

Polarimetric SAR Tomography of forest scenarios

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Abstract—In this paper a technique is described for the joint exploitation of several multi-polarimetric SAR images, to the aim of yielding a separate tomographic reconstruction of each of the different Scattering Mechanisms (SM) (such as ground and volume scattering) that contribute to the received signal. Under large hypotheses it will be shown that the data covariance matrix can be expressed as a Sum of Kronecker Products (SKP), after which it follows that K scattering mechanisms are uniquely identified by $K(K-1)$ real numbers. This result provides the basis to perform SM separation by employing not only model based approaches, generally retained in literature, but also purely algebraic approaches, while yielding the best Least Square solution given the hypothesis of K SMs. Experimental results will be shown basing on airborne data relative to the forest site of Remningstorp, Sweden, acquired by DLR's E-SAR in the framework of the ESA campaign BioSAR2007. Furthermore, it will be shown that the SKP structure also provides a straightforward tool for discussing the well posing of the model inversion problem.

I. INTRODUCTION

In this paper it is considered the problem of the retrieval of the vertical structure of forested areas from multi-polarimetric and multi-baseline Synthetic Aperture Radar (SAR) data, depending on the hypotheses that are made about the illuminated scene. SAR Tomography (T-SAR), by virtue of its capability to resolve multiple targets within the same slant range, azimuth resolution cell, constitutes a unique tool for conducting SAR analyses of forested areas. In its most basic formulation, the aim of T-SAR is to retrieve the vertical distribution of the backscattered power within the system resolution cell. A possible solution to this problem is to exploit super-resolution techniques, such as Capon adaptive filtering, MUSIC, SVD analysis, and others, see [1], [2], [3]. A different solution may be found in the works by *Fornaro et al.*, [4], and *Cloude*, [5], where super-resolution is achieved by exploiting a priori information about target location, such as ground topography and canopy top height. In the analysis of forest scenarios, however, retrieving the vertical distribution of the backscattered power may not suffice for the aim of the identification of the different scattering mechanisms that contribute to the received signal, mainly due to two reasons: i) even the employment of super-resolution techniques could not guarantee a sufficiently fine vertical resolution; ii) two different scattering mechanisms could happen to be simultaneously present at the same vertical location. A viable solution to overcome such limitations is to describe the scene through models, in such a way as to identify the scattering mechanisms via parametric estimation techniques [6]. An alternative approach is provided by Polarimetric SAR Interferometry (PolInSAR), due to *Cloude and*

Papathanassiou. In the original formulation of PolInSAR, a direct separation of the phase centers associated to different targets is carried out by decomposing the data into three orthogonal polarizations [7]. The applicability of such a direct approach, however, is subjected to the assumption about the presence of orthogonal deterministic scattering mechanisms, which is not verified in forested areas [8], [9]. Therefore, in [8] the authors formulated a more sophisticated model-based inversion algorithm for forest parameter estimation. A similar approach may be found in the work by *Treuhaft and Siqueira*, [10], where not only the spatial structure but also the polarimetric response of the targets is parameterized through physical models.

In this paper, the problem of SM separation and characterization from multi-polarimetric and multi-baseline (MPMB) SAR surveys will be faced from an algebraic point of view, under the assumption that the data covariance matrix can be represented through a Sum of Kronecker Products (SKP). As a result, a new general methodology will be defined that encompasses not only model based approaches, but also model free and hybrid approaches, while allowing to find the K SMs that best fit the data covariance matrix in the Least Square sense.

Furthermore, it will be shown that the exploitation of the SKP structure also allows a straightforward discussion of the conditions that are required in order for model based separation of two scattering mechanisms (SMs) to be a well posed problem, in both the cases of single-polarimetric multi-baseline (SPMB) data and multi-polarimetric multi-baseline (MPMB) data.

II. DATA MODEL

Consider a data-set of $3N$ fully polarimetric SAR images acquired along N distinct tracks, properly focused, calibrated, co-registered and phase flattened [11], and let $y_n(\mathbf{w}_i)$ denote a complex valued pixel¹ from the n -th track in the polarimetric channel identified by the polarimetric projection vector \mathbf{w}_i [7]. Dealing with natural scenarios, it is sensible to assume that the target signature changes randomly from one pixel to another, either in the case of distributed or point-like targets. For this reason, the data will be assumed to be a realization of a zero-mean complex process, to be characterized through its second order moments. To do this, the received signal will be assumed to be contributed by K distinct Scattering Mechanisms (SMs), representing ground, volume, ground-trunk scattering, or other

¹The dependency on the slant range, azimuth coordinates has been made implicit to simplify the notation.

[12]. In formula:

$$y_n(\mathbf{w}_i) = \sum_{k=1}^K s_k(n; \mathbf{w}_i) \quad (1)$$

where $s_k(n; \mathbf{w}_i)$ represents the contribution of the k -th SM to the n -th track in the polarimetric channel \mathbf{w}_i . Three fundamental hypotheses will be retained:

H1): statistical uncorrelation among different SMs:

$$E[s_k(n; \mathbf{w}_i) s_h^*(m; \mathbf{w}_j)] = 0 \quad (2)$$

$\forall n, m, i, j$ if $k \neq h$.

H2): invariance of the interferometric coherences associated with each SM with respect to the choice of the polarimetric channel:

$$\frac{E[s_k(n; \mathbf{w}_i) s_k^*(m; \mathbf{w}_i)]}{\sqrt{E[|s_k(n; \mathbf{w}_i)|^2] E[|s_k(m; \mathbf{w}_i)|^2]}} = \gamma_k(n, m) \quad (3)$$

Note that the interferometric coherences $\gamma_k(n, m)$ in (3) are intended as being the result of both the spatial and temporal decorrelation phenomena that affect the k -th SM [13].

