Joint Channel Decoding in Non-cooperative Block-faded Orthogonal Access Schemes

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***Abstract*— In this paper, we study the performance of non-cooperative wireless multiple access systems with noisy separated channels, where *correlated* sources communicate to an access point (AP) in presence of block-faded links. Our goal is to explore the potential benefits which can be obtained when source correlation is exploited at the AP, comparing the performance with that obtained by using distributed source coding (DSC) at the nodes. We consider both the average bit error probability and the outage probability as performance indicators, and we derive a theoretical approach to evaluate their limits. Our results show that the improvement brought by the exploitation of the correlation at the AP is more evident when the correlation becomes sufficiently high. Moreover, some simulation results are presented for two classes of channels codes: serially concatenated convolutional codes (SCCCs) and low-density parity-check (LDPC) codes. Our results show that SCCCs can exploit better the correlation in scenarios with high values of the correlation coefficient (e.g., 0.999).**

***Index Terms*— Source correlation, block fading, joint source channel coding (JSCC), low-density parity check (LDPC) and turbo codes, distributed source coding (DSC), transmit diversity.**

I. Introduction

In this paper, we focus on distributed radio communication systems where two nodes need to transmit to a common remote destination. This model applies to many scenarios, such as, for example, cellular networks, wireless local area networks with one access point (AP), ad-hoc wireless networks, wireless sensor networks, etc. In these cases, collaboration between the nodes might be considered, leading to the so-called *collaborative diversity* [1]. In a cooperative system, each user is assigned one or more partners. The partners overhear each other, process the received signals, and retransmit proper messages to the destination in order to provide extra information, with respect to the signal sent by a single source, to the AP. Even in the presence of noisy inter-partner channels, the virtual transmit-antenna array formed by cooperating nodes provides additional diversity and may improve the system performance in terms of error rate and throughput. In classical cooperation scenarios, therefore, the idea is that of making the nodes cooperate among themselves to implement a distributed channel coding scheme where different nodes retransmit, in some sense, the same information.

In many application scenarios, however, the information which resides in different nodes is *intrinsically correlated*. In other words, even without implementing any cooperation among the nodes, the same or, more generally, "similar" information is transmitted by the nodes. A significant application example where this situation typically appears is given by *wireless sensor networks* [2]. In particular, if the data transmitted by the nodes are generated on the basis of a common physical phenomenon observed by the sensors, it is likely that these data are almost the same, e.g., the correlation is around 0.999. The design of efficient transmission of correlated signals, observed at different nodes, to one or more collectors is one of the main design challenges in these networks. In the case of one collector node, this system model is often referred to as reach-back channel [3]–[5]. In its simplest form, this problem can be summarized as follows: two independent nodes have to transmit correlated sensed data to a collector node by using the minimum possible energy, i.e., by exploiting in some way implicit correlation among the data. In the case of orthogonal additive white Gaussian noise (AWGN) channels the separation between source and channel coding is known to be optimal [6], [7]. This means that the theoretical limit can be achieved by first compressing each source up to the Slepian-Wolf (SW) limit and then utilizing two independent capacity achieving channel codes (one per source) [8]. In this case, no cooperation among the source nodes is required.

If, however, the transmissions are not carried out through separate AWGN channels, then the SW approach is no longer optimal and alternative schemes can bring significant advantages. In [9], a Gaussian multiple access (GMAC) scheme is considered. First, the nodes exchange information through time division-based transmission acts and taking into account the correlation, i.e., they transmit much less, with respect to the entire information, by relying on a distributed source coding (DSC)-based approach. Then, once each source node has the entire information (relative to both source nodes), it compresses and retransmits it to the destination node (the AP), thus achieving a sort of "beamforming gain" with respect to a scenario with no cooperation. This approach requires, basically, that the information is first compressed, thus exploiting the correlation between the sources, and then duplicated in order to obtain a coding and diversity, or beamforming, gain.

An alternative solution to exploit the correlation in this scenario is based on joint source channel coding (JSCC) schemes, where no cooperation among nodes is required and the correlated sources are not source encoded but only channel encoded. The absence of direct cooperation between the source nodes is attractive in scenarios where the communication links between the source nodes may be noisy. If one compares a JSCC system with a system based on source/channel coding separation with the same information rate, the channel codes used in a JSCC scheme must be less powerful (i.e., they have higher rates). This weakness can be compensated by exploiting the correlation between the sources at the decoder, which jointly recovers the information signals by both source nodes, so that the final performance can approach the theoretical limits. For this reason, this approach is also referred to as joint channel decoding (JCD). This approach has attracted the attention of several researchers in the recent past, also because of its implementation simplicity [10]–[14]. Note that, in the JSCC approach, the sources are encoded independently of each other (i.e., for a given source neither the realization from the other source nor the correlation model are available at the encoder) and transmitted through the channel. Correlation between the sources must be instead assumed to be known at the (common) receiver.

