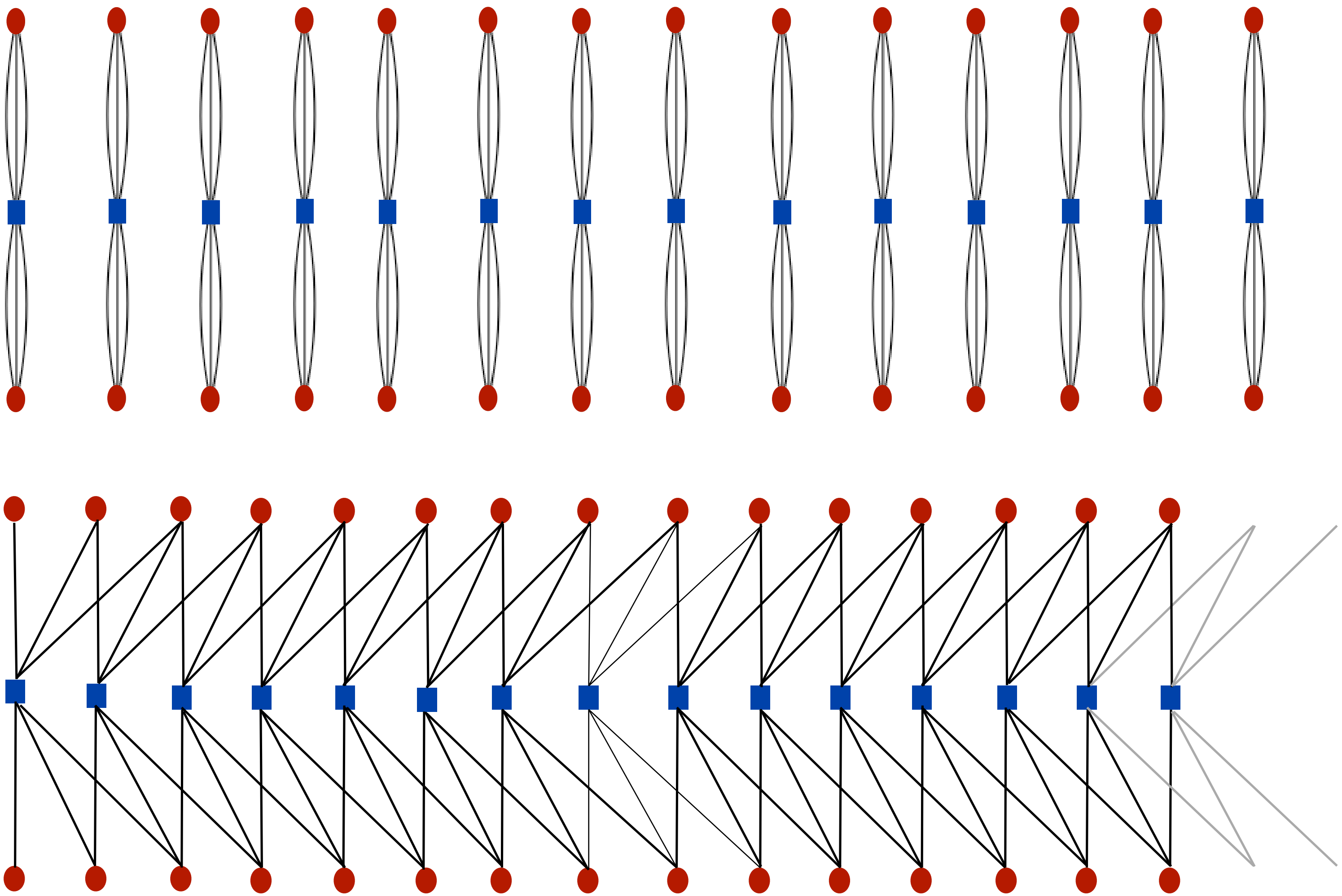


- Cycles deteriorate the performance of iterative decoders

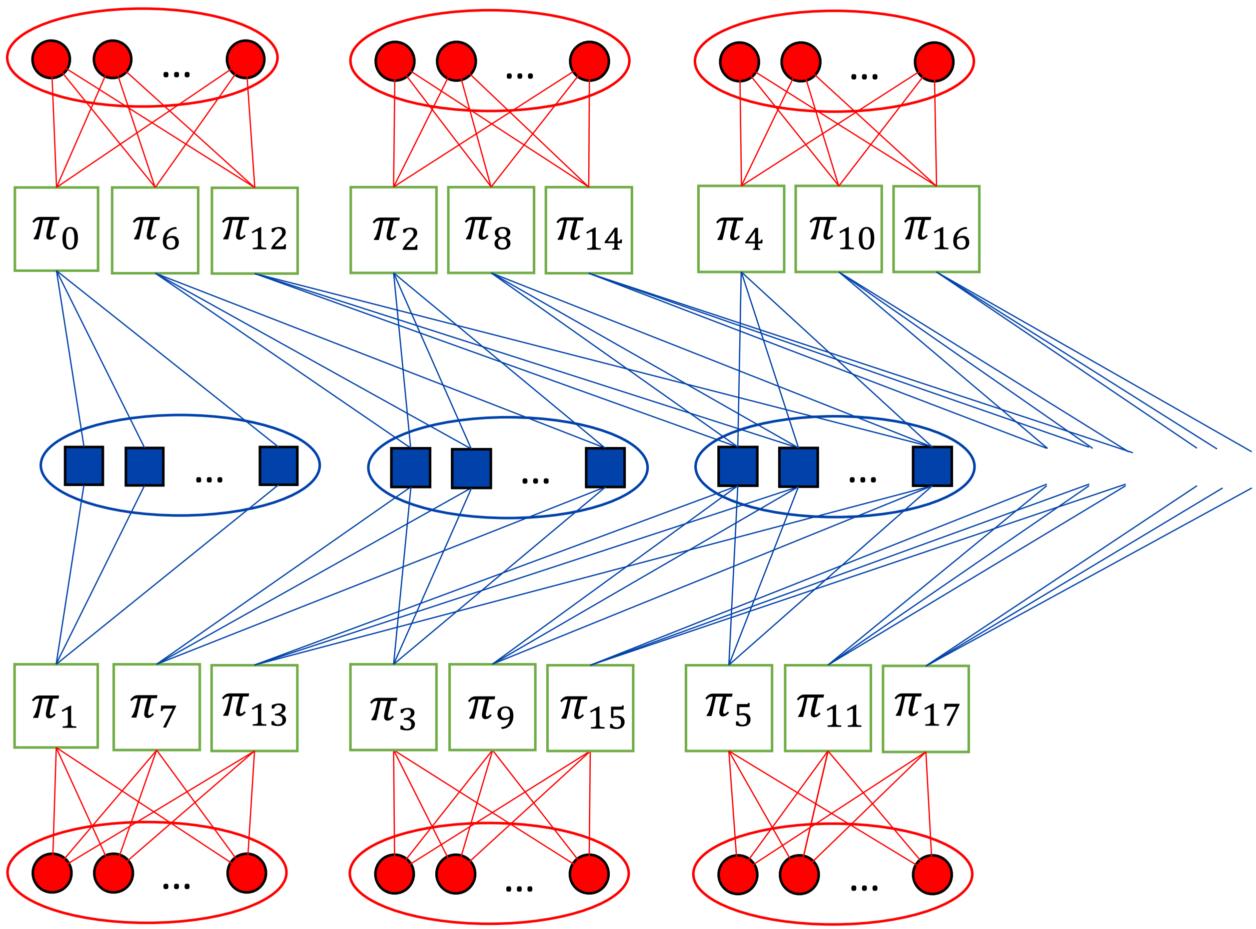
- The *girth* g of a code is the length of the shortest cycle(s) in its Tanner graph

[3] R. G. Gallager, "Low-Density Parity-Check Codes", PhD thesis, Dept. Elect. Eng., MIT, Cambridge, MA, USA, Jul. 1963.
[4] R. Tanner, "A recursive approach to low complexity codes", IEEE Trans. Inf. Theory, vol. 27, no. 5, pp. 533–547, Sep. 1981.