H3): invariance of the polarimetric signature associated with each SM with respect to the choice of the track, up to a scale factor that depends on the SM:

$$E[s_k(n; \mathbf{w}_i) s_k^*(n; \mathbf{w}_j)] = c_k(\mathbf{w}_i, \mathbf{w}_j) \sigma_k^2(n) \quad (4)$$

Under such hypotheses, the second order moments of the data can be expressed as:

$$E[y_n(\mathbf{w}_i) y_m^*(\mathbf{w}_j)] = \sum_{k=1}^K c_k(\mathbf{w}_i, \mathbf{w}_j) \sigma_k(n) \sigma_k(m) \gamma_k(n, m) \quad (5)$$

Without any loss of generality, it will be hereinafter assumed that the polarimetric signatures of each SM are re-scaled such that $\sigma_k^2(1) = 1 \forall k$. Considering only spatial decorrelation phenomena, the validity of hypothesis *H2* is subjected to the condition that the spatial structure of each SM can be described independently on the choice of the polarimetric channel. This is surely true as long as geometrical descriptors are used, such as terrain slope or volume top height, for example. However, this hypothesis is not strictly verified if electromagnetic descriptors are considered, such as subsurface penetration or volume extinction, since such parameters may undergo a variation from one polarimetric channel to another. Accordingly, retaining hypothesis *H2* is equivalent to considering only the average electromagnetic properties of each SM across the three polarimetric channels. An analogous conclusion can be drawn as for temporal decorrelation. Hypothesis *H3* is equivalent to assuming that the temporal and geometrical separation among different tracks result in at most a variation of the total backscattered power associated with each SM. The validity of this hypothesis is clearly subjected to the conditions that: a proper polarimetric calibration has been performed [14]; only a small variation of the look angle occurs from one

track to another; the average polarimetric signature of each SM is temporally homogeneous. The latter point entails that the cases where events like floods, fires, frosts, deforestation occur only in some of the acquisitions are to be considered out of the range of validity of the analysis within this paper, as well as the case where acquisitions from different seasons have to be processed jointly. To conclude, it is important to note that hypotheses *H1*, *H2*, *H3* are fully consistent with the Random Volume over Ground (RVoG) model often assumed in single baseline PolInSAR applications, see for example [8], [15], which constitutes experimental evidence that the range of validity of the analysis within this paper actually encompasses realistic scenarios. In particular, no hypotheses are required about which SM is dominant. In other words, the model in this section is valid in both the cases where ground returns dominate volume returns and vice-versa. Further details are found in [16].

III. SM SEPARATION FROM MPMB DATA: AN ALGEBRAIC PERSPECTIVE

It follows after (5) that the contribution of each scattering mechanism to the covariance matrix of the MPMB data can be expressed as the Kronecker product between two matrices, of which the first accounts for the polarimetric signature and the second accounts for the coherence loss due to the spatial structure and the temporal behavior of the corresponding scattering mechanism. In formula:

$$\mathbf{W}_K = E[\mathbf{y}\mathbf{y}^H] = \sum_{k=1}^K \mathbf{C}_k \otimes \mathbf{R}_k \quad (6)$$

where \mathbf{C}_k is a 3×3 matrix such that $\{\mathbf{C}_k\}_{ij} = c_k(\mathbf{w}_i, \mathbf{w}_j)$ and \mathbf{R}_k is a $N \times N$ matrix such that $\{\mathbf{R}_k\}_{nm} = \gamma_k(n, m)$. By virtue of their physical meaning, the matrices \mathbf{C}_k , \mathbf{R}_k will be hereinafter referred to as *polarimetric signatures* and *structure matrices*, respectively. Although still unexploited in SAR literature, the Sum of Kronecker Products (SKP) structure offers the possibility to discuss the problem of mechanism separation and characterization from a very general point of view, including not only model based approaches, commonly retained in the analysis of forested scenarios, but also model free, and hybrid approaches. The key to the exploitation of the SKP structure is the following noticeable result, due to *Van Loan and Pitsianis*:

Lemma: let $\mathbf{A} = \sum_{k=1}^K \mathbf{X}_k \otimes \mathbf{Y}_k$, where \mathbf{X}_k , \mathbf{Y}_k are two arbitrary matrices with dimension $N_x \times N_x$ and $N_y \times N_y$. Then the elements of \mathbf{A} may be rearranged into a $N_x^2 \times N_y^2$ matrix $\wp(\mathbf{A})$ such that:

$$\wp(\mathbf{A}) = \sum_{k=1}^K \mathbf{x}_k \mathbf{y}_k^T \quad (7)$$

where $\text{vec}(\cdot)$ is the matrix to vector operator, $\mathbf{x}_k = \text{vec}(\mathbf{X}_k)$ and $\mathbf{y}_k = \text{vec}(\mathbf{Y}_k)$.

The proof is found in [17]. It is important to note that a one to one correspondence exists between \mathbf{A} and $\wp(\mathbf{A})$ for any arbitrary matrix \mathbf{A} , so that it is always possible to define the

inverse rearrangement operator \wp^{-1} such that $\wp^{-1}(\wp(\mathbf{A})) = \mathbf{A}$.

The application of (7) to the MPMB covariance matrix in (6) yields:

$$\wp(\mathbf{W}_K) = \sum_{k=1}^K \mathbf{c}_k \mathbf{r}_k^T \quad (8)$$

where $\mathbf{c}_k = \text{vec}(\mathbf{C}_k)$, $\mathbf{r}_k = \text{vec}(\mathbf{R}_k)$, and $\wp(\mathbf{W}_K)$ is a $9 \times N^2$ matrix. Two important results follow:

R1): the rank of $\wp(\mathbf{W}_K)$ is equal to the number of SMs that contribute to the received signal.