In this paper, we consider a non-cooperative JSCC/JCD scenario where transmissions are carried out through separate block-faded channels, i.e., when the delay requirement is short compared to the coherence time of the channel. Perfect channel state information (CSI) is assumed at the receiver while no CSI is available at the transmitters. In this setting, we first consider communications with no channel coding and we show that, in the presence of block fading, letting the nodes to transmit the whole information streams without any source correlation allows to get better performance then classical schemes based on DSC. This result is reminiscent of the transmit diversity case, i.e., when correlation is one if the same information is transmitted by different antennas we get better performance with respect to a single antenna system. Afterwards, we consider the same non-cooperative JSCC scenario in presence of ideal JCD and we show that, in the presence of block fading, non-cooperative JSCC/JCD schemes may potentially outperform schemes based on the separation principle.

II. Scenario and Achievable Rates

We consider two nodes which transmit two binary information signals $\boldsymbol{x} = [x_0, \ldots, x_{k-1}]$ and $\boldsymbol{y} = [y_0, \ldots, y_{k-1}]$, where k is the signal length (equal for both sources). The information signals are assumed to be temporally white with $P(x_i = 0) = P(x_i = 1) = 0.5$, $P(y_i = 0) = P(y_i = 1) = 0.5$, for $i = 0, \ldots, k-1$, and we define $\rho \triangleq P(x_i = y_i) > 0.5$—the correlation coefficient between the sequences $\boldsymbol{x}$ and $\boldsymbol{y}$ is $2\rho - 1$. The signals $\boldsymbol{x}$ and $\boldsymbol{y}$ are delivered to one collector node (the AP). We denote by $\boldsymbol{s}_x = [s_{x,0}, \ldots, s_{x,n_x-1}]$ and $\boldsymbol{s}_y = [s_{y,0}, \ldots, s_{y,n_y-1}]$ the samples transmitted by the nodes, n_x and n_y being the number of channel uses. In Fig. 1, we depict the proposed scenario, where SN stands for "Source Node". The AWGN sequences $\boldsymbol{w}_x$ and $\boldsymbol{w}_y$ are assumed to be independent, owing to the fact that the sources are assumed to transmit over orthogonal channels (e.g., using time division multiple access). While in this section we consider achievable rates for a scenario with AWGN channels, we then extend our approach to scenarios with Rayleigh block faded channels.

Let us introduce the following quantities: $r_x = k/n_x$, $r_y = k/n_y$ are the transmission rates at the two source nodes; $E_x = \mathbb{E}(|s_{x,i}|^2)$ $(i = 0, \ldots, n_x - 1)$ and $E_y = \mathbb{E}(|s_{y,j}|^2)$ $(j = 0, \ldots, n_y - 1)$ are the energies per transmitted symbol by the

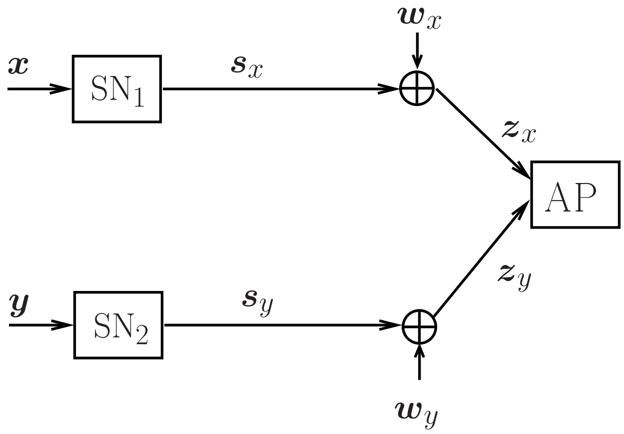

Fig. 1. Proposed multi-access communication scenario: two source nodes communicate directly to the AP.

sources; and $\frac{N_0}{2} = \mathbb{E}(|w_{x,i}|^2) = \mathbb{E}(|w_{y,j}|^2)$ is the variance of the AWGN samples (equal in both links). The SNRs at the AP in the two links are denoted as $\gamma_x = E_x/N_0$ and $\gamma_y = E_y/N_0$. Finally, the conditional entropy of the correlated sources is $H = -\rho \log_2(\rho) - (1-\rho)\log_2(1-\rho)$.

In this scenarios, it is well known that DSC allows to reduce the amount of information to be sent by the couple of nodes by exploiting the correlation between the sources. In particular, denoting by $r_{\mathrm{s},x}$ and $r_{\mathrm{s},y}$ the compressing rates, the following Slepian-Wolf bounds can be obtained:

$$\begin{aligned} r_{\mathrm{s},x} &> H \\ r_{\mathrm{s},y} &> H \\ r_{\mathrm{s},x} + r_{\mathrm{s},y} &> 1 + H. \end{aligned} \tag{1}$$