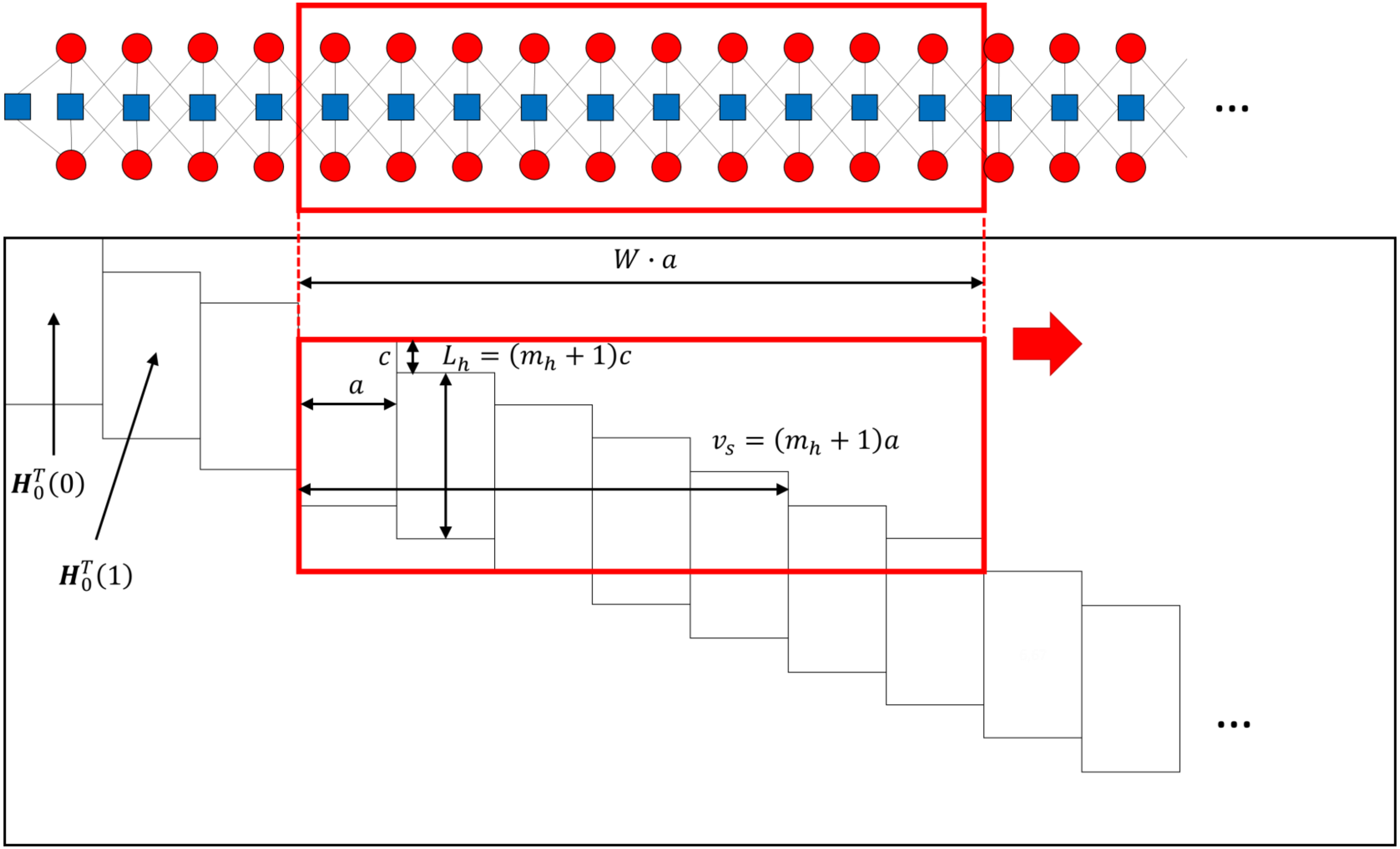
Spatially Coupled LDPC Codes - Coupling



Spatially Coupled LDPC Codes – Lifting



Sliding Window Decoding



[5] M. Papaleo, A.R. Iyengar, P.H. Siegel, J.K. Wolf, G.E. Corazza, "Windowed erasure decoding of LDPC convolutional codes," in Proc. IEEE Information Theory Workshop (ITW), Cairo, Egypt, Jan. 2010, pp. 1-5.

Complexity and Latency of a Sliding Window Decoder

- The decoding latency Λ_{SW} only depends on the window size and on a

- The average decoding complexity Γ_{SW} also depends on the average number of decoding iterations, the number of quantization bits and on the column weight

$$\begin{cases} \Lambda_{SW} = Wa = \alpha(m_h + 1)a = \alpha v_s \\ \Gamma_{SW} = W I_{avg} f(c, a, q, w_c) \end{cases}$$

- Spatially coupled LDPC codes achieve capacity over a broad family of channels under belief propagation decoding[6]
- This happens for $L \to \infty$, $M \to \infty$, $W \to \infty$

[6] S. Kudekar, T. J. Richardson, and R. L. Urbanke, "Spatially coupled ensembles universally achieve capacity under belief propagation", in IEEE Trans. Inf. Theory, Dec. 2013, vol. 59, pp. 7761–7813.