R2): let $\wp(\mathbf{W}_K) = \sum_{k=1}^K \lambda_k \mathbf{u}_k \mathbf{v}_k^H$ be the Singular Value Decomposition (SVD) of $\wp(\mathbf{W}_K)$, the singular values λ_k being sorted in descending order, and define $\tilde{\mathbf{c}}_k = \lambda_k \mathbf{u}_k$, $\tilde{\mathbf{r}}_k = \text{conj}(\mathbf{v}_k)$. It then follows that:

$$\begin{aligned} \mathbf{W}_K &= \sum_{k=1}^K \tilde{\mathbf{C}}_k \otimes \tilde{\mathbf{R}}_k \\ &= \sum_{k=1}^K \mathbf{C}_k(\boldsymbol{\alpha}) \otimes \mathbf{R}_k(\boldsymbol{\alpha}) \end{aligned} \quad (9)$$

where $\tilde{\mathbf{C}}_k, \tilde{\mathbf{R}}_k$ are obtained by reshaping the vectors $\tilde{\mathbf{c}}_k$ and $\tilde{\mathbf{r}}_k$ into matrices of 3×3 and $N \times N$ elements, respectively. By construction, both $\tilde{\mathbf{C}}_k$ and $\tilde{\mathbf{R}}_k$ are Hermitian matrices that give rise to an orthogonal basis under the Frobenius inner product, namely $\text{trace}(\tilde{\mathbf{C}}_k \tilde{\mathbf{C}}_h) = \text{trace}(\tilde{\mathbf{R}}_k \tilde{\mathbf{R}}_h) = 0 \forall k \neq h$. The matrices $\mathbf{C}_k(\boldsymbol{\alpha}), \mathbf{R}_k(\boldsymbol{\alpha})$ are obtained from $\tilde{\mathbf{C}}_k, \tilde{\mathbf{R}}_k$ as:

$$\begin{aligned} \mathbf{C}_k(\boldsymbol{\alpha}) &= \sum_{n=1}^K \{\boldsymbol{\alpha}^{-T}\}_{nk} \tilde{\mathbf{C}}_n \\ \mathbf{R}_k(\boldsymbol{\alpha}) &= \sum_{n=1}^K \{\boldsymbol{\alpha}\}_{nk} \tilde{\mathbf{R}}_n \end{aligned} \quad (10)$$

where $\boldsymbol{\alpha}$ is an arbitrary non-singular $K \times K$ matrix and $\{\boldsymbol{\alpha}\}_{nk}$ denotes the nk -th element of $\boldsymbol{\alpha}$.

The importance of results R1, R2 lies in the demonstration that the covariance matrix of the MPMB data may be formed by infinite different combinations of structure matrices and polarimetric signatures, obtained by letting $\boldsymbol{\alpha}$ vary over the set of the non singular $K \times K$ matrices. Accordingly, the problem of identifying K SMs is equivalent to the problem of finding a non-singular $K \times K$ matrix $\hat{\boldsymbol{\alpha}}$ such that $\mathbf{C}_k(\hat{\boldsymbol{\alpha}}) = \mathbf{C}_k$ and $\mathbf{R}_k(\hat{\boldsymbol{\alpha}}) = \mathbf{R}_k$ for every k . The practical consequences of this result will be shown in Section IV, where a general methodology for SM separation will be proposed basing on the exploration of the range of values assumed by $\boldsymbol{\alpha}$. A conclusive remark is required in order to discuss the effective dimensionality of the matrix $\boldsymbol{\alpha}$. By definition the polarimetric signatures and the structure matrices of each scattering mechanism must be Hermitian, semi-positive definite matrices. It is easy to see that the condition that the matrices $\mathbf{C}_k(\boldsymbol{\alpha}), \mathbf{R}_k(\boldsymbol{\alpha})$ in (10) are Hermitian entails that $\boldsymbol{\alpha}$ is a real valued matrix. Furthermore, it is always possible to re-scale the matrices $\tilde{\mathbf{C}}_k$ and $\tilde{\mathbf{R}}_k$ such

that $\{\tilde{\mathbf{R}}_k\}_{1,1} = 1 \forall n, k$, which results in the condition that $\sum_n \{\boldsymbol{\alpha}\}_{nk} = 1 \forall k$. It then follows that the matrix $\boldsymbol{\alpha}$ is described by at most $K(K-1)$ real numbers.

IV. A GENERAL METHODOLOGY FOR SM SEPARATION FROM MPMB DATA

Define $\widehat{\mathbf{W}}$ as the sample estimate of the MPMB covariance matrix, and \mathbf{W}_K as the matrix that describes the expected MPMB covariance matrix according to some model where K SMs are considered. Then, the problem of SM separation and characterization may be posed as the problem of finding the matrix $\widehat{\mathbf{W}}_K$ that best approximates $\widehat{\mathbf{W}}$, according to some criterion. In the analysis of forest scenarios the Least Square (LS) criterion, either weighted or unweighted, is often assumed, see for example [8], [10], [6]. Accordingly, SM separation will be assumed to be carried out through the estimator:

$$\widehat{\mathbf{W}}_K = \arg \min \left\{ \left\| \widehat{\mathbf{W}} - \mathbf{W}_K \right\|_F \right\} \quad (11)$$

where $\|\cdot\|_F$ is the matrix Frobenius norm. Assume now that \mathbf{W}_K is modeled as a sum of K Kronecker products, consistently with hypotheses i), ii), iii) discussed at the beginning of Section III. In this case the minimizer of (11) can be found in a closed form fashion by noting that [17]:

$$\left\| \widehat{\mathbf{W}} - \mathbf{W}_K \right\|_F = \left\| \wp(\widehat{\mathbf{W}}) - \wp(\mathbf{W}_K) \right\|_F \quad (12)$$