By assuming that source coding (compression) is followed by channel coding within each SN, the actual channel code rates $r_{\mathrm{c},x}$ and $r_{\mathrm{c},y}$, expressed as

$$\begin{aligned} r_{\mathrm{c},x} &= r_{\mathrm{s},x} r_x \\ r_{\mathrm{c},y} &= r_{\mathrm{s},y} r_y \end{aligned} \tag{2}$$

satisfy the following Shannon bounds:

$$\begin{aligned} r_{\mathrm{c},x} &< \tfrac{1}{2}\log_2(1+\gamma_x) \\ r_{\mathrm{c},y} &< \tfrac{1}{2}\log_2(1+\gamma_y). \end{aligned} \tag{3}$$

As discussed in Section I, compressing each source up to the SW limit and then utilizing two independent capacity achieving channel codes allows to achieve the ultimate performance limit. By introducing $\lambda_x = \frac{1}{2}\log_2(1+\gamma_x)$ and $\lambda_y = \frac{1}{2}\log_2(1+\gamma_y)$, the achievable rates r_x and r_y have to satisfy the following inequalities:

$$\begin{aligned} r_x &< \tfrac{\lambda_x}{H} \\ r_y &< \tfrac{\lambda_y}{H} \\ r_x r_y (1+H) - \lambda_x r_y - \lambda_y r_x &< 0. \end{aligned} \tag{4}$$

III. Schemes without Channel Coding

We now derive a simple theoretical upper bound on the bit error rate (BER) in scenarios with *no channel coding*. To elaborate, let us assume binary transmissions, i.e. $s_x = (2x -$

$1)\sqrt{E_x} = \pm\sqrt{E_x}$, $s_y = (2y-1)\sqrt{E_x} = \pm\sqrt{E_y}$ and denote by u_x and u_y the received samples, i.e:

$$\begin{array}{c} u_x = s_x + w_x \\ u_y = s_y + w_y \end{array} \quad (5)$$

Denoting by $\boldsymbol{v} = \{x,y\}$ the couple of bits transmitted by the two nodes, the maximum a posteriori (MAP) estimation $\tilde{\boldsymbol{v}} = \{\tilde{x},\tilde{y}\}$ may be easily derived as:

$$\tilde{\boldsymbol{v}} = \underset{\boldsymbol{v}}{\operatorname{argmax}} Pr\{\boldsymbol{v}|\boldsymbol{u}\} \quad (6)$$

where $\boldsymbol{u} = \{u_x, u_y\}$.

By using the Bayes rule and by putting away the constant terms (i.e., the terms which do not depend on $\boldsymbol{v}$), one obtains

$$\begin{aligned} \tilde{\boldsymbol{v}} &= \underset{\boldsymbol{v}}{\operatorname{argmax}} Pr\{\boldsymbol{u}|\boldsymbol{v}\} Pr(\boldsymbol{v}) \\ &= \underset{\boldsymbol{v}}{\operatorname{argmax}} \left[\ln Pr\{u_x|v_x\} + \ln Pr\{u_y|v_y\} + \ln Pr(\boldsymbol{v})\right] \quad (7) \end{aligned}$$

where the last equality follows from the fact that the logarithm is a monotonically increasing function and the two channels are orthogonal. By using the model in (5), it is now straightforward to get from (7) the decoding rule:

$$\tilde{\boldsymbol{v}} = \underset{\boldsymbol{v}}{\operatorname{argmax}} [u_x s_x + u_y s_y + N_0 \times \ln Pr\{\boldsymbol{v}\}].$$

where $Pr\{\boldsymbol{v}\}$ is the a-priori probability of $\boldsymbol{v}$, i.e., $Pr\{\boldsymbol{v}\} = \frac{\rho}{2}$ if $x = y$, and $Pr\{\boldsymbol{v}\} = \frac{1-\rho}{2}$ if $x \neq y$. Given the above, it is straightforward to derive the error probability $P_e(\boldsymbol{v},\tilde{\boldsymbol{v}})$ that the detected sequence $\tilde{\boldsymbol{v}}$ is different from $\boldsymbol{v}$ as:

$$P_e(\boldsymbol{v},\tilde{\boldsymbol{v}}) = \frac{1}{2}\operatorname{erfc}\left\{\sqrt{\gamma_{eq}(\boldsymbol{v},\tilde{\boldsymbol{v}})} + \frac{1}{4\sqrt{\gamma_{eq}(\boldsymbol{v},\tilde{\boldsymbol{v}})}} \ln\left[\frac{Pr(\boldsymbol{v})}{Pr(\tilde{\boldsymbol{v}})}\right]\right\} \quad (8)$$

where $\gamma_{eq}(\boldsymbol{v},\tilde{\boldsymbol{v}}) = (x \oplus \tilde{x})\gamma_x + (y \oplus \tilde{y})\gamma_y$, $\oplus$ being the XOR operator, and $Pr(\tilde{\boldsymbol{v}})$ is the a-priori probability of $\tilde{\boldsymbol{v}}$, i.e., $Pr\{\tilde{\boldsymbol{v}}\} = \frac{\rho}{2}$ if $\tilde{x} = \tilde{y}$, and $Pr\{\tilde{\boldsymbol{v}}\} = \frac{1-\rho}{2}$ if $\tilde{x} \neq \tilde{y}$.

