An Optimization-theoretic Approach to Transmit Power Control in Wireless Sensor Networks

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***Abstract*—In this paper, we present an innovative transmit power control scheme, based on optimization theory, for wireless sensor networks (WSNs) which use carrier sense multiple access (CSMA) with collision avoidance (CA) as medium access control (MAC) protocol. In particular, we focus on schemes where several remote nodes send data directly to a common access point (AP). Under the assumption of finite overall network transmit power and low traffic load, we will derive the optimal transmit power allocation strategy that minimizes the packet error rate at the AP. This approach is based on modeling the CSMA/CA MAC protocol through a finite state machine and takes into account the adjacency matrix (associated with the transmit power distribution) of the network. It will be then shown that the transmit power allocation problem reduces to a convex constrained minimization problem. Our results show that, under the assumption of low traffic load, the power allocation strategy, which guarantees the lowest delay, requires the maximization of network connectivity, which can be equivalently interpreted as maximization of the number of non-zero entries of the adjacency matrix. The obtained theoretical results are confirmed by simulations for unslotted Zigbee WSNs.**

I. Introduction

Wireless sensor networks (WSNs) are an interesting research topic, both in military [1], [2] and civilian scenarios [3]. In particular, remote/environmental monitoring, surveillance of reserved areas, etc., are important fields of application of WSNs. These applications often require very low power consumption and low-cost hardware [4]. One of the most common standards for wireless networking with low transmission rate and high energy efficiency has been proposed by the Zigbee Alliance [5]. In this context, an interesting research direction for the design of WSNs is related to network architectures able to guarantee high energy efficiency. In particular, since the overall energy available in a WSN is typically limited (all nodes are battery-equipped), the research community has focused on the derivation of transmit power allocation strategies which allow to maximize a specific performance indicator, yet guaranteeing high energy savings.

In [6], the authors present a comparison between three power control schemes based on the analysis of the received signal-to-noise ratio in dense relay networks. In particular, one of these opportunistic schemes aims at extending the lifetime of the relays, in order to maximize the lifetime of the entire network. In [7], the authors introduce a power allocation scheme which minimizes the estimation mean-square error at the fusion center of a network where sensors transmit to the fusion center over noisy wireless links. In [8], the authors jointly optimize the data source quantization at each sensor, the routing scheme, and the power control strategy in a WSN in order to derive an efficient solution for the problem of overall network optimization. Finally, in [9] the authors present an opportunistic power allocation strategy which is based on local and decentralized estimation of the links' quality. In this scenario, only nodes experiencing channel conditions above a pre-fixed quality threshold are allowed to transmit, in order to avoid a waste of energy.

In this paper, we propose an innovative transmit power control scheme for Zigbee WSNs which builds upon optimization theory. This approach relies on the assumptions of (i) low traffic load and (ii) finite overall network transmit power, and its goal is the minimization of the packet error rate (PER) at the access point (AP). Modeling the carrier sense multiple access with collision avoidance (CSMA/CA) medium access control (MAC) protocol through a finite state machine, it is possible to allocate the transmit powers at the sensors in order to maximize the number of ones of the adjacency matrix, i.e., to maximize the number of active connections between the nodes in the network. In all cases, we will assume that the sensors transmit directly to the AP. This assumption holds in industrial and home monitoring applications, where the nodes' position is a priori determined. In more general scenarios, the positions of the nodes could be unknown, then one can consider estimating the positions of the nodes are estimated and applying our optimized transmit power control. The proposed optimization approach will guarantee a lower PER than that in a scenario where all nodes transmit at the same power, yet guaranteeing relevant energy savings.

The structure of this paper is the following. In Section II, the optimization-theoretic model, upon which this work is based, is presented, describing the simplified model and the optimized transmit power allocation strategy. In Section III, the Zigbee standard and its implementation in the Opnet simulator are described. In Section IV, the performance, in terms of PER and delay, is presented, focusing on the impact of the adjacency matrix, the traffic load, and the used power allocation strategy. Finally, Section V concludes the paper.