Whereas the matrix $\wp(\widehat{\mathbf{W}})$ is in general expected to have full rank, since it is estimated from the data, by construction the matrix $\wp(\mathbf{W}_K)$ is constraint to have at most rank K . Therefore, the minimizer of (11) can be determined as $\widehat{\mathbf{W}}_K = \wp^{-1}(\hat{\wp}_K)$, where $\hat{\wp}_K$ is the solution of the following low-rank matrix approximation problem:

$$\hat{\wp}_K = \arg \min_{\text{rank}(\wp_K) \leq K} \left\{ \left\| \wp(\widehat{\mathbf{W}}) - \wp_K \right\|_F \right\} \quad (13)$$

that can be solved easily by taking the first K terms of the SVD of $\wp(\widehat{\mathbf{W}})$, by virtue of the Eckart-Young-Mirsky theorem [18]. In formula, letting $\wp(\widehat{\mathbf{W}}) = \sum_{k=1}^K \lambda_k \mathbf{u}_k \mathbf{v}_k^H$ be the SVD of $\wp(\widehat{\mathbf{W}})$, the minimizer of (13) is obtained as:

$$\hat{\wp}_K = \sum_{k=1}^K \lambda_k \mathbf{u}_k \mathbf{v}_k^H \quad (14)$$

Furthermore, it can be shown that the solution is unique if and only if $\lambda_k \neq \lambda_{k+1}$ [18]. Under this condition problem (11) admits a unique solution as well, that may be written as:

$$\widehat{\mathbf{W}}_K = \sum_{k=1}^K \mathbf{C}_k(\boldsymbol{\alpha}) \otimes \mathbf{R}_k(\boldsymbol{\alpha}) \quad (15)$$

where the matrices $\mathbf{C}_k(\boldsymbol{\alpha}), \mathbf{R}_k(\boldsymbol{\alpha})$ are obtained according to (10), and $\boldsymbol{\alpha}$ is an arbitrary non-singular $K \times K$ matrix. It is important to note that the choice of $\boldsymbol{\alpha}$ does affect only the matrices $\mathbf{C}_k(\boldsymbol{\alpha})$ and $\mathbf{R}_k(\boldsymbol{\alpha})$, whereas the sum of

the Kronecker Products between $\mathbf{C}_k(\boldsymbol{\alpha})$ and $\mathbf{R}_k(\boldsymbol{\alpha})$ results identically equal to the *global* minimizer of problem (11) by construction, see also [17]. This important property entails that the solution provided by (15) is the best LS solution given the hypothesis of K SMs. In this sense, (15) may be thought of as a sort of scattering mechanism filter, that allows to deal only with the K dominant SMs in the data. At this point, the problem is how to find a matrix $\hat{\boldsymbol{\alpha}}$ such that the matrices $\mathbf{C}_k(\hat{\boldsymbol{\alpha}})$ and $\mathbf{R}_k(\hat{\boldsymbol{\alpha}})$ are physically meaningful. This task can be accomplished by requiring that: i) both $\mathbf{C}_k(\hat{\boldsymbol{\alpha}})$ and $\mathbf{R}_k(\hat{\boldsymbol{\alpha}})$ are Hermitian, semi-positive definite matrices for every k : this condition ensures the physical validity of the solutions; ii) some criterion is optimized, in such a way as to select a unique solution among the set of all the physically valid solutions. Different approaches may arise depending on the choice of the criterion to be optimized.

1) *Model based SM separation*: the solution is selected in such a way that the matrices $\mathbf{C}_k(\hat{\boldsymbol{\alpha}})$, $\mathbf{R}_k(\hat{\boldsymbol{\alpha}})$ are as close as possible to some physically based model. Possible choices are, for example, the Freeman models, [19], for the polarimetric signatures or the RVoG model for the structure matrices [10].

2) *Model free SM separation*: the solution is selected according to mathematical optimization criteria. For example, SM separation may be achieved by maximizing the diversity among the structure matrices, that can be easily measured according to the Frobenius inner product. In formula:

$$\hat{\boldsymbol{\alpha}} = \arg \max \left\{ \sum_{k,h \neq k} 1 - \frac{|\text{trace}(\mathbf{R}_k(\boldsymbol{\alpha}) \mathbf{R}_h(\boldsymbol{\alpha}))|}{\|\mathbf{R}_k(\boldsymbol{\alpha})\|_F \|\mathbf{R}_h(\boldsymbol{\alpha})\|_F} \right\} \quad (16)$$

subject to the constraint of semi-positive definitiveness.

Further details are found in [16].