Note that the same model applies also with a more efficient quaternary modulation scheme (QPSK), where two coded symbols are transmitted at the same time in the real and imaginary part of the complex transmitted sample.

An upper bound (union bound) on the bit error probability P_b can be obtained by summing the pairwise error probabilities properly weighted by the respective a-priori probabilities, i.e.:

$$\begin{aligned} P_b \leq\ & \operatorname{erfc}\left\{\sqrt{\gamma_x + \gamma_y}\right\} \\ & + \frac{\rho}{4}\left\{\operatorname{erfc}\left[\sqrt{\gamma_x} + \frac{1}{4\sqrt{\gamma_x}}\ln\left(\frac{\rho}{1-\rho}\right)\right]\right. \\ & \left. + \operatorname{erfc}\left[\sqrt{\gamma_y} + \frac{1}{4\sqrt{\gamma_y}}\ln\left(\frac{\rho}{1-\rho}\right)\right]\right\} \\ & + \frac{1-\rho}{4}\left\{\operatorname{erfc}\left[\sqrt{\gamma_x} - \frac{1}{4\sqrt{\gamma_x}}\ln\left(\frac{\rho}{1-\rho}\right)\right]\right. \\ & \left. + \operatorname{erfc}\left[\sqrt{\gamma_y} - \frac{1}{4\sqrt{\gamma_y}}\ln\left(\frac{\rho}{1-\rho}\right)\right]\right\}. \quad (9) \end{aligned}$$

Define the log-likelihood ratio $L_p \triangleq \ln(\frac{\rho}{1-\rho})$ and consider the following inequalities:

$$1 \pm \frac{1}{2\gamma_x} L_p \leq \left(1 \pm \frac{L_p}{4\gamma_x}\right)^2. \quad (10)$$

$$1 \pm \frac{1}{2\gamma_y} L_p \leq \left(1 \pm \frac{L_p}{4\gamma_y}\right)^2. \quad (11)$$

It can be shown that this inequalities are tight for high SNRs values γ_x and γ_y, in particular for $\gamma_x >> L_p/4$ and $\gamma_y >> L_p/4$. We now show that this is verified in normal operative conditions. Consider without loss of generality γ_x. For high ρ values, e.g. for $\rho = 0.99$, we get $L_p/4 \cong 1$. Hence, the inequality (10) is tight for $\gamma_x >> 0$ dB, and this is verified, for sure, for reasonably low values of the BER. For lower values of ρ, it follows that $L_p/4 < 1$ and, therefore, the condition $\gamma_x >> L_p/4$ is verified even more accurately. It can then be concluded that the inequalities (10) and (11) are very tight in all situations of practical interest. Using (10) and (11) into (9), one obtains:

$$\begin{aligned} P_b \leq\ & \operatorname{erfc}\left\{\sqrt{\gamma_x + \gamma_y}\right\} + \frac{\rho}{4}\left\{\operatorname{erfc}\left[\sqrt{\gamma_x\left(1 + \frac{1}{2\gamma_x}L_p\right)}\right]\right. \\ & \left. + \operatorname{erfc}\left[\sqrt{\gamma_y\left(1 + \frac{1}{2\gamma_y}L_p\right)}\right]\right\} \\ & + \frac{1-\rho}{4}\left\{\operatorname{erfc}\left[\sqrt{\gamma_x\left(1 - \frac{1}{2\gamma_x}L_p\right)}\right]\right. \\ & \left. + \operatorname{erfc}\left[\sqrt{\gamma_y\left(1 - \frac{1}{2\gamma_y}L_p\right)}\right]\right\}. \quad (12) \end{aligned}$$

Moreover, by considering the Chernoff-Rubin bound for the error function, i.e., $\operatorname{erfc}(x) \leq 2e^{-x^2}$, from (12) and taking into account the definition of L_p, one can write

$$\begin{aligned} P_b \leq\ & 2e^{-(\gamma_x+\gamma_y)} + \frac{1}{2}e^{-\gamma_x}e^{\frac{L_p}{2}}\left(1-\rho+e^{-L_p}\rho\right) \\ & + \frac{1}{2}e^{-\gamma_y}e^{\frac{L_p}{2}}\left(1-\rho+e^{-L_p}\rho\right) \\ =\ & 2e^{-(\gamma_x+\gamma_y)} + \left(e^{-\gamma_x} + e^{-\gamma_y}\right) \times \sqrt{\rho(1-\rho)}. \quad (13) \end{aligned}$$