II. Optimization-theoretic Model

A. Definition of a Simplified Model for Zigbee Sensor Networks

In the following, we first introduce some key parameters of a Zigbee sensor network. Then, we present a simplified version of its MAC protocol and, under the assumption of low traffic load, we propose a simplified analytical model for the estimation of the following main network performance indicators: PER at the AP and average delay.

First of all, each sensor node[1] is characterized by two main parameters: (i) its position on a two-dimensional plane and (ii) its transmit power, as stated in the following definition.

Definition 1: A *sensor* is represented by a couple $s = (x, P)$, where $x \in \mathbb{R}^2$ is the sensor position and $P \in \mathbb{R}$ is its transmit power.

We assume that the detection operation is described by an ideal threshold model, as stated in the following assumption.

***Assumption** 1 (Threshold reception):* Given two sensors $s_1 = (x_1, P_1)$ and $s_2 = (x_2, P_2)$, there exists a *minimum power function* $\Pi(x_1, x_2)$ such that sensor s_2 receives the transmission of sensor s_1 if and only if

$$P_1 \geq \Pi(x_1, x_2).$$

For instance, this assumption holds because of the propagation loss (according to the Friis formula) and assuming that a threshold detector is used at the receiver [10]. In fact, in this case the power P_r received by sensor s_2 can be expressed as:

$$P_\mathrm{r} = P_1 G_\mathrm{t} G_\mathrm{r} \left(\frac{\lambda}{4\pi r}\right)^{\alpha} \quad (1)$$

where G_t and G_r are the gains of the transmit and receive antennas, r is the distance, λ is the wavelength, and α is the pathloss exponent. According to the ideal threshold detector model, sensor s_2 receives a transmission from sensor s_1 if and only if $P_r > P_\mathrm{min}$, where P_min is the (pre-defined) receiver reception threshold. In this case,

$$\Pi(x_1, x_2) = \frac{P_\mathrm{min}}{G_\mathrm{t} G_\mathrm{r}} \left(\frac{4\pi r}{\lambda}\right)^{\alpha}.$$

A sensor network can be introduced as a set of sensors, characterized by their positions and transmit powers, together with an associated minimum power function.

Definition 2: A *sensor network* of N elements is an ordered set $\mathcal{S} = (c, \Pi, s_1, s_2, \ldots, s_N)$, where $s_1, s_2, \ldots, s_N$ are sensors, $c \in \mathbb{R}^2$ is the position of the AP, and $\Pi : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ is the associated minimum power function.

Definition 3 (Adjacency matrix): Given a sensor network $\mathcal{S} = (c, \Pi, s_1, s_2, \ldots, s_N)$, with $s_i = (x_i, P_i)$, $i = 1, \ldots, N$, its associated *adjacency matrix* is given by

$$A(\mathcal{S}) \in \mathbb{R}^{N \times N}$$

where

$$A_{ij} = A(\mathcal{S})_{ij} = \begin{cases} 1 & \text{if } P_i \geq \Pi(x_i, x_j) \\ 0 & \text{otherwise.} \end{cases}$$

[1]For the sake of simplicity, we will simply use the term “sensor” to refer to a wireless node with sensing capabilities.

The complement of $A(\mathcal{S})$ corresponds to

$$\bar{A}(\mathcal{S}) = \begin{cases} 1 & \text{if } A_{ij} = 0 \\ 0 & \text{otherwise.} \end{cases}$$

The number of ones in the adjacency matrix is given by the *adjacency* of sensor network $\mathcal{S}$ and is denoted by $|A(\mathcal{S})|$. The complementary adjacency is given by the number of zeros in the adjacency matrix and is denoted by $|\bar{A}(\mathcal{S})|$.

For each $i = 1, \ldots, N$,we define the following two sets:

$$\begin{aligned} \mathcal{R}_i &\triangleq \{j = 1, \ldots, N | A_{ji} = 1\} \\ \mathcal{T}_i &\triangleq \{j = 1, \ldots, N | A_{ij} = 1\} \end{aligned}$$

which represent the sets of indices of the sensors which s_i can receive from and transmit to, respectively. We denote by $\overline{\mathcal{R}}_i$ and $\overline{\mathcal{T}}_i$ the complements of these two sets.

In order to make the theoretical analysis feasible, the Zigbee sensor network is described by the following simplified model.