A. On the single baseline case

Consider the problem of the separation of ground and volume scattering basing on single baseline ($N = 2$), fully polarimetric acquisitions. Assuming the PS condition to hold, the matrices $\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2$ in have the form:

$$\tilde{\mathbf{R}}_k = \begin{bmatrix} 1 & \tilde{\gamma}_k \\ \tilde{\gamma}_k^* & 1 \end{bmatrix} \quad (17)$$

Accordingly, the equations relative to the structure matrices can be recast by considering only the complex coherences, namely:

$$\begin{aligned} \gamma_g(a, b) &= a\tilde{\gamma}_1 + (1-a)\tilde{\gamma}_2 \\ \gamma_v(a, b) &= b\tilde{\gamma}_1 + (1-b)\tilde{\gamma}_2 \end{aligned} \quad (18)$$

from which it follows that γ_g, γ_v are bound to lie on the line l in the complex plane that passes through $\tilde{\gamma}_1, \tilde{\gamma}_2$. At this point the problem can be given a straightforward geometrical interpretation. The semi-positive definitiveness condition for the structure matrices is satisfied by requiring that $|\gamma_g| \leq 1, |\gamma_v| \leq 1$, and thus only the portion of l inside the unitary circle has to be considered. Further requiring the semi-positive definitiveness condition for the polarimetric signatures, it follows that the physical solution for γ_g, γ_v are

bound to belong to two segments along the line l within the unit circle. Accordingly, the problem of identifying two SMs basing on multi-polarimetric, single-baseline data is equivalent to the problem of finding two points along two segments. This conclusion is exactly the same as the one drawn by *Cloude and Papathanassiou* in [15], although in that paper a totally different methodology is used, and only the single baseline case is considered. By virtue of this result, the general methodology proposed in Section IV can be thought of as a multi-baseline extension of the procedures usually exploited in single-baseline PolInSAR that is able to jointly exploit *all* the available interferograms, in such a way as to ensure the physical validity of the solution and optimize the desired criteria by considering the whole data-set at once.

V. EXPERIMENTAL RESULTS

An experiment on real data has been performed basing on a data set of $N = 9$ P-Band fully polarimetric SAR images of the forest site of Remningstorp, Sweden, acquired by DLR's E-SAR from March until May 2007, in the framework of the ESA campaign BioSAR 2007. The Remningstorp data-set has been the object of a thorough model based Tomographic analysis, see [6], [20], which provided many experimental evidences that indicate that contributions from the ground dominate those from the canopy in every polarimetric channel.

SM separation has been carried out according to Section IV. As a result, it has turned out that more than 90% of the information carried by the data can be represented by retaining just two Kronecker products², therefore providing an experimental validation of the model in (6). Two options have been evaluated for SM separation: i) selection of the solution that maximizes the diversity among the structure matrices, see (16); ii) selection of the solution corresponding to the centre of the region of positive definitiveness. In both cases, the estimated structure matrices $\mathbf{R}_1(\hat{a}, \hat{b})$ and $\mathbf{R}_2(\hat{a}, \hat{b})$ have then been analyzed by evaluating the corresponding Capon spectra. The maximum diversity solution has given rise to two very well defined spatial structures, see Fig. (1), which have shown a very good agreement with LIDAR³ based terrain and canopy elevation estimates provided by the Swedish Defence Research Agency (FOI). The first SM can be reasonably associated with ground scattering, whereas the second appear to be a small volume at a high elevation. Solution ii), instead, has given rise to a different profile for the second SM, see Fig. (2), that appears now as a large volume that extends from the ground to the LIDAR top height⁴. The following conclusion may be drawn: i) the first and the second SMs can be reasonably

²To be intended as: $1 - \frac{\|\hat{\mathbf{W}} - \sum_{k=1}^2 \hat{\mathbf{C}}_k \otimes \hat{\mathbf{R}}_k\|_F}{\|\hat{\mathbf{W}}\|_F} > 0.9$, where $\|\cdot\|_F$ is the Frobenius matrix norm.

³LIDAR measurements have been resampled onto the SAR slant range, azimuth coordinates and spatially averaged, having care to include in the computation only those samples where a return from the canopy was actually present.

⁴Note that overlapping between ground and volume contributions is not un-physical, as volume contributions may originate from understory as well.

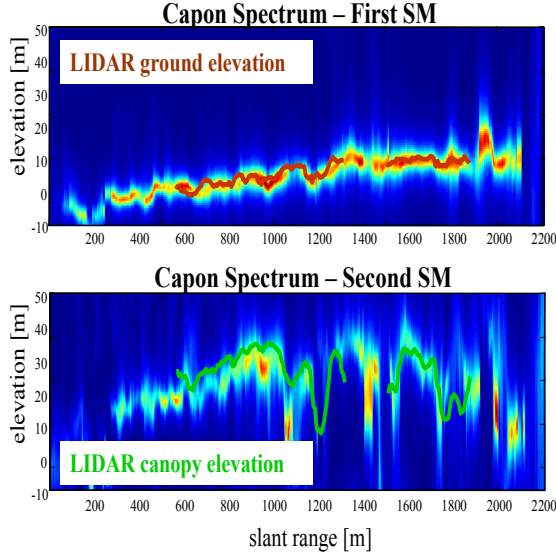


Fig. 1. Top: Capon Spectrum of the first SM. Bottom: Capon Spectrum of the second SM. The solution has been selected by maximizing the diversity between the structure matrices.