Consider now the presence of independent Rayleigh faded links. Assuming that the two links are characterized by the same average SNR, $\Gamma \triangleq \mathbb{E}[\gamma_x] = \mathbb{E}[\gamma_y]$, the SNRs have the following exponential distribution [15]:

$$f_{\gamma_x}(t) = f_{\gamma_y}(t) = \frac{1}{\Gamma}\exp\left(-\frac{t}{\Gamma}\right)U(t) \quad (14)$$

where $U(t)$ is the unit step function. Denoting by $\gamma_{b,s} = \gamma_x + \gamma_y$ the sum of the SNRs, owing to the independence between the communication links it is straightforward to obtain the pdf of $\gamma_{b,s}$ as $f_{\gamma_{b,s}}(t) = (t/\Gamma^2)\exp(-t/\Gamma)U(t)$. An upper bound on the BER can be derived by averaging over the distribution of the fading terms appearing in (13), obtaining:

$$P_b \leq \frac{2}{(\Gamma+1)^2} + \left(\frac{2}{\Gamma+1}\right) \times \sqrt{\rho(1-\rho)}. \quad (15)$$

The upper bound on the BER given above allows to separate the effects of the SNR and the correlation between the two sources. As an example, if $\rho = 1$, i.e., the two sources are identical, the system is equivalent to a classical transmission diversity system where the slope of the BER curve is two. On the other hand, for $\rho = 0.5$, if we neglect the term which depends on the squared SNR, the slope of the BER curve is one, as in systems with no diversity.

In order to evaluate the gain which can be obtained by exploiting the source correlation at the receiver, consider an ideal scheme where the information is compressed to its minimum data rate. When no CSI is available at the transmitter, the natural choice is to have the same compression rates at the transmitters which yields, from (1) $r_{s,x} = r_{s,y} = (1+H)/2$. In this ideal DSC scheme, correlation is completely lost on account of data compression and the performance is equivalent to that of a single link affected by Rayleigh fading with an average SNR $\Gamma_{\mathrm{id}} = 2\Gamma/H$. The BER for the scenario with DSC can then be written as:

$$P_{\mathrm{b}} = \frac{1}{2}\int_0^{\infty} \mathrm{erfc}\left(\sqrt{t}\right)\frac{1}{\Gamma_{\mathrm{id}}}\exp\left(-\frac{t}{\Gamma_{\mathrm{id}}}\right)\mathrm{d}t. \qquad (16)$$

We remark that, in the DSC case, the BER in (16) refers to the "compressed" bits. Therefore, bit errors might have a catastrophic impact on the uncompressed data stream. In the absence of DSC, this problem does not exist. It is worth noting that, if $\rho = 1$ then $H(x,y) = 1$ and, consequently, $\Gamma_{\mathrm{id}} = 2\Gamma$. Note also that, for $\rho = 1$, if no compression of data is considered (i.e., the scenario of interest in this paper) the same data are transmitted into 2 independent channels, with an energy per channel equal to 1/2-th of that in the DSC case. This situation is the same of that of classical multi-antenna systems which achieve 2-degree diversity by means of repetition codes. Of course, for $\rho < 1$ we expect that a certain degree of diversity can be still obtained.

In Fig. 2, we compare the BER of DSC and JSCC schemes. In particular, both simulation and theoretical results are shown, fixing the correlation coefficient ρ to 0.95 in (a) and 0.999 in (b). As for the DSC case, the analysis in (16) is exact and hence we only show theoretical results. Conversely, for the JSCC case the theoretical BER is obtained by integrating the union bound (9) over the joint distribution of γ_{x} and γ_{y}. In this case, by comparing the theoretical results with simulation results, it can be observed that the derived upper bounds on the BER are very tight. Moreover, it can be noticed that the performance of the DSC schemes is always worse than that of JSCC schemes. Obviously, the higher the correlation level among the sources, the more pronounced this effect. Note that for high values of ρ, e.g., $\rho = 0.999$, the BER curves in the JSCC case are characterized by higher slopes than in the DSC cases, at least for low-to-medium SNRs. This effect appears also for lower values of ρ, e.g., $\rho = 0.95$, even if it can be hardly perceived form the BER curves in Fig. 2 (a). The rationale behind this behavior is as follows. As one can see from (15), the upper bound on the BER is given by two distinct terms: the first term is inversely proportional to the square SNR and, therefore, the slope of the BER curve is two, as in systems with diversity; the second term is instead characterized by a unitary slope, as in systems with no diversity. However, for high values of ρ the second term is very small and becomes significant only for high SNRs. This justifies the facts that the slopes of the BER curves are not constant and they approach one for increasing SNRs. However, as shown in Fig. 2, exploiting the correlation at the receiver provides a gain, with respect to DSC schemes, even in the presence of a unitary slope, e.g., for high SNRs. We refer to this gain as "diversity gain due to correlation," since it is due to the fact that the negative influence of a strongly faded link is partially recovered by the other (less faded) links.