***Assumption** 2 (Simplified model):*

1) Poisson generation: the traffic generated by each sensor in the network is modeled as a homogeneous Poisson process [11]. The processes associated with different sensors are independent of each other and have intensity g (dimension: [pck/s]).
2) Limited CCA: before transmitting, each sensor waits for a random backoff time, with average T_{B_1} (dimension: [s]), then s_i checks if the channel is clear. This clear channel assessment (CCA) is limited only to those sensors whose indices lie in the set $\mathcal{R}_i$. In other words, the sensing is limited only to those sensors that can effectively (i.e., with sufficiently high received power) transmit to the i-th sensor. The CCA has a duration equal to T_CCA.
3) Infinite number of backoffs: if the channel is found busy, the current sensor transmission is delayed by a random backoff time with average T_{B_2} (dimension: [s]), which is longer than T_{B_1}, as will be shown in Subsection III-A. During the backoff period the traffic generation at the transmitting sensor does not stop. There is no limit on the total number of subsequent backoffs that a single packet transmission can incur.
4) Constant transmission length: each transmission has the same length $T_\mathrm{trans} = L/R$, where L is the packet length (dimension: [b/pck]) and R is the transmission data-rate (dimension: [b/s]).
5) Transmission turnaround time (TAT): after sensing, if the channel is found idle, each sensor waits a turnaround time, denoted as T_TAT (dimension: [s]), before starting its transmission.

For each sensor s_i ($i = 1, \ldots, N$) the following counting processes can be defined.

- $G_i(t)$: the number of times that sensor s_i has checked if the channel is clear in the time interval $[0, t]$.
- $B_i(t)$: the number of times that a packet transmission of sensor s_i has been delayed, through the backoff mechanism, in $[0, t]$.

- $T_i(t)$: the number of times that a sensor s_i has transmitted in $[0,t]$ (counting both successful and unsuccessful transmission acts).
- $E_i(t)$: the number of transmission errors incurred by sensor S_i in $[0,t]$.

For a counting process $P(t)$, define the steady state intensity as follows:

$$F[P] \triangleq \lim_{t\to\infty,\tau\to 0} \frac{\mathbb{E}\left[P(t+\tau)-P(t)\right]}{\tau} \tag{2}$$

where $\mathbb{E}[\cdot]$ denotes the expected value. We recall that for stationary Poisson traffic generation processes, the steady state intensity is denoted by g. In the following, we assume that the limit at the righthand side of (2) exists for all previously defined counting processes: this is equivalent to assuming that the network reaches a "steady state." Under this hypothesis, the following equilibrium conditions must be satisfied:

$$\begin{aligned} F[T_i] &= g && (3)\\ F[G_i] &= g + F[B_i]. && (4)\end{aligned}$$

Condition (3) states that, at steady state, the intensity of transmissions must be equal to the intensity of traffic generation. Condition (4) states that, at steady state, the intensity of channel sensing has to be equal to the sum of the intensities of packet generations and backoffs.

The backoff traffic intensity can be expressed as follows:

$$F[B_i] = F[G_i]\chi_i$$

where χ_i represents the ratio between the numbers of backoffs and transmission attempts. In this way, the processes $\{T_i(t)\}$ and $\{B_i(t)\}$ satisfy the following relations:

$$\begin{aligned} &F[G_i] = F[B_i] + F[T_i] = \frac{g}{1-\chi_i}\\ &F[B_i] = \chi_i g.\end{aligned}$$

The term χ_i can be equivalently interpreted as the probability, for the i-th sensor, to assess that the channel is busy during the CCA. In order to derive a simple expression for χ_i, it is assumed that the processes $\{T_i(t)\}$ are uncorrelated and Poisson. This simplification is meaningful in low traffic conditions. In fact, in this case $F[B_i] << g$ and the processes $\{T_i(t)\}$ are statistically very similar to Poisson traffic generation processes. However, as it will be shown in Section IV, the estimated PER obtained with these simplifications is close to that predicted by (realistic) simulations also in relatively high traffic conditions.