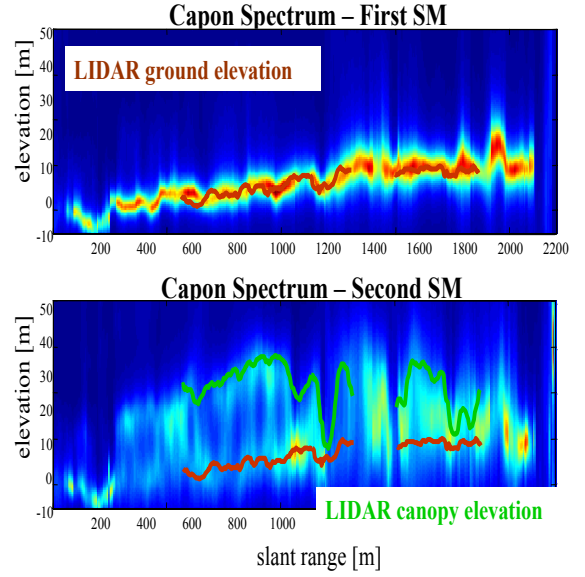


Fig. 2. Top: Capon Spectrum of the first SM. Bottom: Capon Spectrum of the second SM. The solution has been selected by taking the centre of the region of positive definiteness.

associated to ground and volume scattering, respectively; ii) although resulting in *exactly* the same covariance matrix, the two solutions appear to give rise to quite different profiles: iii) the top and bottom heights appear to make sense in both cases. Accordingly, it is not possible to determine which solutions better characterizes the imaged scene without exploiting a priori information. Still, the results provided in this section have shown that the imaged scene can be well characterized by retaining just two SMs, and that such SMs are associated with a perturbed double bounce scattering from ground-trunk interactions and volume backscattering, even in absence of physical models.

VI. WELL POSING

This section is devoted to discussing the well posing of model based inversion from both Single and Multi Polarimetric, Multi Baseline observations (hereinafter referred to as SPMB and MPMB). To this aim, two SMs will be assumed, representing the contributions from the ground and from the canopy.

A. Model inversion from SPMB data

Considering only 2 SMs and one polarimetric channel equations (5) defaults to:

$$\begin{aligned} \gamma(n, m) &= c\gamma_g(n, m) + (1 - c)\gamma_c(n, m) \\ \gamma_g(n, m) &= \exp(j\varphi_g(n, m))\tau_g(n, m)v_v(n, m) \\ \gamma_c(n, m) &= \exp(j\varphi_g(n, m))\tau_v(n, m)v_v(n, m) \end{aligned} \quad (19)$$

where: $\gamma_g(n, m)$, $\gamma_c(n, m)$ are the complex coherences for the ground and the volume relative to the $nm - th$ interferogram; c is the ground fractional backscattered powers with respect to the total backscattered power; $\varphi_g(n, m)$

represents the ground phase in the $nm - th$ interferogram; $v_g(n, m)$, $v_v(n, m)$ are the geometric coherences that arise from the spatial structures of the ground and the volume [13]; $\tau_g(n, m)$, $\tau_v(n, m)$ are real valued parameters that represent the coherence losses for the ground and the volume due to temporal decorrelation [13]. Note that the contribution of any other decorrelation sources (coregistration errors, noise, or other) can be accounted for in the expression of $\tau_g(n, m)$, $\tau_v(n, m)$. After equations (19) the estimation of the ground and the canopy can be performed by characterizing the geometric coherences, $v_g(n, m)$ and $v_v(n, m)$, through appropriate physical models that describe their structural parameters, such as terrain slope, canopy elevation, extinction coefficient, or other. Therefore, the well posing of the problem will be discussed by comparing the number of independent real equations (IREs) that characterize the model of the observables with respect to the number of real unknowns that are required to solve such model. The number of unknowns required to describe the spatial structure of the scene clearly depends on the model through which the scene is represented. A conveniently simple, yet widely and successfully employed, solution is offered by the Random Volume over Ground (RVoG) model [10], that is essentially based on two assumptions: i) the canopy can be modeled as an uniform volume that extends from the ground to an unknown elevation, and is characterized by an unknown extinction coefficient; ii) the ground undergoes no coherence loss throughout the acquisitions⁵, namely $v_g(n, m) = 1$. Consistently with the RVoG model, in the following it will be assumed that at least 3 real unknowns are required for

⁵This hypothesis is usually enforced by employing Common Band Filtering techniques [21].

the retrieval of the vertical structure, represented by the ground fractional backscattered power, c , and two structural parameters.

A difficulty in evaluating the number of IREs that can be extracted from the observables arises due to non-linearity of model (19). This problem can be solved quite easily by linearizing the expression of the coherences about some reference values of the unknowns, in such a way as to evaluate the number of IREs through standard methods from linear algebra. Such an approach, however, does not provide insights into the mechanisms that rule the well posing of model inversion. For this reason, the evaluation of the number of IREs will be carried out in the following through a direct approach. Since $N(N-1)/2$ different complex interferograms may be formed out of N acquisitions, it follows that up to $N_e = N(N-1)$ IREs are available for model inversion. However, it may be proved through a simple example that a lower number of IREs is actually available. Consider an ideal case where temporal decorrelation is absent, the ground phases have been perfectly compensated for, and the baseline sampling is uniform. In this case the coherence of each interferogram depends only on spatial decorrelation, from which it follows that $\gamma(n, m) = \gamma(n-m)$, as a consequence of the fact that spatial decorrelation depends only on the baseline difference and a uniform baseline sampling has been assumed. Therefore, only $N-1$ different complex interferograms are actually available, for a total of $N_e = 2(N-1)$ IREs⁶. Clearly, additional IREs arise by adopting an irregular baseline sampling. This choice, however, is likely to result in the inversion problem to be well posed but ill conditioned, thus requiring a careful design of the baseline set. Therefore, the hypothesis of uniform baseline sampling will be retained in the following as a prudential choice, whereas the case of irregular baseline sampling is left open for future researches. If also the ground phases and/or ground and canopy temporal decorrelation are accounted for, then the condition $\gamma(n, m) = \gamma(n-m)$ is lost, resulting in a higher number of IREs. In this case, however, the set of the unknowns is given not only by the structural parameters, but also by a number of nuisance parameters, since ground phases and temporal decorrelation have to be estimated and compensated for. If no model is assumed for the ground phases and temporal decorrelations, then the number of real nuisance parameters is equal to $N(N-1)/2$ for the ground phases, $N(N-1)/2$ for ground temporal decorrelation, and $N(N-1)/2$ for canopy temporal decorrelation. Since at most $N(N-1)$ IREs are available, it follows that only one of these three factors can be accounted for in the framework of a SPMB analysis. At this point the well posing of the problem of separating the ground from the canopy can be discussed by considering the following scenarios:

