On the design of incentive mechanisms in wireless networks: a game theoretic approach

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• New design challenges
• Applications
  - Channel access
  - Flow control
• Conclusions
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Mobile communications trend

• Mobile communications grow exponentially
• Future wireless networks must manage dynamically and efficiently a large set of devices
• Networks are migrating towards more distributed approaches, shifting intelligence from the core network towards the edges of the network

<table>
<thead>
<tr>
<th>Year</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>140%</td>
</tr>
<tr>
<td>2010</td>
<td>159%</td>
</tr>
<tr>
<td>2011</td>
<td>133%</td>
</tr>
<tr>
<td>2012 (estimate)</td>
<td>110%</td>
</tr>
<tr>
<td>2013 (estimate)</td>
<td>90%</td>
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<tr>
<td>2014 (estimate)</td>
<td>78%</td>
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</table>
A new design methodology

Terminals are more autonomous, more powerful, and more programmable

**Issue**: what if they are programmed to accomplish a personal objective?

→ a new design approach:

**Distributed schemes for strategic users**: the designer must provide the incentive for the users to take efficient decisions
Game theory is the branch of mathematics studying interactions between decision-makers.

Common assumption: users are selfish and strategic, they act to maximize their own utility.

Nash Equilibrium (NE)
- Existence?
- Computation?
- Uniqueness?
- Efficiency?
• New design challenges

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Slotted-Aloha MAC protocol

- Time is slotted and slots are synchronized
- The users contend for the channel
- A packet is received if it does not collide
- User i selects the transmission probability $p_i$
- User i’s throughput: $T_i(p) = p_i \prod_{j \neq i} (1 - p_j)$

Users adopt the always transmit strategy → network collapse
Users pay for their resource usage

Assumptions:
- i’s utility: $U_i(p) = \theta_i \ln T_i(p) - c_i p_i$
- Design objective: max sum-utility

Design problem: compute the optimal unit price $c_i$

Results:
- Given $c_i$, the unique NE is $p_i^{NE} = \frac{\theta_i}{c_i}$
- Optimal pricing policy is $c_i = \sum_k \theta_k$
Intervention scheme

An intervention device is placed in the system, it can affect users’ resource usage.

Intervention rule: a function of the users’ actions → users’ utilities can be shaped.

Design problem: compute the optimal rule.

Results:
- For the affine intervention rule class, the NE and the optimal rule are analytically computed.
Sum utility

\[ \theta_i = 1 \]

- **Cooperation**
- **Pricing**
- **Intervention**

![Graph showing the relationship between sum utility and the number of users.](image)
Imperfect monitoring case

The proposed schemes charge / intervene based on the actions adopted by the users

**Problem:** what if the users’ actions are not perfectly observable?

Imperfect monitoring model: \( \hat{p}_i = \left[ p_i + n_i \right]^1 \)

where: \( n_i \sim \mathcal{U} \left[ -\epsilon_i, \epsilon_i \right] \)

**Results:**
- The NE and the best policies (pricing & intervention) are analytically computed
Sum utility, imperfect monitoring

Imperfect monitoring, $\theta_i = 1$; $\varepsilon_i = 0.1$

- **Cooperation**
- **Pricing**
- **Intervention**

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**Graph:**
- **Y-axis:** Sum utility
- **X-axis:** Number of users
- Data points show a decrease in sum utility as the number of users increases, with different slopes for each strategy.
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Flow control

- n users
- \( d_i \) rate user i
- service rate \( \mu \)
- M/M/1 queue
- arrival rate \( \lambda = \sum_{i=1}^{n} d_i \)

Utility user i:

\[
U_i(d, t_i) = \frac{\text{throughput}^{t_i}}{\text{average delay}} = d_i^{t_i} (\mu - \lambda)
\]

Utility designer:

\[
U_0(d, t) = \sqrt[n]{\prod_{i=1}^{n} U^+_i(d, t_i)}
\]

Optimal policy:

\[
d_i^*(t) = \frac{t_i \mu}{n + \sum_{k=1}^{n} t_k}
\]
The intervention device sends an additional flow of packets with rate given by the intervention rule $f(d)$

**Design problem:** compute the optimal rule $f(d)$

**Results:**
- For the affine intervention rule class, the NE and the optimal rule are analytically computed
Complete information results

Complete information, $\mu = 5 \text{ Mbps}, \quad T_i = [0.1, 1]$
In the initialization phase the device asks the users to report their types…will they be honest?