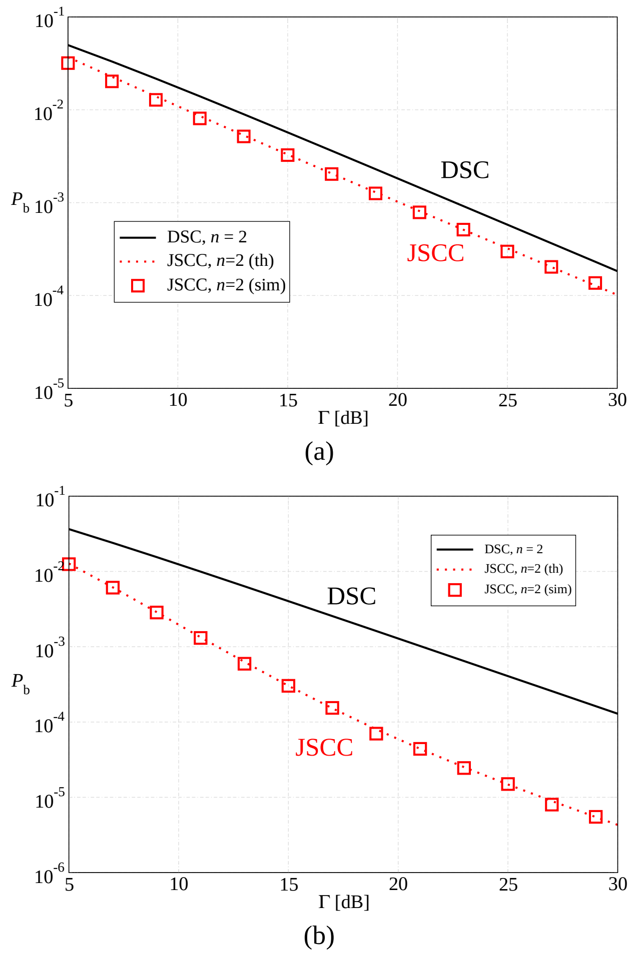

Fig. 2. BER performance, as a function of the SNR in the uncoded scenario. The *correlation coefficient* ρ is fixed to 0.95 in (a) and 0.999 in (b). The performance of JSCC schemes is compared with that of DSC schemes.

In order to evidence the diversity gain due to correlation, in Fig. 3 we show the SNR, required to achieve a BER equal to 5×10^{-3}, as a function of the correlation coefficient ρ. The performance of JSCC schemes is compared with that associated with DSC schemes. As one can see, in both scenarios the system performance is basically fixed for values of the correlation coefficient lower than 0.8. For both classes of schemes, the SNR gain becomes relevant for values of ρ higher than 0.8 and is an increasing function of the number of sources. However, JSCC schemes outperform DSC schemes: at very high correlation values (e.g., 0.97), the predicted SNR gain is higher than 5 dB.

IV. Schemes with Channel Coding

Consider the case where no distributed compression is performed at the transmitters and correlation is fully exploited at the receiver, i.e., $r_{s,x} = r_{s,y} = 1$ (channel rates are equal to transmission rates). The receiver exploits correlation by means of JCD, which allows to achieve the ultimate capacity

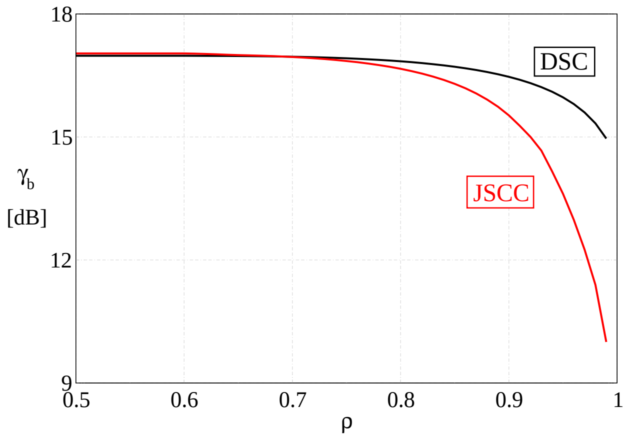

Fig. 3. SNR, as a function of the *correlation coefficient* ρ, required to achieve a BER (at the AP) equal to 5×10^{-3}. The performance of JSCC schemes is compared with that of DSC schemes.

(4) in presence of ideal CSI [14], [16]. However, in the considered scenario no CSI is present at the transmitters, i.e., link adaptation is not permitted. Accordingly, the natural choice is to have common information rates at the two sources, i.e., $r = r_x = r_y$. From (4), setting $r = r_x = r_y$ the common rate constraints for the JCD scheme turn out to be:

$$\begin{array}{r} r < \frac{\lambda_x}{H} \\ r < \frac{\lambda_y}{H} \\ r < \frac{\lambda_x+\lambda_y}{1+H} \end{array} \Longleftrightarrow r < \min\left\{\frac{\lambda_x}{H}, \frac{\lambda_y}{H}, \frac{\lambda_x+\lambda_y}{1+H}\right\}. \quad (17)$$