Under the above simplifications, χ_i equals the probability of finding at least one packet transmission event, during a time interval equal to the transmission length T_{trans}, in the set of independent Poisson processes $\{G_j(t)\}_{j\in\mathcal{R}_i}$. In other words, one can write:

$$\chi_i = \lim_{t\to\infty} \mathcal{P}\left\{\max_{j\in\mathcal{R}_i}\{T_j[t+T_{\text{trans}}]-T_j[T_{\text{trans}}]\}>0\right\}$$

that is

$$\begin{aligned} \chi_i &= 1-\prod_{j\in\mathcal{R}_i}(1-e^{-F[T_j]T_{\text{trans}}})\\ &\simeq \sum_{j\in\mathcal{R}_i} gT_{\text{trans}}\\ &= g|\mathcal{R}_i|T_{\text{trans}} && (5)\end{aligned}$$

where we have used (3) and approximated $\prod_{j\in\mathcal{R}_i}(1-e^{-gT_{\text{trans}}})$ with $\sum_{j\in\mathcal{R}_i} gT_{\text{trans}}$. The latter simplification holds in low traffic conditions, where $gT_{\text{trans}} << 1$. The notation $|\mathcal{R}_i|$ stands for the number of elements of the set $\mathcal{R}_i$. From (5), the following simplified expressions for network sensing and backoff intensities can then be obtained:

$$\begin{aligned} &F[G_i] \simeq (1+\textstyle\sum_{j\in\mathcal{R}_i} gT_{\text{trans}})g\\ &F[B_i] \simeq (\textstyle\sum_{j\in\mathcal{R}_i} gT_{\text{trans}})g.\end{aligned}$$

In general, the number of transmission errors accumulated by sensor s_i can be written in the following form:

$$F[E_i] = \gamma_i F[G_i] + \lambda_i F[T_i] + \eta_i F[T_i] + \kappa_i F[T_i] \tag{6}$$

where the four terms at the righthand side can be characterized as follows. The term $\gamma_i F[G_i]$ represents the intensity of transmission errors occurred because the channel was occupied by a packet transmission that could not be detected by the i-th sensor during a CCA interval. The term $\lambda_i F[T_i]$ represents the intensity of transmission errors due to interference from other sensors that cannot receive s_i. The term $\eta_i F[T_i]$ represents the intensity of transmission errors made because another sensor can begin transmitting when s_i is waiting the turnaround time between the CCA and the transmission act. Finally, the term $\kappa_i F[T_i]$ represents the intensity of transmission errors due to the fact that other sensors can begin transmitting in the first subinterval, of length T_{TAT}, of a transmission act from sensor s_i. In fact, if some other sensor begins transmitting during the turnaround time, it cannot detect the previous starting instant of a transmission by s_i. The last two terms appearing in (6) take into account transmission errors which are independent of the network connectivity and are significant in the overall network error analysis.

Under the assumption of low traffic load and with the simplification that all relevant processes are Poisson and independent, the coefficient γ_i in (6) can be computed as follows:

$$\begin{aligned} \gamma_i &= \lim_{t\to\infty} \mathcal{P}\{\max_{j\in\overline{\mathcal{R}}_i}\{T_j[t+T_{\text{trans}}]-T_j[T_{\text{trans}}]\}>0\}\\ &= 1-\prod_{j\in\overline{\mathcal{R}}_i}(1-e^{-F[T_j]T_{\text{trans}}})\\ &\simeq \sum_{j\in\overline{\mathcal{R}}_i} F[T_j]T_{\text{trans}} \simeq |\overline{\mathcal{R}}_i|T_{\text{trans}}g.\end{aligned}$$

Similarly, the coefficient λ_i in (6) can be expressed as

$$\begin{aligned} \lambda_i &= \lim_{t\to\infty} \mathcal{P}\{\max_{j\in\overline{\mathcal{T}}_i}\{G_j[t+T_{\text{trans}}]-G_j[T_{\text{trans}}]\}>0\}\\ &= 1-\prod_{j\in\overline{\mathcal{T}}_j}(1-e^{-F[G_j]T_{\text{trans}}})\\ &\simeq \sum_{j\in\overline{\mathcal{T}}_j} F[G_j]T_{\text{trans}} \simeq |\overline{\mathcal{T}}_j|T_{\text{trans}}g.\end{aligned}$$