Yes, if the scheme is *incentive compatible (IC)***!

**Results:**

- We characterized the maximum efficiency IC scheme
- We derived sufficient condition for its existence
- We proposed two suboptimal IC schemes
  - Convergent algorithm
  - Communication free mechanism
Incomplete information results

Incomplete information, $\mu = 5 \text{ Mbps}, T = [0.1, 1]$

- Red: Cooperative users
- Green dashed: Intervention suboptimal algo
- Blue dashed: Intervention com free mechanism
- Dotted: Bayesian Nash Equilibrium

Utility designer vs Number of users
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Conclusions

• Networks require more distributed approaches, in which terminals are more autonomous and smart

• New design challenges: provide the incentive for the users to comply

• Applications to relay network, channel access, flow control

• Sometimes we can reach optimal performance (e.g., channel access perfect monitoring, flow control complete information), sometimes we can not

• But an accurate design is always able to prevent higher inefficiencies

2. L. Canzian, A. Zanella, and M. Zorzi, "Overlapped NACKs: Improving Multicast Performance in Multi-access Wireless Networks”, in *Proc. IEEE PerGroup 2010*


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Mathematical details: intervention perfect monitoring

The intervention device jams i’s packets with probability given by the intervention rule:

\[ f_i^I(p_i) = [r_i(p_i - \tilde{p}_i)]^1_0 \]

**Design problem:** compute the optimal rule \( r_i, \tilde{p}_i \)

If \( r_i \geq \frac{1}{\tilde{p}_i} \), the best NE is: \( p_i = \tilde{p}_i \)

Optimal rule: \( r_i \geq \frac{1}{\tilde{p}_i} \), \( \tilde{p}_i = \frac{\theta_i}{\sum_k \theta_k} \)
Threshold vs. - imperfect monitoring scenario

Imperfect monitoring, everybody is aware of the errors, $\theta_i = 1$
The action of the users and the device – imperfect monitoring
Flow control

Optimal action profile vs. NE action profile
complete info scenario

Optimal action profile
\[ d_i^*(t) = \frac{t_i \mu}{n + \sum_{k=1}^{n} t_k} \]

NE action profile
\[ d_i^{NE^0}(t) = \frac{t_i \mu}{1 + \sum_{k=1}^{n} t_k} \]
The effect of the affine intervention rule
complete info scenario

\[
f(d) = \left[ \sum_{i=1}^{n} c_i (d_i - \tilde{d}_i) \right] _0^{d_0^M}
\]
Given the mechanism 
\[ (R, M, m^S, \pi) \]

User interaction modeled through the game
\[ \Gamma = (\mathcal{N}, \Phi, \Delta, T, P_t, \{\overline{U}_i(\cdot, \cdot, t)\}_{i=1}^n) \]

Report strategy
\[ \phi_i : T_i \rightarrow R_i \]

Action strategy
\[ \delta_i : M_i \times T_i \rightarrow D_i \]
Flow control

Maximum efficiency mechanism

Lemma \((T, M, d^S, \pi)\) is a maximum efficiency incentive compatible direct mechanism if

\[1: \text{the optimal action profile } d^*(t) \text{ of the game } \Gamma_t \text{ is sustainable without intervention in } \Gamma_t\]

\[2: \text{the suggested action profile is the optimal action profile of game } \Gamma_t, \text{ i.e., } d^S(t) = d^*(t);\]

\[3: \text{the intervention rules selected with positive probability sustain without intervention } d^*(t)\]

\[4: \text{users have incentives to report their real types when they adopt the suggested action profile, i.e.,}\]

\[
\sum_{t-i \in T-i} P_t(t | \tau_i) U_i \left( d^*_0, d^S(t), t \right) \geq \sum_{t-i \in T-i} P_t(t | \tau_i) U_i \left( d^*_0, d^S_i(\hat{\tau}_i, t_{-i}), \hat{\delta}_i(d^S_i(t_{-i}, \hat{\tau}_i)), t \right)
\]