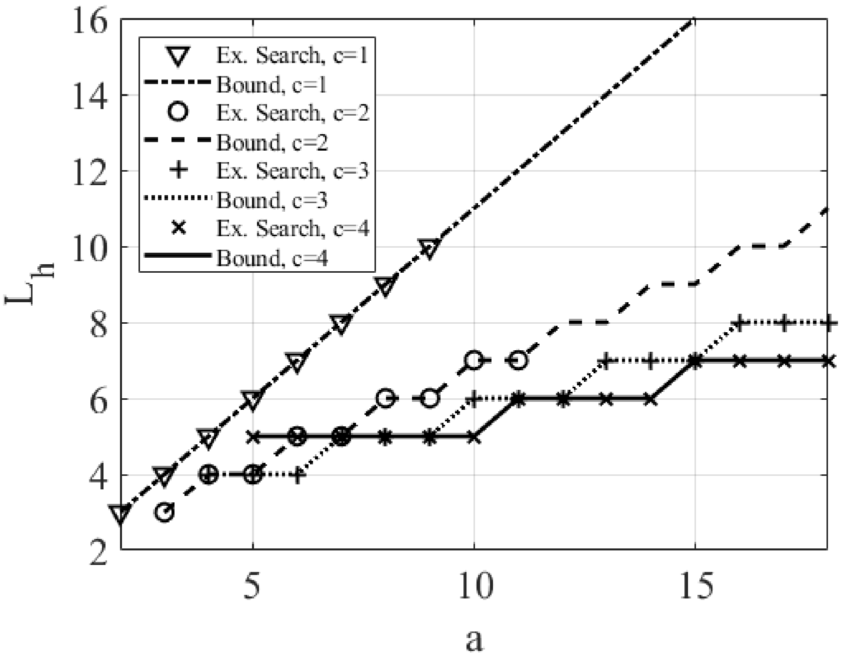
Main Contributions

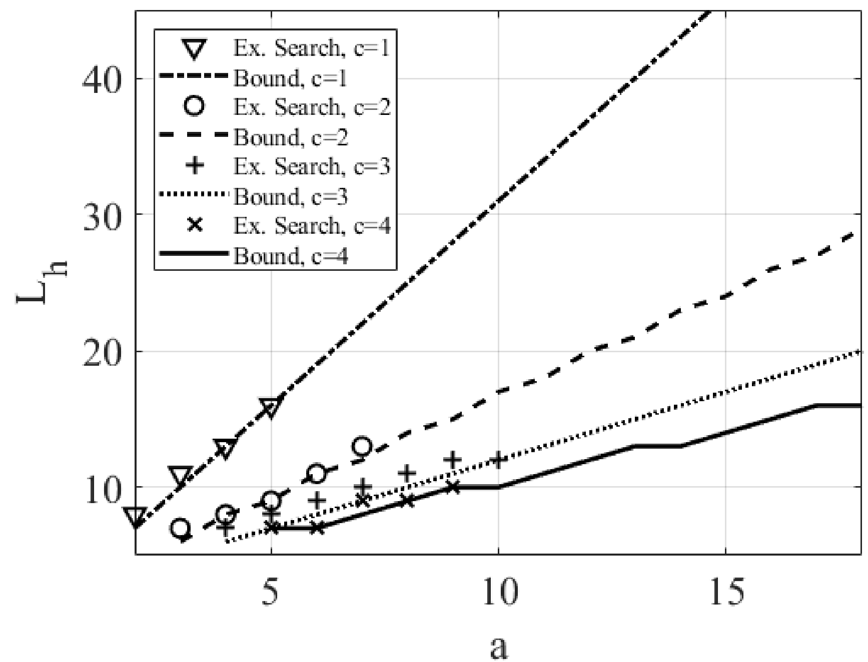
1. **COMPACT CODES**
   - Lower bounds on the constraint length of some families of SC-LDPC codes which allow to avoid cycles up to a given length
   - Design methods permitting to achieve or approach these bounds

2. **LOW-WEIGHT CODEWORDS AND CYCLES**
   - Codewords can be associated to a number of cycles in the code Tanner graph
   - Heuristic method based on the removal of some cycles yielding improved minimum distance properties and error rate performance

3. **DECODING PARAMETERS:**
   - Heuristic method to find the best possible trade-off between window size and number of iterations of sliding window decoders

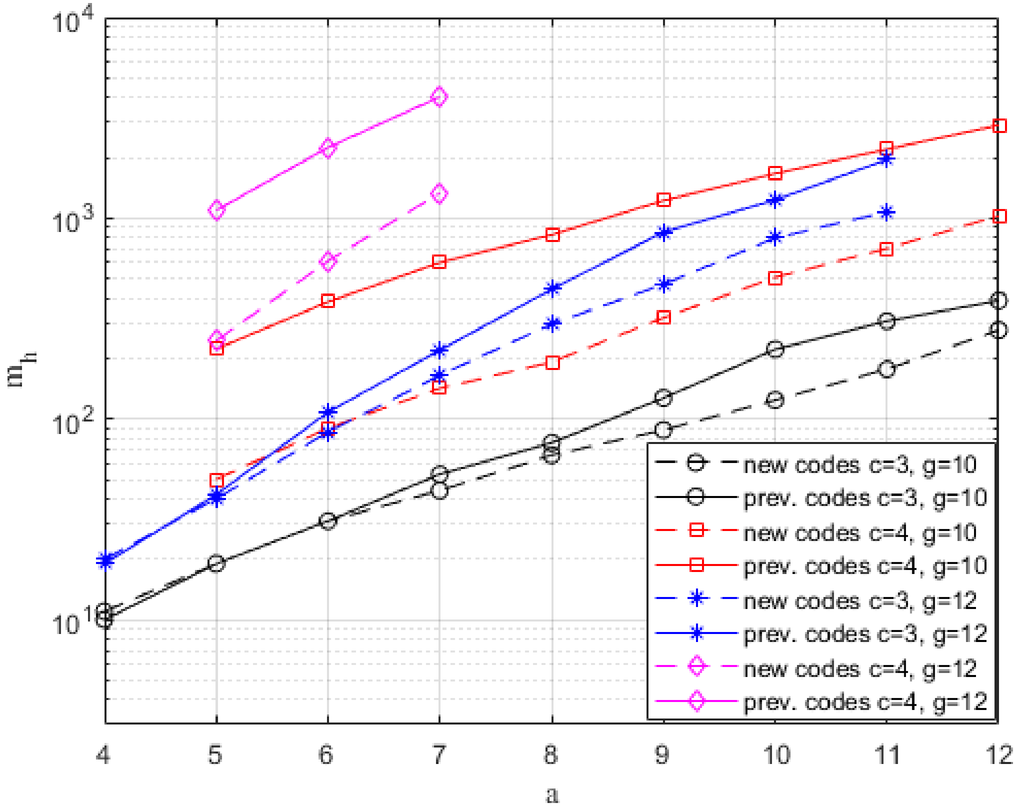
Constraint Length: Bounds vs. Exhaustive Search





