“K-Nearest Neighbors: A Powerful Tool to Design Radar Detectors”

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Classical Radar Detection

- Binary hypothesis testing problem

$$\begin{cases} H_0: & \boldsymbol{z} = \boldsymbol{n} \\ H_1: & \boldsymbol{z} = \alpha \boldsymbol{v} + \boldsymbol{n} \end{cases}$$

- Kelly's pioneering work: one-step and two-step generalized likelihood ratio tests (GLRTs)

$$t_{\text{Kelly}} = \frac{|\boldsymbol{z}^H \boldsymbol{S}^{-1} \boldsymbol{v}|^2}{\boldsymbol{v}^H \boldsymbol{S}^{-1} \boldsymbol{v} \left(1 + \boldsymbol{z}^H \boldsymbol{S}^{-1} \boldsymbol{z}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\text{Kelly}}$$

$$t_{\text{AMF}} = \frac{|\boldsymbol{z}^H \boldsymbol{S}^{-1} \boldsymbol{v}|^2}{\boldsymbol{v}^H \boldsymbol{S}^{-1} \boldsymbol{v}} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\text{AMF}}.$$

General *model-based* problem formulation

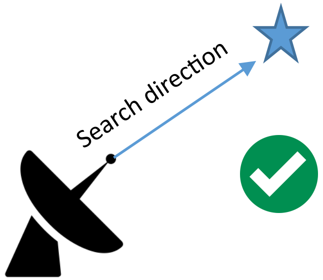
$$\begin{cases} H_0: & \boldsymbol{x} \sim \mathcal{D}^0 \\ H_1: & \boldsymbol{x} \sim \mathcal{D}^1 \end{cases}$$

no target vs target

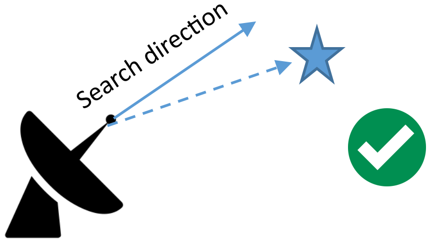
Several approaches designed for different $\mathcal{D}^0$ and $\mathcal{D}^1$