\[\forall i \in \{1, ..., n\}, \forall \tau_i \in T_i, \forall \hat{\tau}_i \in T_i,\]

\[1 \text{ is valid, } 2 \text{ and } 3 \text{ say how to select the mechanism, } 4 \text{ is valid if, } \forall \tau_k \in T_i \text{ and } \forall t_{-i} \in T_{-i}\]

\[
\left( \frac{n + \sum_{j \neq i} t_j + \tau_{k+1}}{n + \sum_{j \neq i} t_j + \tau_k} \right)^{\tau_{k+1}} \left( \frac{\tau_k}{\tau_{k+1}} \right) = 1
\]
Proposition

$$
\overline{d}^S = \arg\max_{d^S} \sum_{t \in T} P_t(t) U_0 \left( d_0^*, d^S(t), t \right)
$$

subject to:

$$
\sum_{t-i \in T-i} P_t(t \mid \tau_i) U_i \left( d_0^*, d^S(t-i, \tau_i), t \right) \geq \sum_{t-i \in T-i} P_t(t \mid \tau_i) U_i \left( d_0^*, d^S_i(t-i, \hat{\tau}_i), \hat{\delta}_i(d_i^S(t-i, \hat{\tau}_i)), t \right)
$$

\forall i \in \{1, \ldots, n\}, \forall \tau_i \in T_i, \forall \hat{\tau}_i \in T_i, \forall \hat{\delta}_i : D_i \to D_i

and \quad \forall t \in T,

$$
\pi(f \mid t) = \begin{cases} 
1 & \text{for a certain } f \in \mathcal{F}^{\overline{d}^S, t} \\
0 & \text{otherwise}
\end{cases}
$$

describe an optimal mechanism, and the affine intervention rules is optimal with respect to \Gamma
Algorithm 2 Flow control suboptimal algorithm.

1. **Initialization**: $\forall t \in T$, $d^S(t) = d^*(t)$, $\pi(\tilde{f} \mid t) = 1$ for a certain $\tilde{f} \in \mathcal{F}^{d^S,t}$ and $\pi(f \mid t) = 0$ for $f \neq \tilde{f}$.
2. **For** $s = 1 : m$
3.  **For** $l = 1 : m$
4.   **If** $W_i(\tau_s, \tau_s) < W_i(\tau_s, \tau_l)$
5.    $d^S_i(\tau_l, t_{-i}) \leftarrow \min \left\{ d^S_i(\tau_l, t_{-i}) + \epsilon_i, d^{NE^0}_i(\tau_l, t_{-i}) \right\}$, $\pi(\tilde{f} \mid t) \leftarrow 1$ for a certain $\tilde{f} \in \mathcal{F}^{d^S,t}$ and $\pi(f \mid t) = 0$ for $f \neq \tilde{f}$, $\forall t_{-i} \in T_{-i}$
6. **Repeat** from 2 until 3 is unsatisfied $\forall s$ and $l$
A priori mechanism: independent on users reports

Proposed a priori mechanism:
Suggested action profile $\bar{d}$ and intervention rule

$$f(d) = \left[ \sum_{i=1}^{n} c_i (d_i - \bar{d}_i) \right]_{d_0}^{d_0^M}, \quad c_i > \frac{\tau_m (\mu - \sum_{k=1}^{n} \bar{d}_k) - \bar{d}_i}{\bar{d}_i}, \quad d_0^M \geq \mu$$

Where $\bar{d}$ is the solution of the convex problem:

$$\arg\min_{d} -\ln \left( \mu - \sum_{i=1}^{n} d_i \right) \mathbb{E}_t \left[ \prod_{i=1}^{n} d_i^{t_i} \right]$$

$$d_i \geq 0, \quad d_i \leq \mu, \quad \forall i \in \mathcal{N}$$
Manager’s expected utility vs. low type probability incomplete information scenario

\[ n = 4, \mu = 5, T_i = [0, 1] \]