[7] M. Battaglioni, A. Tasdighi, G. Cancellieri, F. Chiaraluce, and M. Baldi, “Design and analysis of time-invariant SC-LDPC convolutional codes with small constraint length”, IEEE Trans. Commun., vol. 66, no. 3, pp. 918–931, Mar. 2018.
[8] M. Baldi, M. Battaglioni, F. Chiaraluce, and G. Cancellieri, “Time-invariant spatially coupled low-density parity-check codes with small constraint length”, in Proc. IEEE BlackSeaCom, pp. 1–5, Varna, Bulgary, Jun. 2016.

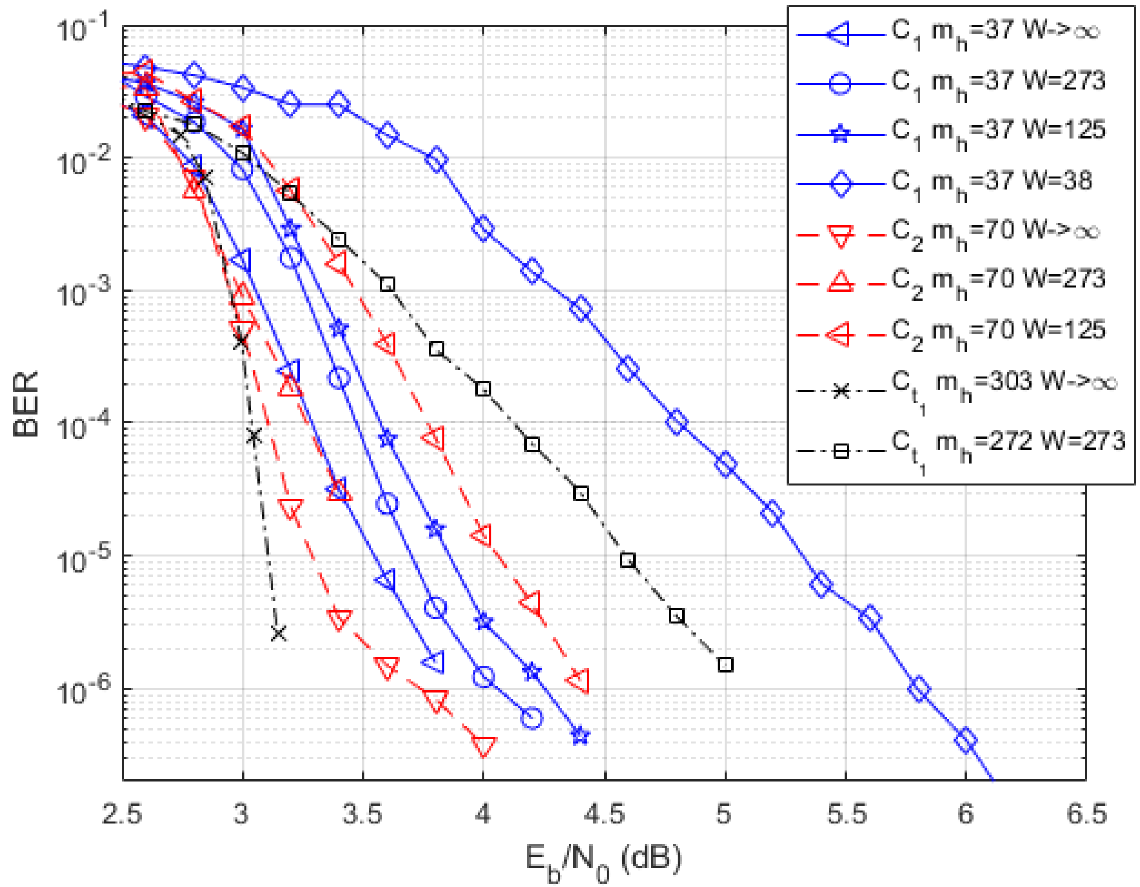
Design Based on Sequentially Multiplied Columns



[9] I. E. Bocharova, F. Hug, R. Johannesson, B. D. Kudryashov, and R. V. Satyukov, "Searching for voltage graph-based LDPC tailbiting codes with large girth", IEEE Trans. Inf. Theory, vol. 58, no. 4, pp. 2265–2279, Apr. 2012.