to promote

Selectivity: detect targets from a specific direction

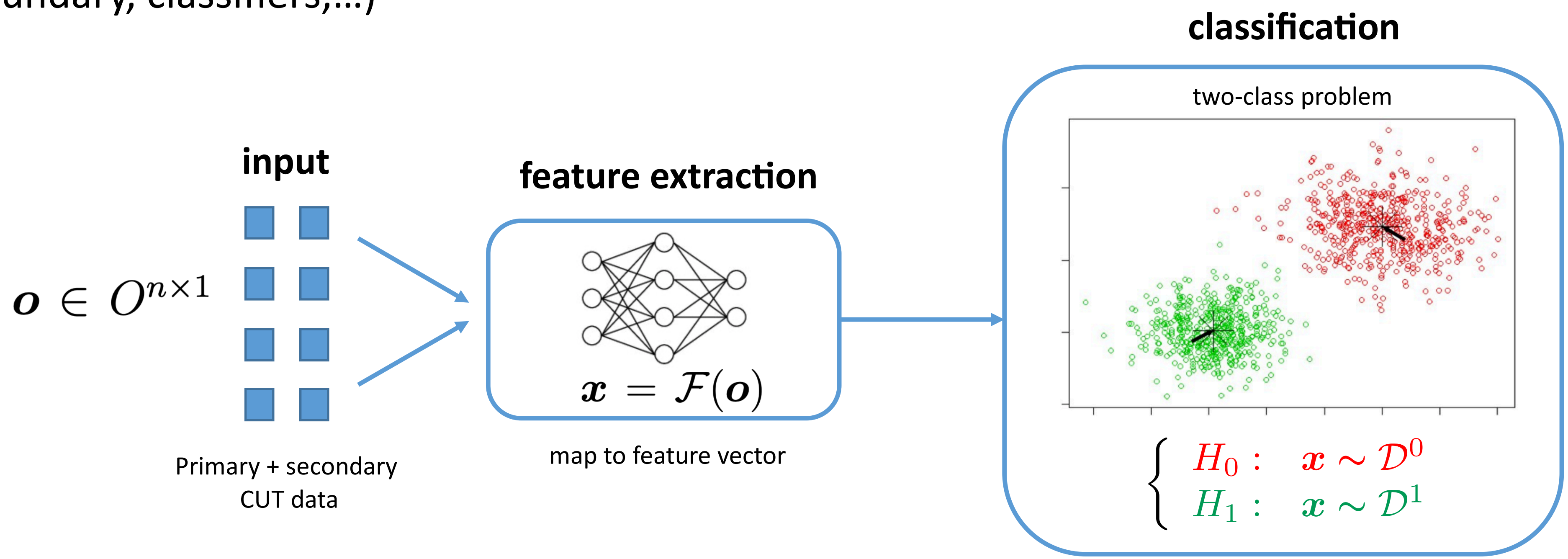


Robustness: reveal targets in presence of mismatches

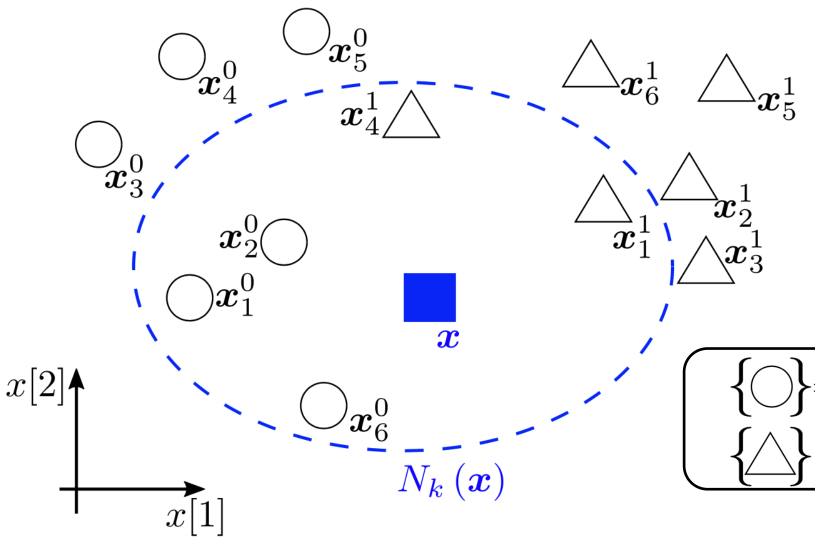


Radar Detection under the lens of Machine Learning

- intuitive interpretation using general concepts from machine learning (data clusters, decision region boundary, classifiers,…)

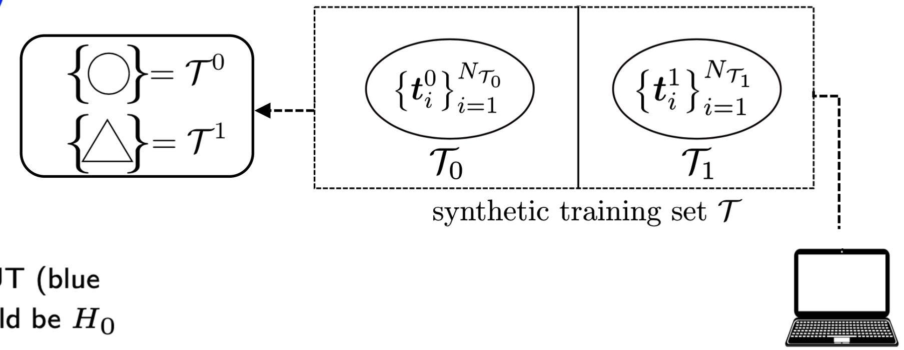


- **K-Nearest Neighbors (KNN):** simplest ML algorithm for classification, easily interpretable



$N_{\mathcal{T}} = 6$ ($N_{\mathcal{T}_0} = N_{\mathcal{T}_1}$), $k = 5$ NN of CUT (blue square) in $N_k(\boldsymbol{x})$; for $M = 3$, decision would be H_0

<u>model is NOT learned</u> (KNN is *non-parametric*, and it does not make any assumption about the statistical distribution of training data)



<u>Pseudorandom generation of dataset</u>

- ✓ Avoid data collection (problem of data scarcity)
- ✓ Not limited to "seen" data
- ✓ Decouple detection capability of the algorithm from the advantage of a real dataset

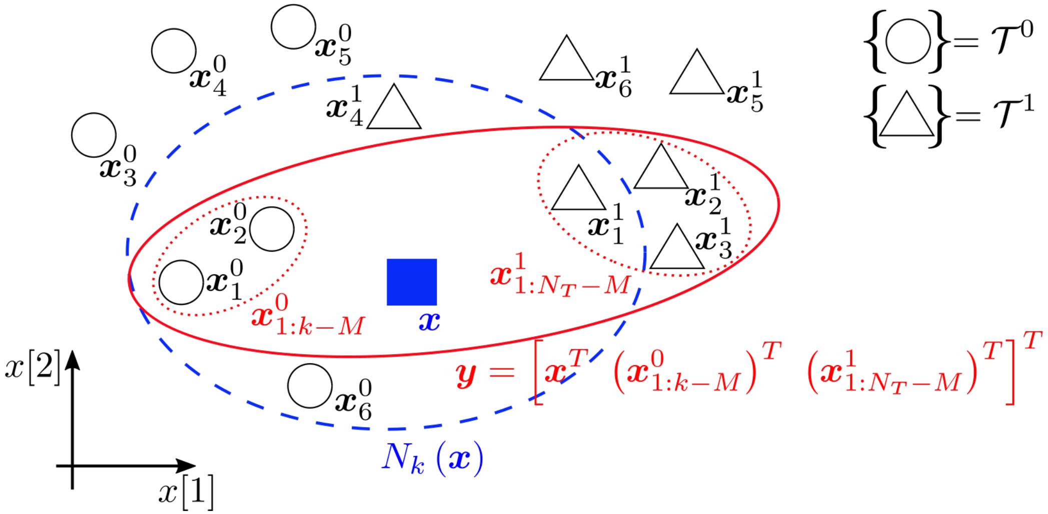
<u>combine model-based & data driven</u>