Similarly to the uncoded case, in order to evidence the advantage of exploiting correlation at the receiver, the JCD scenario is compared with a DSC scheme which applies the separation principle, i.e., information data is first compressed up to the SW limit and then transmitted into the channel utilizing two independent capacity achieving channel codes. As previously discussed, such a scheme is known to achieve the ultimate capacity in presence of ideal CSI. When no CSI is available at the transmitter, the natural choice is to set the compression rates $r_{s,x} = r_{s,y} = (1+H)/2$ (as in the uncoded case) and to transmit by using common channel rates, i.e., $r_{c,x} = r_{c,x}$. In this setting, the transmitting rate $r = r_x = r_y$ must fulfill:

$$\begin{array}{r} r < \frac{2}{1+H}\lambda_x \\ r < \frac{2}{1+H}\lambda_y \end{array} \Longleftrightarrow r < \frac{2}{1+H}\min\{\lambda_x, \lambda_y\}. \quad (18)$$

Let us now introduce the feasible two-dimensional channel SNR regions as the ensemble of SNR pairs (γ_x, γ_y) which satisfy (4) for JCD schemes and (18) for DSC schemes. In the considered block-faded scenario without CSI an outage event is declared if *at least one* of the users cannot communicate at the target rate r (similar to the definition of outage in the single link scenario), i.e., if the SNR pair (γ_x, γ_y) does not belong to the feasibility region. To derive the outage probabilities $P_o^{(DSC)}$ and $P_o^{(JCD)}$ for the DSC and JCD cases, respectively, it is convenient to derive the distribution of $\lambda = 0.5\log_2(1+\gamma)$, γ being the SNR of each link which has exponential distribution given in (14). It is straightforward to get:

$$f_\lambda(t) = \frac{\ln(2)}{\Gamma} \times 2^{2t+1} \exp\left(-\frac{2^{2t}-1}{\Gamma}\right) U(t).$$

Accordingly, $P_o^{(DSC)}$ may be easily evaluated from (18) as:

$$\begin{aligned} P_o^{(DSC)} &= 1 - \int_{(1+H)r/2}^{\infty} f_\lambda(x)\,dx \int_{(1+H)r/2}^{\infty} f_\lambda(y)\,dy \\ &= 1 - \exp\left(-2\times\frac{2^{r(1+H)}-1}{\Gamma}\right) \end{aligned} \quad (19)$$

while for the JCD case one obtains:

$$\begin{aligned} P_o^{(JCD)} &= 1 - \int_{rH}^{r} f_\lambda(x)\,dx \int_{(1+H)r-x}^{\infty} f_\lambda(y)\,dy \\ &\quad - \int_{r}^{\infty} f_\lambda(x)\,dx \int_{rH}^{\infty} f_\lambda(y)\,dy \\ &= 1 - \int_{rH}^{r} \exp\left(\frac{-2^{2r(1+H)-2x}+1}{\Gamma}\right) f_\lambda(x)\,dx \\ &\quad - \exp\left(\frac{-2^{2r}+1}{\Gamma}\right) \exp\left(\frac{-2^{2rH}+1}{\Gamma}\right). \end{aligned} \quad (20)$$

In Fig. 4, we compare the outage probability of DSC and JCD schemes obtained by evaluating (19) and (20). The correlation coefficient ρ is fixed to 0.95 in (a) and 0.999 in (b). Simulation results for the JCD are also shown in Fig. 4. In the simulation setting, each node transmits packets of length $k = 1000$ using (i) a serial concatenated convolutional code (SCCC) characterized by a recursive systematic rate-$1/2$ constituent code and (ii) a low-density parity-check (LDPC) code characterized by a parity-check matrix with two diagonals (with rate $1/2$). Details on these channel codes can be found in [16]. In both cases, the correlation is exploited at the AP by using a proper iterative algorithm between the two component decoders [13], [14], [16]. As one can see, the performance of the realistic simulations are slightly away from the theoretical results. This is due to the fact that the considered codes are small (i.e., $k = 1000$), whereas the analytical results hold in an asymptotic regime, i.e., when $k \to \infty$. Moreover, one can observe that for large values of the correlation coefficient (e.g., $\rho = 0.999$) the “optimized” SCCC is able to better use the correlation among data, whereas the LDPC code performance degrades.

Finally, in Fig. 5 the theoretical SNR required to obtain an outage probability equal to 5×10^{-2} is shown as a function of the correlation coefficient ρ. The performance of JCD schemes is compared with that of DSC schemes. The same considerations carried out in Fig. 3 still hold also for the outage probability, i.e., the SNR gain becomes relevant for values of ρ higher than 0.8. Moreover, JCD schemes outperform DSC schemes: at very high correlation values (e.g., 0.97), the predicted SNR gain is higher than 7 dB.