[10] M. H. Tadayon, A. Tasdighi, M. Battaglioni, M. Baldi, and F. Chiaraluce, "Efficient search of compact QC-LDPC and SC-LDPC convolutional codes with large girth", IEEE Commun. Lett., vol. 22, no. 6, pp. 1156–1159, Jun. 2018.

Error Rate Performance



[11] J. L. Fan, “Array codes as LDPC codes”, in Constrained Coding and Soft Iterative Decoding, pp. 195–203. Springer, 2001.

[12] R. M. Tanner, D. Sridhara, A. Sridharan, T. E. Fuja, and D. J. Costello, “LDPC block and convolutional codes based on circulant matrices”, IEEE Trans. Inf. Theory, vol. 50, no. 12, pp. 2966–2984, Dec. 2004.

Connection between Low-weight Codewords and Cycles

- In codes defined by a parity-check matrix with **column weight 2**, any **minimal codeword** can be unambiguously associated to **ONE cycle** in the Tanner graph

- When the column weight is larger than 2, we have to deal with more complicated mechanisms

- Any **minimal codeword** can be decomposed in **a number of cycles** overlapping in some positions

- A punctual removal of these cycles yields improvements of the code minimum distance and error rate performance

Code Design

1. Estimate the code minimum distance and look for its low-weight codewords[13]

2. Find the cycles associated to low-weight codewords

3. Modify ***H*** in such a way that all the cycles with the same length as those in step 2. are removed and estimate the code minimum distance

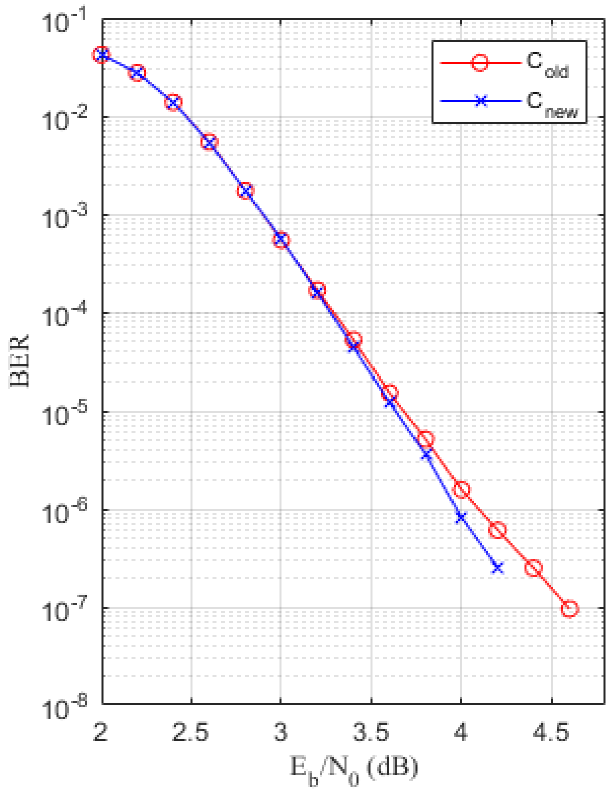
4. Repeat step 3. until the minimum distance achieves the upper bound[14] or all the cycles in the decomposition of a low-weight codeword are unavoidable

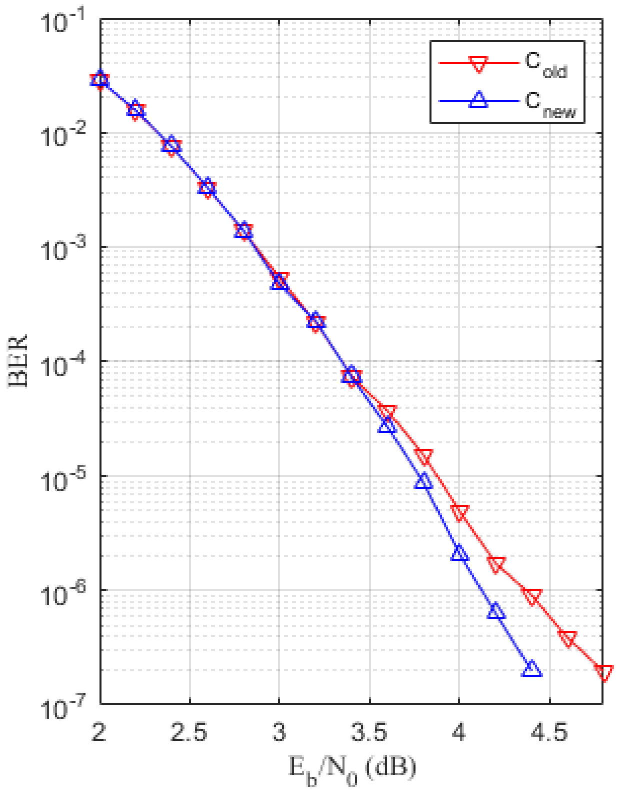
[13] D. J. C. MacKay. (2008) Source code for approximating the MinDist problem of LDPC codes. [Online]. Available: http://www.inference.eng.cam.ac.uk/mackay/MINDIST_ECC.htm