KNN Workflow

Input data from CUT $\boldsymbol{o} = [\boldsymbol{z}^\top \; \boldsymbol{r}_1^\top \; \cdots \; \boldsymbol{r}_K^\top]^\top$ → Feature vector **x** → $\bar{\ell} = \frac{1}{k} \sum_{\{i:\boldsymbol{x}_i \in N_k(\boldsymbol{x})\}} \ell_i \underset{H_0}{\overset{H_1}{\gtrless}} \eta$ (Either 0 or 1)

KNN-based Detectors: Theoretical Characterization

- KNN can be theoretically characterized (not usual for ML approaches)



$$P(\bar{\ell} > \eta) = 1 - \begin{pmatrix} N_\mathcal{T} \\ k-M \end{pmatrix} \begin{pmatrix} N_\mathcal{T} \\ N_\mathcal{T}-M \end{pmatrix} \times E\left[I_\mathcal{Y}(\boldsymbol{y}) \left(p_0\left(\boldsymbol{x}, \boldsymbol{x}^0_{1:k-M}\right)\right)^{N_\mathcal{T}-k+M} \left(p_1\left(\boldsymbol{x}, \boldsymbol{x}^1_{1:N_\mathcal{T}-M}\right)\right)^M \right]$$

returns P_{fa} or P_d according to the hypothesis actually in force

vectors can have any distribution, only two probabilities (p_0, p_1) matter

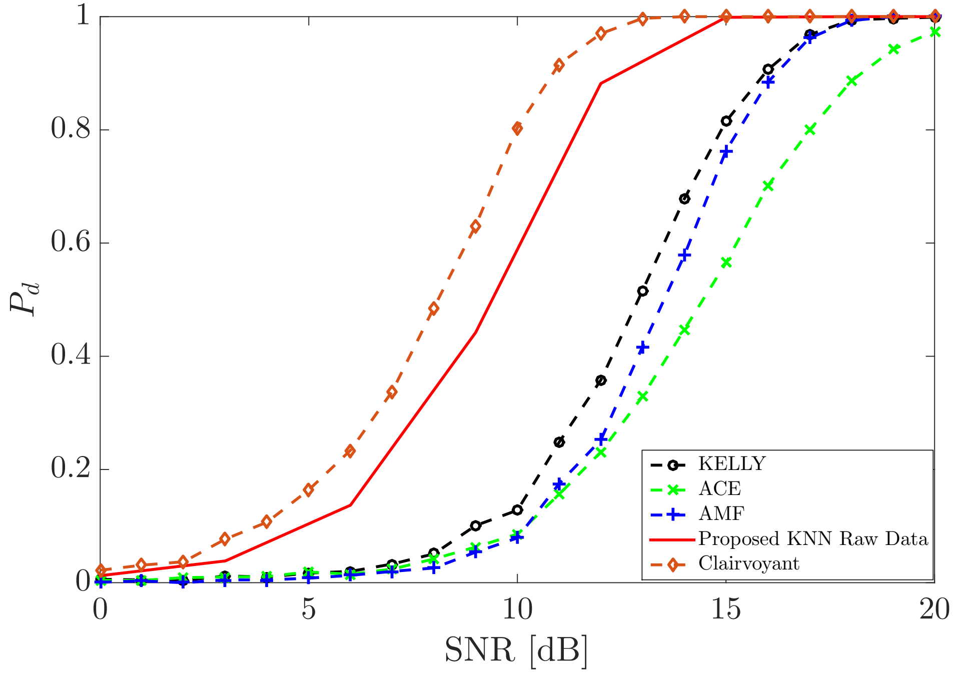
A. Coluccia, A. Fascista, G. Ricci: "A k-nearest neighbors approach to the design of radar detectors," *Signal Proces.*, 2020