The coefficient η_i in (6) can be estimated as

$$\begin{aligned}\eta_i &= \lim_{t\to\infty} \mathcal{P}\{\max_{j=1,\dots,N}\{T_j[t+T_{\mathrm{TAT}}]-T_j[t]\}>0\}\\ &= 1-\prod_{j=1,\dots,N}(1-e^{-F[G_i]T_{\mathrm{TAT}}}) \simeq NgT_{\mathrm{TAT}}.\end{aligned}$$

Finally, the coefficient k_i in (6) is given by

$$\begin{aligned}\kappa_i &= \lim_{t\to\infty} \mathcal{P}\{\max_{j=1,\dots,N}\{T_j[t+T_{\mathrm{TAT}}]-T_j[t]\}>0\}\\ &= 1-\prod_{j=1,\dots,N}(1-e^{-F[T_i]T_{\mathrm{TAT}}}) \simeq NgT_{\mathrm{TAT}}.\end{aligned}$$

Using the expressions found above for the coefficients γ_i, λ_i, η_i, and κ_i in (6), the transmission error intensity can be approximated as

$$F[E_i] \simeq \left[(|\overline{\mathcal{T}}_i|+|\overline{\mathcal{R}}_i|)T_{\mathrm{trans}}+2NT_{\mathrm{TAT}}\right]g^2.$$

Therefore, the overall network error intensity can be estimated as follows:

$$\sum_{i=1}^{N} F[E_i] \simeq (2|\bar{A}(\mathcal{S})|T_{\mathrm{trans}}+2N^2T_{\mathrm{TAT}})g^2$$

and the error probability, i.e., the ratio between the overall network error intensity and the generation intensity (given by Ng), becomes

$$\begin{aligned}P_{\mathrm{er}} &= \frac{\sum_{i=1}^{N} F[E_i]}{Ng}\\ &\simeq \left(2\frac{|\bar{A}(\mathcal{S})|}{N^2}T_{\mathrm{trans}}+2T_{\mathrm{TAT}}\right)Ng. \qquad (7)\end{aligned}$$

Expression (7) shows that, under the considered simplifying assumptions, the error probability grows linearly with the network complementary adjacency $|\bar{A}(\mathcal{S})|$.

In the following, we find an estimate of the average network delay. First of all, we remark that if, after the first backoff, the channel is found idle, the total delay is given by

$$D_{\min} = T_{\mathrm{B}_1}+T_{\mathrm{CCA}}+T_{\mathrm{TAT}}+T_{\mathrm{trans}}.$$

This is the minimum average delay that a packet incurs if the channel is found idle at the first transmission attempt. If the channel is found busy, the sensor waits for a backoff time with average T_{B_2}, then senses again the channel. If, at the second transmission attempt the channel is found idle, the overall delay can be expressed as $D_{\min}+D_{\mathrm{BO}}$, where

$$D_{\mathrm{BO}} = T_{\mathrm{B}_2}+T_{\mathrm{CCA}}.$$

Under the low traffic load assumption, the probability of having more than one backoff during a single transmission act is negligible, and, therefore, the average transmission delay can be estimated as

$$\begin{aligned}D_i &= D_{\min}+D_{\mathrm{BO}}\gamma_i = D_{\min}+D_{\mathrm{BO}}\sum_{j\in\mathcal{R}_i}F[T_j]T_{\mathrm{trans}}\\ &\simeq D_{\min}+D_{\mathrm{BO}}|\mathcal{R}_i|gT_{\mathrm{trans}}\end{aligned}$$

where T_{BO} is the average backoff time. The average network delay then can be expressed as

$$\begin{aligned}D &= \frac{\sum_{i=1}^{N} D_i}{N}\\ &\simeq D_{\min}+D_{\mathrm{BO}}\frac{|A(\mathcal{S})|}{N}T_{\mathrm{trans}}g. \qquad (8)\end{aligned}$$

Expression (8) for the delay shows that the network delay depends linearly on the network adjacency.