[14] R. Smarandache and P. O. Vontobel, "Quasi-cyclic LDP\C codes: influence of proto- and Tanner-graph structure on minimum Hamming distance upper bounds", IEEE Trans. Inform. Theory, vol. 58, no. 2, pp.585–607, Feb. 2012.

Error Rate Performance

- Based on the heuristic procedure, we have removed the cycles causing the occurrence of low-weight codewords from the code Tanner graph





[15] M. Zhang, Z. Wang, Q. Huang, and S. Wang, “Time-invariant quasi-cyclic spatially coupled LDPC codes based on packings”, IEEE Transactions on Communications, vol. 64, no. 12, pp. 4936–4945, Dec. 2016.

Complexity Constrained Scenarios

- The decoding complexity of a sliding window decoder is

$$\Gamma_{SW} = W I_{avg} f(c, a, q, w_c)$$

- Complexity may need to be kept limited due to scarcity of hardware and software resources

- We have proposed a heuristic method to find the best possible trade-off between **window size** and **number of iterations** of sliding window decoders

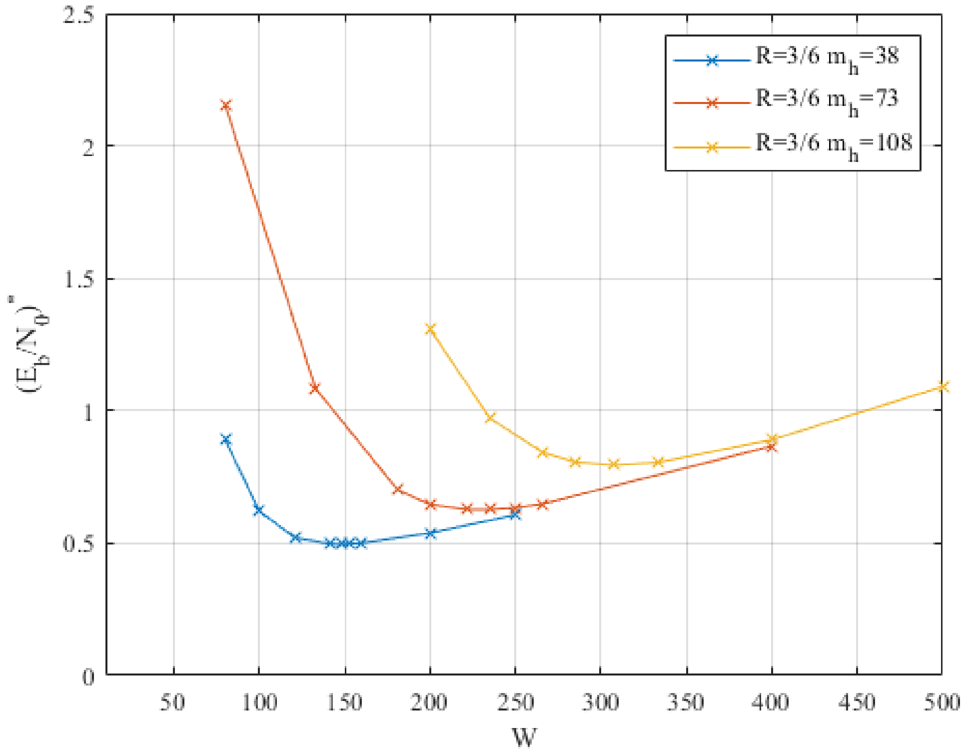
- We have considered **PEXIT analysis**, which allows to analyze the convergence behavior of protograph-based codes[17]

[16] M. Battaglioni, M. Baldi, and E. Paolini, "Complexity-constrained spatially coupled LDPC codes based on protographs", in Proc. ISWCS, pp. 49–53, Bologna, Italy, Sep. 2017.

[17] G. Liva and M. Chiani, "Protograph LDPC codes design based on EXIT analysis", in Proc. IEEE Global Telecommunications Conference (GLOBECOM) 2007, Washington, DC, USA, Nov. 2007, pp. 3250–3254.