KNN-based Adaptive Detection in Gaussian Noise

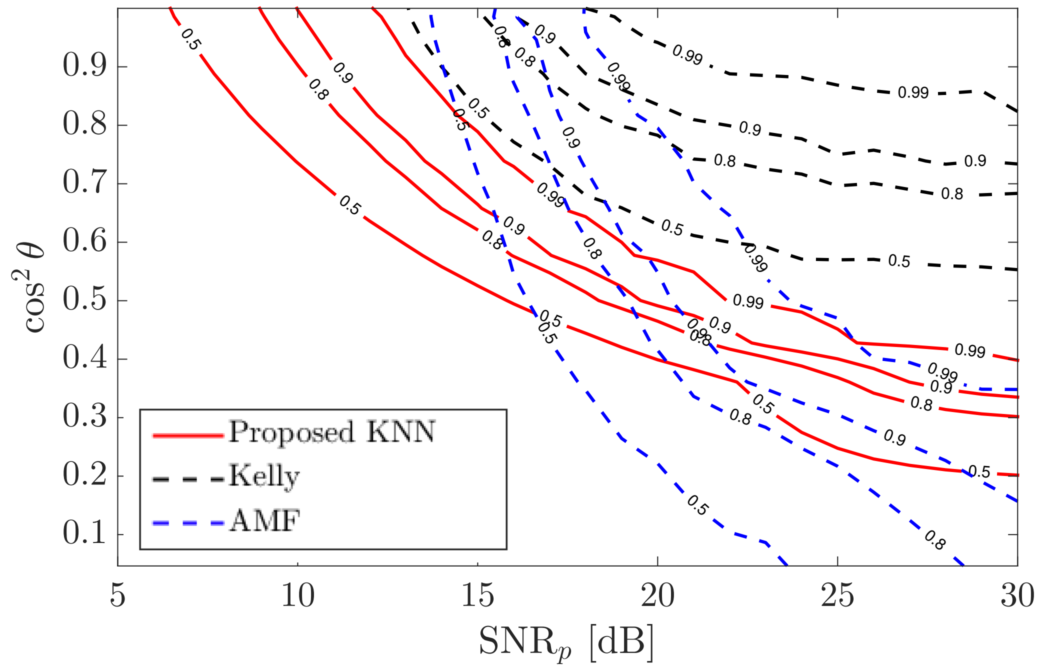
First approach: use as feature vector **x** the “whitened data” under test