1) *Absence of temporal decorrelation*: If temporal decorrelation can be neglected, as it may happen in case of simul-

taneous or low frequency (P-Band, VHF) acquisitions, after model (19) it is easy to see that the observable amplitudes are constrained to the condition $|\gamma(n, m)| = |\gamma(n-m)|$, whereas no constraint is placed on the observable phases, due to the presence of the ground phases $\varphi(n, m)$. Accordingly, $N_e = N(N-1)/2 + (N-1)$ IREs can be extracted from the observables, whereas $N(N-1)/2$ nuisance parameters have to be accounted for in order to estimate the ground phases. Therefore $N-1$ IREs can be exploited for model inversion. Since it has been assumed that at least 3 parameters are required for the retrieval of the vertical structure, it follows that at least $N_{min} = 4$ images are required for the inversion problem to be well posed. Each additional image provides 1 additional IRE available for model inversion.

2) *Absence of temporal decorrelation; Ground phase compensated*: In some cases the ground phases can be successfully estimated and compensated for through the employment of phase calibration techniques [22], [23], [24]. A typical case where phase calibration can be performed is when a sufficiently dense grid of stable scatterers, such as those resulting from ground-trunk interactions, may be identified within the forest [20]. The expression of the ground phases once phase calibration has been performed may be written as:

$$\varphi(n, m) = (k_z(n) - k_z(m)) z_g = \varphi_z(n-m) \quad (20)$$

where $k_z(n)$ is the vertical wavenumber of the n -th image [11] and z_g is the ground elevation. It follows that the condition $\gamma(n, m) = \gamma(n-m)$ is valid, and thus the problem defaults to the inversion of 4 real parameters (including ground elevation) from $N_e = 2(N-1)$ IREs. Accordingly, at least $N_{min} = 3$ images are required for the inversion problem to be well posed, whereas each additional image provides 2 additional IREs available for model inversion⁷.

3) *Presence of canopy temporal decorrelation; Ground phase compensated*: Finally, we consider the case where phase calibration has been performed, but canopy temporal decorrelation, $\tau_v(n, m)$, has to be accounted for. Since uniform baseline sampling is assumed, model (19) can be rewritten as:

$$\begin{aligned} \gamma_k(m) &= \exp(j\varphi_z(k)) \cdot \\ &\{c v_g(k) + (1-c) \tau_v(n, m) v_v(k)\} \end{aligned} \quad (21)$$

where $\gamma_k(m)$ has been defined as $\gamma_k(m) = \gamma(m+k, m)$. After (21) it is easy to see that, for every k , the $N-k$ coherences $\{\gamma_k(m)\}_{m=1}^{N-k}$ are bounded to lie on a straight line, l_k , in the complex plane. This entails that the coherences $\{\gamma_k(m)\}_{m=1}^{N-k}$ can be obtained by fixing two complex numbers, required to fit the line l_k in the complex plane, plus $N-k-2$ real numbers, required to determine the positions of the remaining $N-k-2$ coherences along l_k . It follows that $N-k+2$ IREs can be extracted from the coherences $\{\gamma_k(m)\}_{m=1}^{N-k}$, with the only exception of $\gamma_{N-1}(m)$, which

⁷In principle, only $N-1$ ground phases should be considered, under the hypothesis that ground phases are determined only by ground elevation and propagation disturbances. Accordingly, a case here considered has to be regarded as a worst case scenario, which includes a phase varying term related to the nature of the scatterers as well.

⁶In general, all the $N(N-1)/2$ complex interferograms must be exploited in order to best estimate the parameters of interest, regardless of baseline sampling. Within this section, however, the interest is focused only on the well posing of the inversion problem.

provides just 2 IREs. By summing over k , it is readily found that

$$N_e = 2 + \sum_{k=1}^{N-2} (N - k + 2) = \frac{N(N-1)}{2} + 2N - 3 \quad (22)$$

IREs can be extracted from the observables, whereas $N(N-1)/2$ nuisance parameters have to be accounted for in order to estimate the canopy temporal decorrelation. Therefore $2N-3$ IREs are available for the inversion of 4 real parameters (including ground elevation), and thus at least $N_{min} = 4$ images are required for the inversion problem to be well posed, whereas each additional image provides 2 additional IREs available for model inversion.

In each case more sophisticated models can be assumed for both the ground and the canopy by exploiting the additional IREs that arise if $N > N_{min}$. In [20], for example, experimental evidence of the possibility to retrieve the vertical structure from SPMB data has been provided by modeling the scene through 3 structural parameters, plus ground elevation.

B. Model inversion from MPMB data

This section is devoted to analyzing the well posing of the model inversion problem basing on MPMB data. In MPMB model based approaches SM separation is performed by modeling the MPMB covariance matrix basing on explicit electromagnetic models that describe either the structure matrices [8], [6], and/or the polarimetric signatures [19], [10] associated with ground and volume scattering. In formula:

$$\mathbf{W}_2(\theta) = \mathbf{C}_g(\theta) \otimes \mathbf{R}_g(\theta) + \mathbf{C}_v(\theta) \otimes \mathbf{R}_v(\theta) \quad (23)$$

where θ is the set of the unknowns. Parameter estimation is then carried out by looking for the best approximation of the observed covariance matrix, without explicitly exploiting the SKP structure. Still, the properties arising from the SKP structure provide many useful insights that can greatly simplify the discussion of the well posing of the inversion problem.