B. Optimal Transmit Power Allocation

In this subsection, we discuss the following problem:

Problem 1 (PER minimization): Upon the assignment of a total available transmit power P_{tot} for the sensor network $\mathcal{S}$, distribute it among the sensors in the network in order to minimize the packet error probability at the AP.
This problem is equivalent to the one of minimizing the overall transmit power to guarantee a desired packet error probability at the AP.

Under low traffic load assumption, using the estimate (7) on the packet error probability, the solution of Problem 1 is equivalent to the maximization of the adjacency $|A(\mathcal{S})|$ of sensor network $\mathcal{S}$. This fact allows to recast Problem 1 in the following form.

Problem 2 (Network adjacency maximization): Upon the assignment of a total available transmit power P_{tot} for the sensor network $\mathcal{S}$, distribute it among the sensors in the network in order to maximize the network adjacency $|A(\mathcal{S})|$.

Assign to each sensor s_i a transmission power $P_i > 0$, $i = 1,\dots,N$. Then, the network adjacency is given by the following function:

$$\begin{aligned}|A(\mathcal{S})| &= \Gamma(P_1,P_2,\dots,P_N)\\ &\triangleq \sum_{i=1,\dots,N,\ j=1,\dots,N} H(P_i-\Pi(x_j,x_i)) \qquad (9)\end{aligned}$$

where $H(x) = \begin{cases} 1 \text{ if } x\geq 0\\ 0 \text{ otherwise}\end{cases}$ is the Heaviside function.

For each sensor s_i, define $\mathcal{P}_i$ as the following set of transmit power values:

$$\mathcal{P}_i \triangleq \{\Pi(x_j,x_i),\ j=1,\dots,N, \ \text{ with } j\neq i\} \qquad (10)$$

where $\Pi(x_j,x_i)$ is the transmit power with which sensor s_i can reach sensor s_j.

The following property leads to the possibility of limiting the search of possible transmit powers for a sensor s_i to the set $\mathcal{P}_i$.

Proposition 1: For any set of transmit powers $P_i > 0$, $i = 1,\dots,N$, there exists a set of values $\bar{P}_i \in \mathcal{P}_i$, such that

$$\Gamma(\bar{P}_1,\bar{P}_2,\dots,\bar{P}_N) = \Gamma(P_1,P_2,\dots,P_N) \qquad (11)$$

$$\bar{P}_i \leq P_i,\ \forall i=1,\dots,N. \qquad (12)$$

Proposition 1 simply means that, in the ideal threshold detection hypothesis, it is not convenient to allocate to sensor s_i a transmit power that does not belong to the set $\mathcal{P}_i$, since it would employ extra power without gaining extra connectivity.

For instance, in a network composed of 4 sensors, suppose that sensor 1 can reach the AP using a transmit power of 0.5 mW, whereas it needs 1 mW to reach sensor 2, 2 mW to reach sensor 3, and 0.2 mW to reach sensor 4, respectively. In this case, $\mathcal{P}_i = \{0.5\,\mathrm{mW}, 1\,\mathrm{mW}, 2\mathrm{mW}\}$ contains the transmit powers that allow to reach the AP and sensors 2 and 3. The optimal transmit power for the first sensor should be chosen in this set. In fact, for example, it would not be convenient to choose a transmit power equal to 1.5 mW instead of 1 mW because, despite the increased transmit power, the connectivity would be the same (sensor 1 would still reach the AP and sensors 2 and 4)

The power allocation problem can then written in the following form.

Problem 3 (Discrete optimization problem): For each sensor $i = 1, \ldots, N$ choose a transmit power $P_i \in \mathcal{P}_i$ such that the function $\Gamma(P_1, P_2, \ldots, P_N)$ (defined by (9)) is maximized while satisfying the constraint

$$\sum_{i=1}^{N} P_i \leq P_{\mathrm{tot}}.$$

This problem corresponds to a *multiple choice knapsack problem*, which has been extensively studied in the literature [12] and can be solved by standard computational libraries, such as MOSEK [13].