$$\boldsymbol{x} = \boldsymbol{S}^{-1/2}\boldsymbol{z}$$

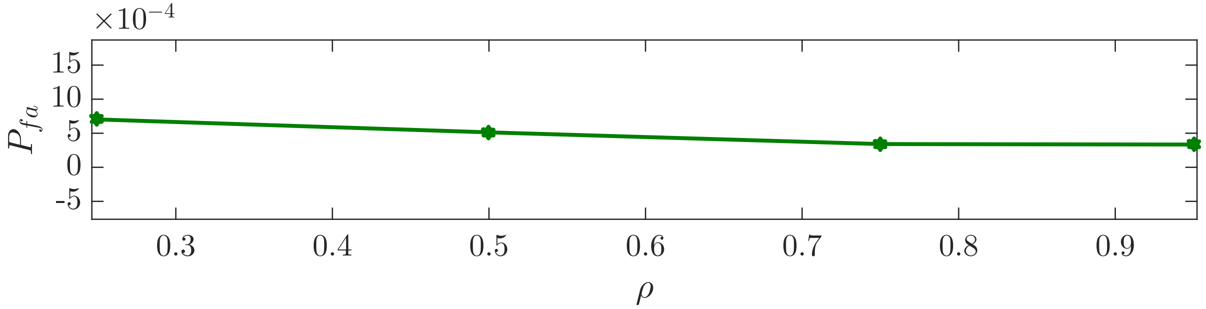
matched scenario, design SNR 15 dB



mismatch performance

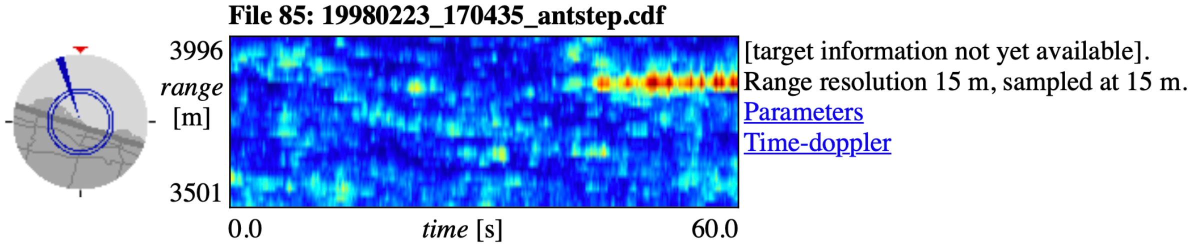


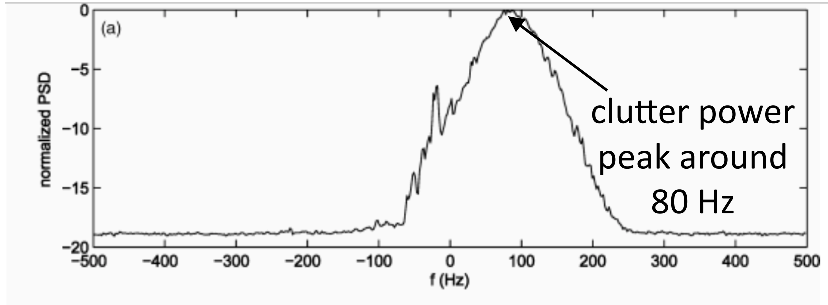
…looses *CFAR property but*



First KNN Approach: Analysis on real IPIX data

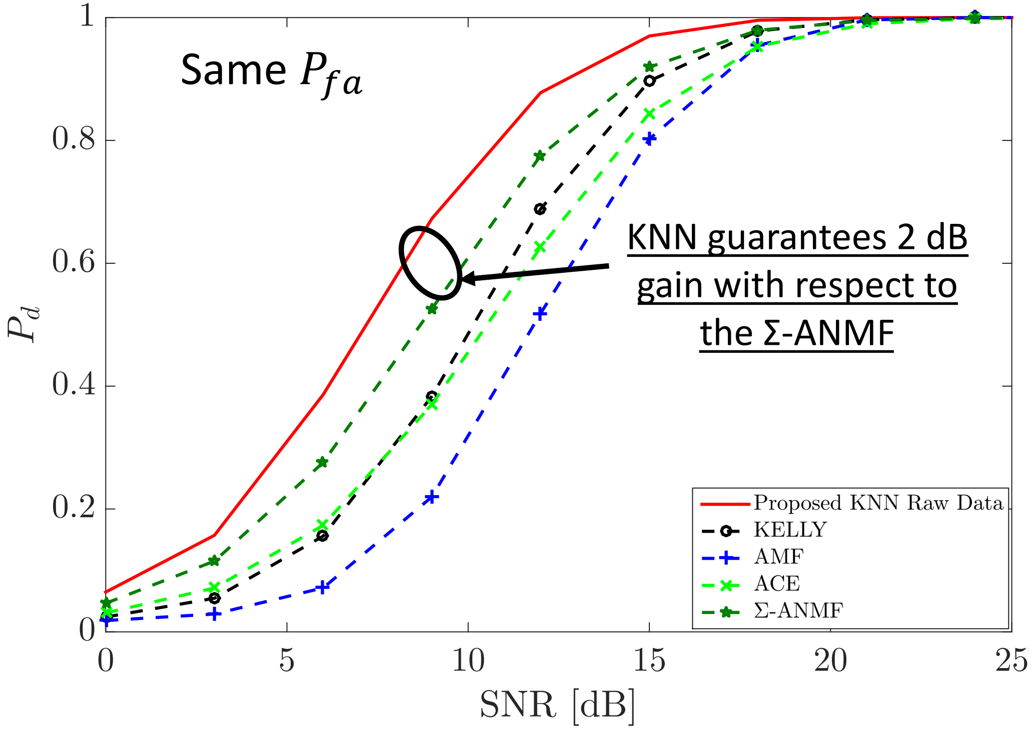
- Training data pseudorandomly generated
- Test dataset





from: A. De Maio, G. Foglia, E. Conte and A. Farina, "CFAR behavior of adaptive detectors: an experimental analysis," IEEE Transactions on Aerospace and Electronic Systems, vol. 41, no. 1, pp. 233-251, Jan. 2005

- Under H_1 we add a synthetic target with normalized Doppler frequency 80 Hz (target embedded in deep clutter)



actual P_{fa} larger than nominal one, but comparable among all the algorithms

Second Approach: KNN with well-known radar statistics

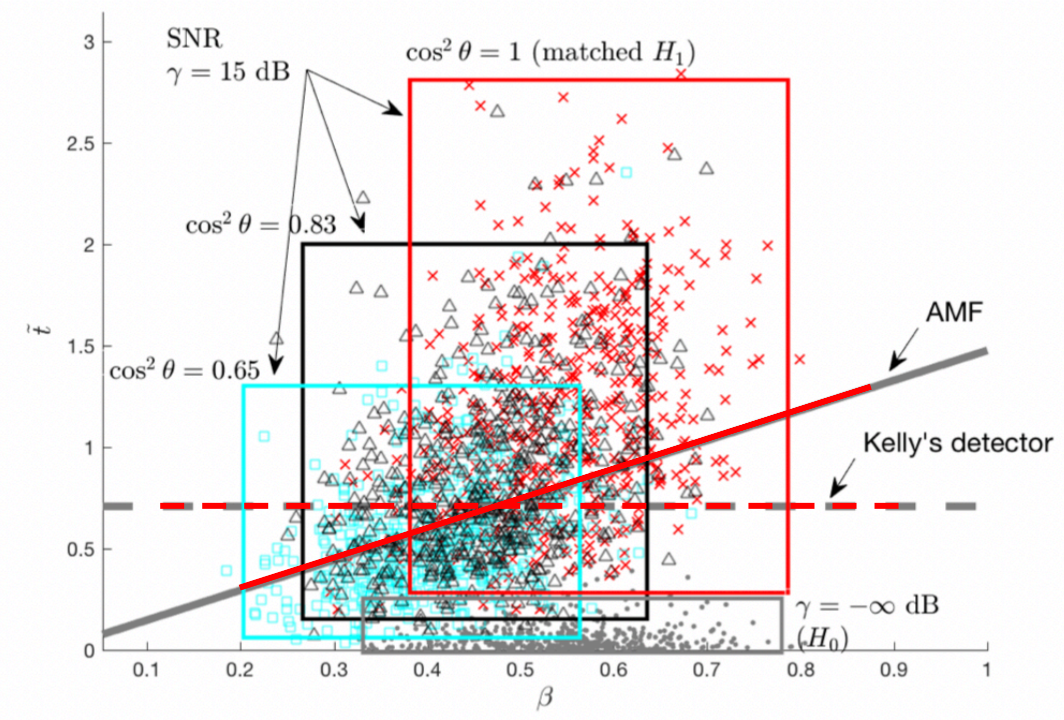
- **Second approach**: use as feature vector **x** a set of well-known radar statistics sharing a common dependency on the *maximal invariant* statistics