It is here discussed the case where only the structure matrices for ground and volume scattering are modeled, whereas no model is assumed for polarimetric signatures. After the results provided above it follows that the structure matrices for ground and volume scattering, $\mathbf{R}_g, \mathbf{R}_v$, are obtained as linear combinations of the matrices $\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2$, extracted from the rearrangement of the data covariance matrix. Since the transformation from $\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2$ to $\mathbf{R}_g, \mathbf{R}_v$ is linear and invertible by definition, it follows that $\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2$ can be written as linear combinations of $\mathbf{R}_g, \mathbf{R}_v$. Therefore, parameter estimation can proceed from the model:

$$\begin{aligned} \tilde{\mathbf{R}}_1 &= r_1 \mathbf{R}_g(\theta) + (1 - r_1) \mathbf{R}_v(\theta) \\ \tilde{\mathbf{R}}_2 &= r_2 \mathbf{R}_g(\theta) + (1 - r_2) \mathbf{R}_v(\theta) \end{aligned} \quad (24)$$

where (r_1, r_2) are two different real numbers to be estimated along with the model parameters θ . Once the pair (r_1, r_2) has been determined the polarimetric signatures can be obtained unambiguously from the matrices $\tilde{\mathbf{C}}_1, \tilde{\mathbf{C}}_2$. It follows that all the information required for the estimation of θ is included

in the matrices $\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2$, which can therefore be interpreted as the new observables of the problem. The evaluation of the number of IREs that arise from $\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2$ may proceed as follows. By analyzing the derivatives of $\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2$ with respect to θ and (r_1, r_2) it is easy to see that the number of IREs that can be extracted from $\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2$ is at least equal to the number of IREs that can be extracted from $\mathbf{R}_g, \mathbf{R}_v$, by virtue of the fact that the transformation between $\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2$ and $\mathbf{R}_g, \mathbf{R}_v$ is linear and invertible. At most 2 additional IREs may arise, since $\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2$ are function of the parameters (r_1, r_2) as well. To the aim of practical applications, however, this point may be overlooked. For this reason, the evaluation of the number of IREs provided by the MPMB data will be carried out simply by analyzing how many real numbers are required to jointly characterize the matrices $\mathbf{R}_g, \mathbf{R}_v$. Results relative to three different scenarios are reported in Table I. It may be appreciated that the exploitation of the polarimetric information makes it possible to analyze extremely complex scenarios, characterized by the presence of temporal decorrelation and non compensated ground phases, that would be inaccessible basing on a single polarimetric channel.

VII. CONCLUSIONS

This paper has considered the problem of the retrieval of the vertical structure of forested areas, basing on single and multi-polarimetric multi-baseline SAR acquisitions. Model based approaches have been discussed from the point of view of the well posing of model inversion, taking into account the role of three kinds of eventual nuisance parameters, associated to the presence of volume temporal decorrelation, ground temporal decorrelation, and uncompensated ground phases. As a result, it has been shown that model inversion can not be carried out basing on single polarimetric data, regardless of the number of available baselines, if more than one kind of nuisance parameters has to be accounted for. Since at least volume temporal decorrelation is likely to occur in real scenarios, it follows that performing phase calibration has to be considered as a mandatory choice if only single polarimetric data are available. The case of multi-polarimetric acquisitions has been discussed by noting that, under very large hypotheses, the covariance matrix of the multi-polarimetric, multi-baseline (MPMB) data can be described through a Sum of Kronecker Products (SKP). Basing on the algebraic properties of the SKP structure it has been shown that the number of independent equations that can be extracted from the data is at least equal to the number of parameters required to characterize the structure matrices associated to ground and volume scattering. By virtue of this result, it has been concluded that all three kinds of nuisance parameters mentioned above can be handled basing on MPMB acquisitions, resulting in the possibility to analyze scenarios that would be inaccessible basing on single polarimetric data.

Basing on the algebraic properties arising from the SKP structure, it has been shown the equivalence between the problem of separating K SMs is equivalent to determining a non singular, real valued square matrix α with dimension

Scenario	S1	S2	S3
Ground phase compensation	Yes	No	No
Canopy temporal decorrelation	No	Yes	Yes
Ground temporal decorrelation	No	No	Yes
Number of IREs (N_e)	$4(N-1)$	$2(N-1) + N(N-1)$	$(N-1) + \frac{3}{2}N(N-1)$
Nuisance parameters	a, b, z_g	a, b, φ_g, τ_c	$a, b, \varphi_g, \tau_c, \tau_g$
Minimum number of tracks (N_{min})	3	3	5

TABLE I

WELL POSING OF THE PROBLEM OF SM SEPARATION FROM MPMB DATA BY EXPLOITATION OF STRUCTURAL MODELS FOR GROUND AND VOLUME SCATTERING.

$K \times K$. This result has been exploited to propose a new general methodology for the processing of MPMB data, based on the exploration of the range of values assumed by α . Such a methodology presents many attractive features: i) it yields is the best LS solution given the hypothesis of K SMs; ii) it makes it possible to exploit not only model based approaches, but also model free, and hybrid approaches; iii) it is consistent the inversion procedures usually exploited in single-baseline PolInSAR in the case where only single baseline data are available.

Future researches will be devoted to the investigation of the best criterion to be exploited for SM separation, and to the definition of an algorithm to carry out SM separation under the hypothesis that the received signal is contributed by more than two SMs.

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