An Optimal Choice of the Decoding Parameters

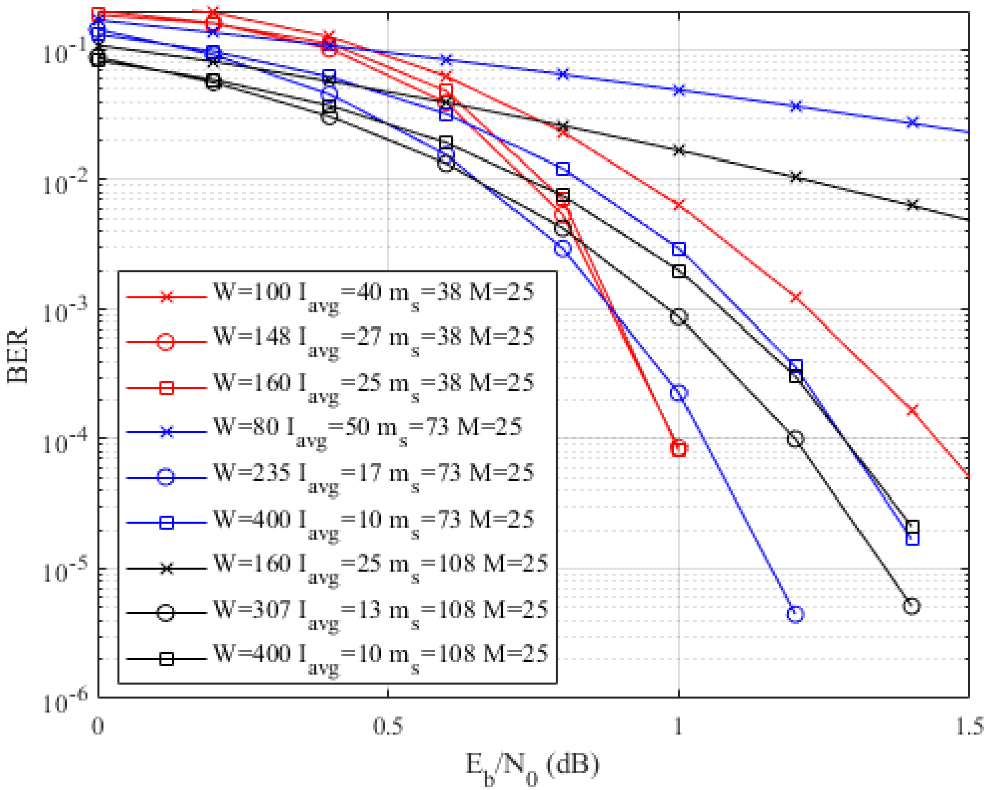
- There is an **optimal choice** of (W, I_{avg}) which minimizes the threshold $\left(\frac{E_b}{N_0}\right)^*$, with $I_{avg} = \left\lfloor \frac{C}{W} \right\rfloor$

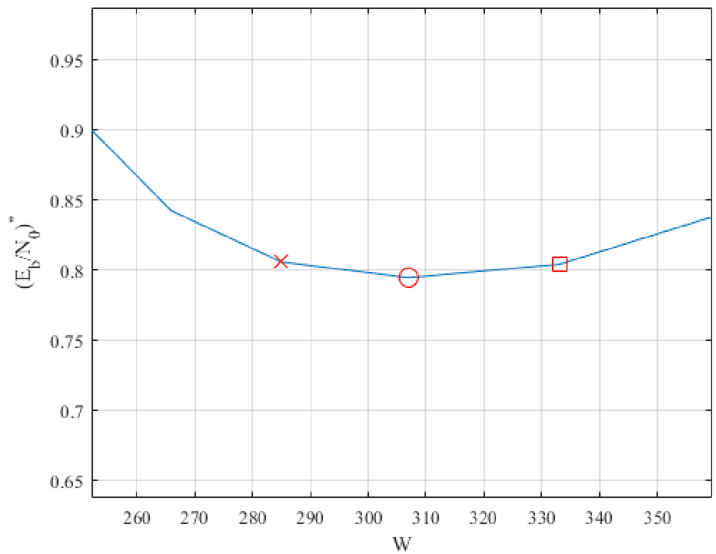


Lower
Larger complexity

Error Rate Performance

- For each code (color) we have considered the minimum threshold (marker '○'), a point to its left (marker 'x') and a point to its right (marker '□')





Conclusions

- Compact codes allowing for significant savings in terms of decoding complexity and latency under sliding window decoding can be designed

- The small window sizes enabled by compact codes makes them interesting for future scenarios

- The performance of these codes can be further improved by careful analysis of the code Tanner graph and choice of the decoding parameters

- Many properties of spatially coupled LDPC convolutional codes are at least as good as those of their QC-LDPC block counterparts