$$\boldsymbol{x} = \left[d_1\tilde{t}b[1]\ d_2\tilde{t}b[2]\ \cdots\ d_m\tilde{t}b[m]\right]^T$$

with $b[j] = f_j(\beta), \quad j = 1, \ldots, m,$

$$t_{\text{AMF}} = \frac{\tilde{t}}{\beta}, \qquad t_{\text{Kelly}} = \frac{\tilde{t}}{1+\tilde{t}}$$

with $\tilde{t} = \frac{t_{\text{Kelly}}}{1-t_{\text{Kelly}}}$ and $\beta = \frac{1}{1+\boldsymbol{z}^H\boldsymbol{S}^{-1}\boldsymbol{z}-\frac{|\boldsymbol{z}^H\boldsymbol{S}^{-1}\boldsymbol{v}|^2}{\boldsymbol{v}^H\boldsymbol{S}^{-1}\boldsymbol{v}}}$



- AMF detector more inclined to decide for H_1 also in presence of mismatches thanks to its positive slope (*robust behavior*)
- horizontal line of Kelly's detector can effectively separate H_0 from H_1 under matched conditions (*selective behavior*)

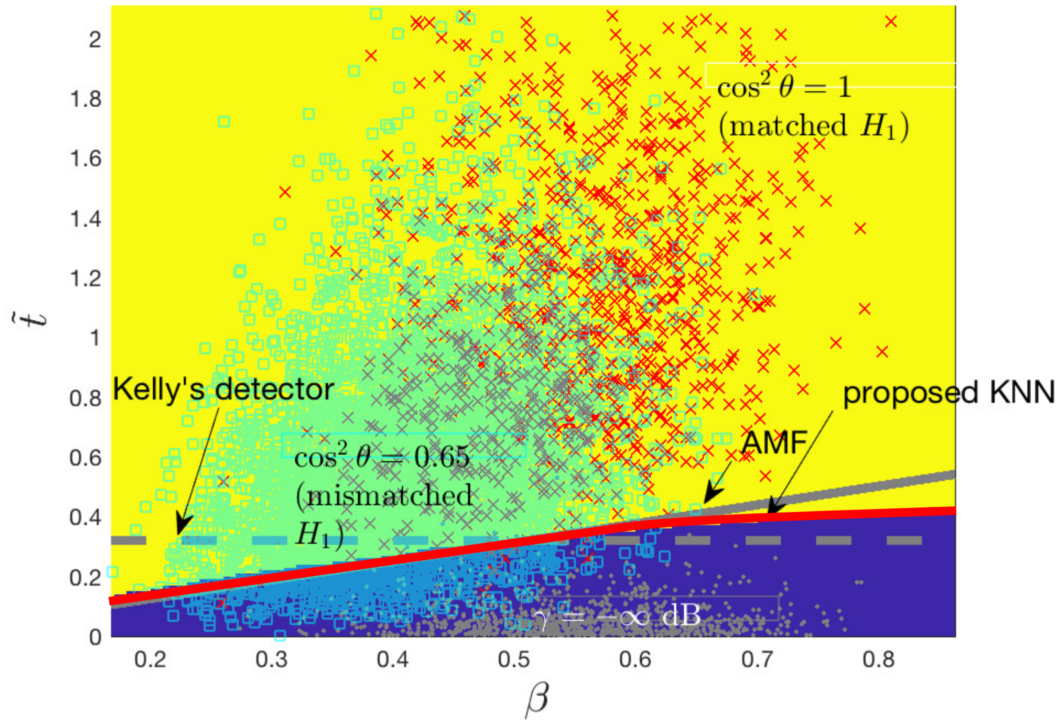
Second Approach: An Example

- **Example**: use a 2D feature vector with Kelly and AMF statistics

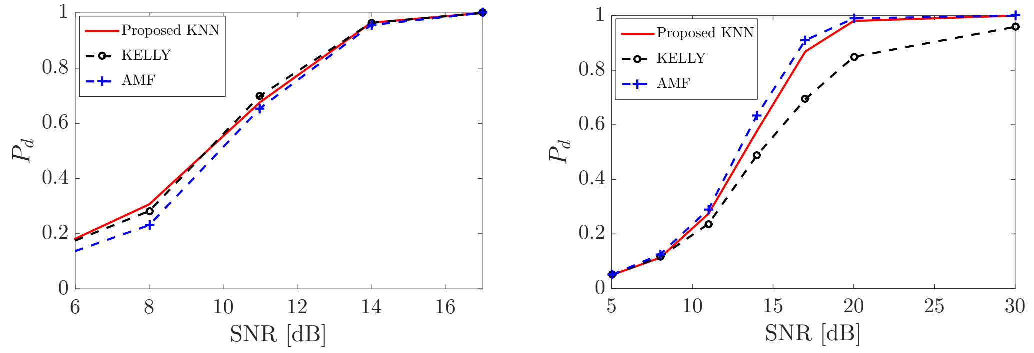
$$\boldsymbol{x} = \begin{bmatrix} d_1 \tilde{t} \\ d_2 \frac{\tilde{t}}{\beta} \end{bmatrix}$$