Note that the decoding complexity of JCD is given by that of the channel decoder associated with the code used at each node (either turbo or LDPC) and that of the iterative

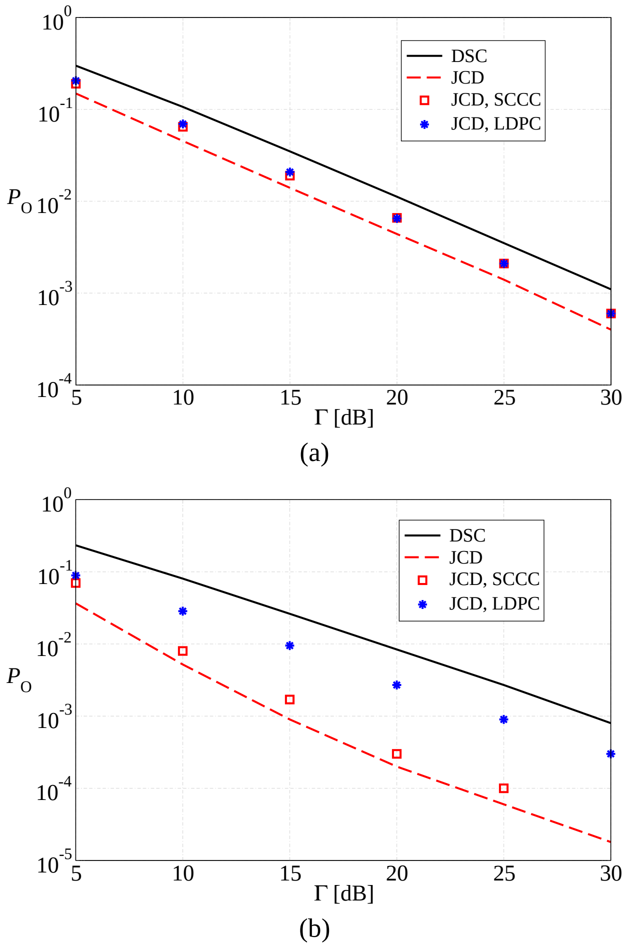

Fig. 4. Outage probability performance, as a function of the SNR, in the coded scenario. The *correlation coefficient* ρ is fixed to 0.95 in (a) and 0.999 in (b). The performance of JCD schemes is compared with that of DSC schemes. Simulation results for SCCC and LDPC codes are presented.

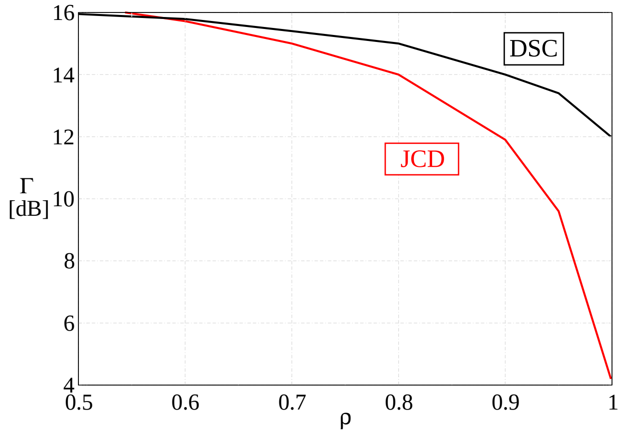

Fig. 5. SNR, as a function of the *correlation coefficient* ρ, required to achieve an outage probability (at the AP) equal to 5×10^{-2}. The performance of JCD is compared with that of DSC.

exchange of information (weighed by the correlation) between the component decoders. Since the correlation-based information weighing is negligible with respect to the complexity of each component decoding algorithm, the total complexity is approximately $2n_{\rm iter}\mathscr{C}_{\rm single-dec}$, where $n_{\rm iter}$ is the number of iterations between the two component decoders and $\mathscr{C}_{\rm single-dec}$ is the complexity of each decoding algorithm. The single decoder complexity $\mathscr{C}_{\rm single-dec}$ depends on the chosen coding/decoding scheme and can be evaluated, for example, in terms of multiplications/additions.

We remark that, even though the source-AP links are independent, the considered scenario is similar to the GMAC scenario presented in [9], where the separation is not optimal. In our case, the fact that the separation is not optimal comes from the fact that the links are unbalanced because of the presence of fading. This asymmetry, which cannot be compensated since no CSI is considered at the sources, tends to favor schemes with joint decoding at the AP. In other words, as in the GMAC scheme considered in [9] and in the uncoded communication scenario considered in Section III, it is more convenient to exploit the source correlation entirely at the AP, even if the links are separated.

V. Conclusions

In this paper, we have analyzed the performance of multiple access schemes where two *correlated* sources communicate to an AP, in the absence of cooperation, through noisy separated block-faded links. Using an information-theoretic approach, we have explored the potential benefits which can be obtained when source correlation is exploited at the AP by using JCD. The performance of JCD transmission scheme has been compared with that of DSC schemes. Our results (in terms of both average bit error probability and outage probability) show that the improvement brought by the exploitation of the correlation at the AP is more evident when the correlation coefficient becomes more relevant. Moreover, some simulation results have been presented for both SCCCs and LDPCs, showing that SCCCs better exploit the correlation when the correlation coefficient is large (e.g., 0.999). This remark the conclusions of [16], i.e., the fact that channel codes need to be properly designed to exploit the source correlation at the AP.

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