with d_1 and d_2 arbitrary (nonnegative) tuning parameters

- Interpretation in the feature space



- the proposed detector is a *non-linear* classifier in the feature space
- its decision region boundary is a mix of Kelly's and AMF ones
  - it has a positive slope very close to the AMF's one until acrossing point
  - then bends to try to get closer to the horizontal line of Kelly's detector
- in doing so, it can strike a balance between Kelly's and AMF performance, while guaranteeing the same P_{fa}



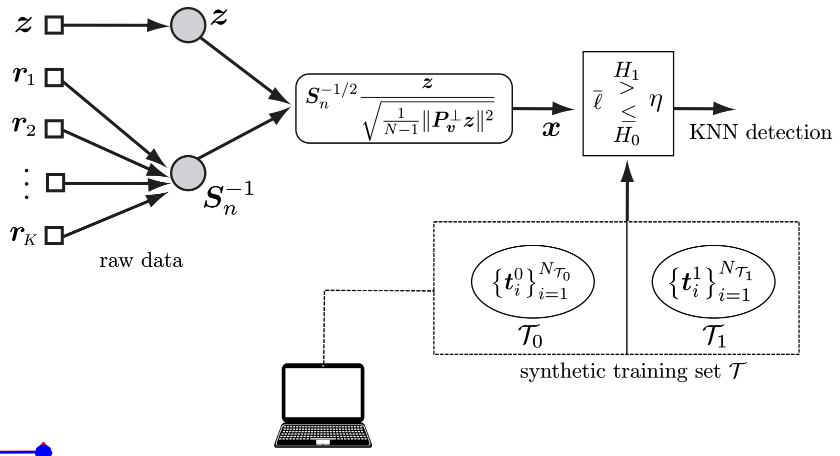
(a) matched conditions $\cos^2\theta = 1$ (b) mismatched conditions $\cos^2\theta = 0.46$

A. Coluccia, A. Fascista, G. Ricci: "Robust CFAR radar detection using a k-nearest neighbors rule", *IEEE ICASSP*, 2020

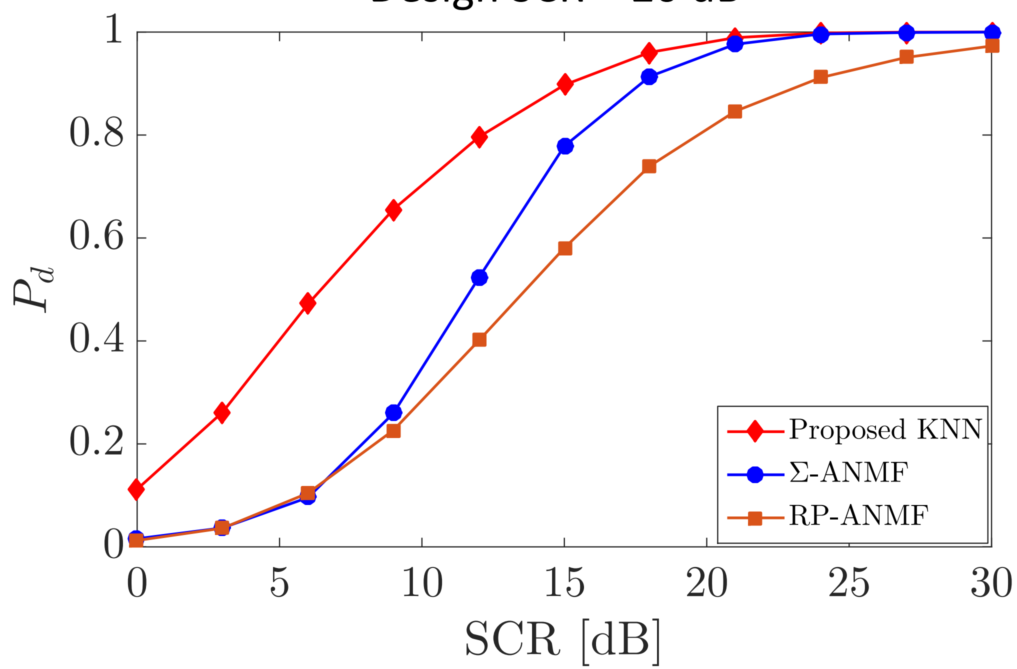
KNN-based Detection of Coherent Targets in non-Gaussian Noise

- Data model: *K-distributed clutter + thermal noise*

$$x = S_n^{-1/2} \frac{z}{\sqrt{\frac{1}{N-1}\|P_v^{\perp} z\|^2}}$$



Design SCR = 20 dB



A. Coluccia, A. Fascista, and G. Ricci, ``*A KNN-based Radar Detector for Coherent Targets in non-Gaussian Noise*'', to be submitted to IEEE SPL

KNN in non-Gaussian Noise: Analysis on Real IPIX Data

- Again training data pseudorandomly generate
- Test dataset

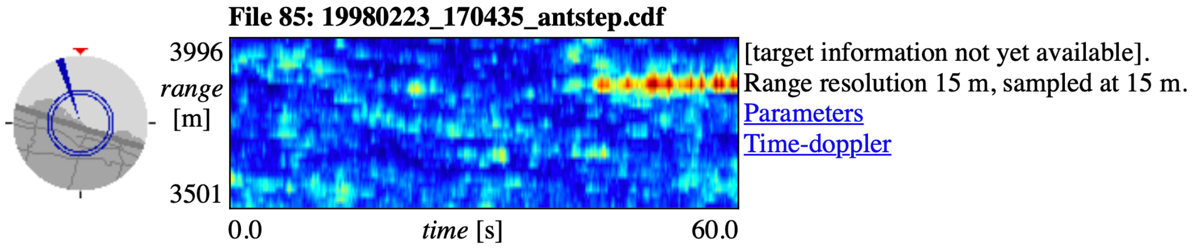
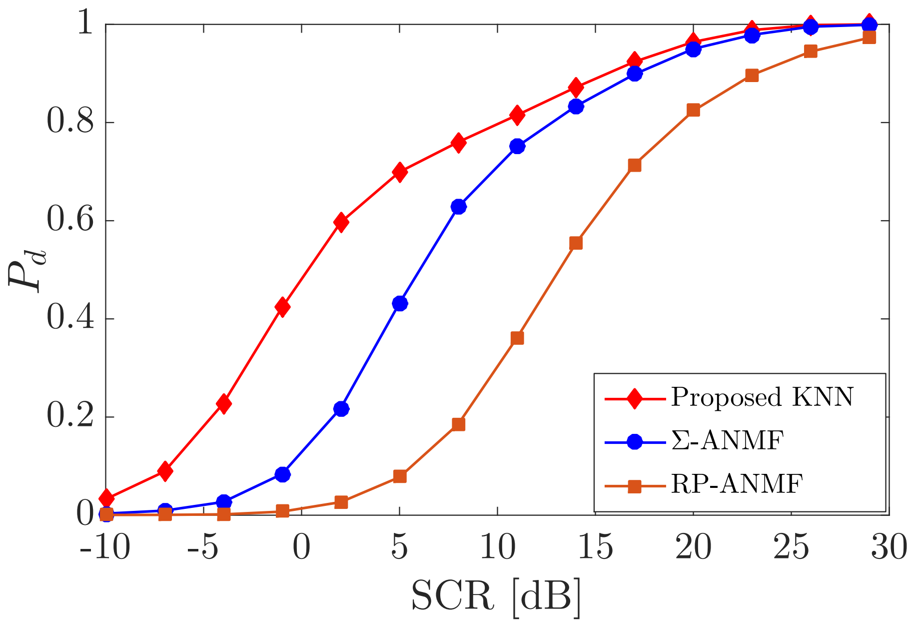


TABLE I
Parameters and CV Distance (From Amplitude ECDF) of K-CDF with Same First- and Second-Order Moments of ECDF

	K-CDF					
	HH			VV		
Resolution	μ	ν	d_{CV}	μ	ν	d_{CV}
30 m	47.572	0.420	12.317	54.457	0.640	9.914
15 m	139.950	0.370	18.767	50.482	0.710	11.190
3 m	17.412	0.820	18.057	3.789	2.030	5.663

from: E. Conte, A. De Maio and C. Galdi, "Statistical analysis of real clutter at different range resolutions," in IEEE Transactions on Aerospace and Electronic Systems, vol. 40, no. 3, pp. 903-918, July 2004



actual P_{fa} different from nominal one for all algorithms

A. Coluccia, A. Fascista, and G. Ricci, ``*A KNN-based Radar Detector for Coherent Targets in non-Gaussian Noise*'', to be submitted to IEEE SPL

Conclusions

- classical radar detection revisited under a machine learning perspective
- In typical radar scenarios, data are scarce and highly heterogeneous → combine model-based & data driven
- First KNN approach with raw data provides a significant gain, also on real data, but looses CFAR property (in practice, quite robust)
- Second KNN approach guarantees the CFAR property and allows the design of novel detectors with hybrid selective/robust behaviors
- ongoing work: a general framework for analysis and design of CFAR detectors in feature spaces

A. Coluccia, A. Fascista and G. Ricci, "*CFAR Feature Plane: A Novel Framework for the Analysis and Design of Radar Detectors*," in *IEEE Transactions on Signal Processing*, vol. 68, pp. 3903-3916, 2020.