III. Simulation Model

A. Zigbee Standard

The increasing need for low-power consumption applications has led to the creation of low-rate wireless personal networks (LR-WPANs). One of the newest standards for WSNs, with significant power savings, is Zigbee [5]. More precisely, the Zigbee Alliance provides instructions only for the upper layers (i.e., from the third to the seventh layer) of the ISO/OSI stack [14]. At the first layers of the ISO/OSI stack (physical, PHY, and MAC), the Zigbee technology is based on the IEEE 802.15.4 standard [15] and guarantees (theoretically) a maximum transmission data-rate equal to 250 kpbs over a wireless communication link. We remark that, in this paper, we focus only on the first two layers of the ISO/OSI stack (especially on the MAC layer): in this case, the Zigbee standard is equivalent to the IEEE 802.15.4 standard.

The IEEE 802.15.4 standard employs a non-persistent CSMA/CA MAC protocol. We remark that the non-persistent CSMA/CA MAC protocol provides a medium access mechanism which tries to avoid packet collisions. A node, before transmitting a new packet, waits for an interval denoted as backoff interval (BI), randomly chosen within a range defined during the network start-up phase by the backoff exponent (BE) and expressed as a multiple of a reference time interval, referred to as backoff unit and denoted as T_{B}. In particular, the backoff interval is a random variable uniformly distributed in $[0, (2^{BE} - 1) \cdot T_{\mathrm{B}}]$. For the first transmission attempt, the Zigbee standard defines $BE = BE_{\min} = 3$. After the corresponding BI has elapsed, the node tries to send its packet again: if it detects a collision, it doubles the previously chosen maximum waiting interval and selects a new value for BI; if, instead, the channel is free, it transmits its packet. This procedure is repeated for two times, after which, for the subsequent three unsuccessful transmission attempts, BE is kept fixed to $BE_{\max} = 5$. After five unsuccessful retransmission attempts, the packet is dropped. This back-off algorithm makes it likely that a node will eventually manage to transmit its packet.

After the back-off period has expired, a node, before effectively starting the packet transmission, needs to sense the channel in order to assess its status. The Zigbee standard provides a CCA technique which allows a node to sense the channel for a specific time interval, referred to as CCA time. If at least another node transmits during this interval, the channel is declared busy and the node, which was sensing the channel, discards the packet and starts a retransmission. On the opposite, if the channel is available, the node, before transmitting, waits for a TAT period, in order to take into account internal radiofrequency interface recalibration.

B. Considered Opnet Model

The simulations have been carried out with the Modeler package of the Opnet simulator [16] and a built-in Zigbee network model designed at the National Institute of Standards and Technologies (NIST) [17]. We have considered star topologies in scenarios where $N = 20$ nodes transmit directly to the AP. The considered topologies are shown in Fig. 1. More precisely:

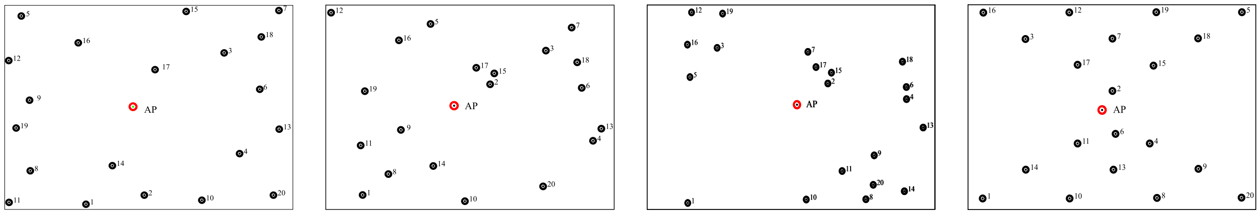
- in Fig. 1 (a), $N = 20$ nodes are randomly deployed over a 100 m^2 square[2] area and are approximately concentrated towards the external perimeter of the surface;
- in Fig. 1 (b), $N = 20$ nodes are deployed over the same surface as before, but present a few clusters and isolated nodes;
- in Fig. 1 (c), $N = 20$ nodes are placed in order to form four small clusters and only one node is isolated from the others;
- in Fig. 1 (d), $N = 20$ nodes are placed over a regular grid and form two " triangular" grids which converge at the AP.

We believe that the considered topologies are representative of a large set of possible WSN topologies. However, we remark that the proposed framework can be applied to a WSN with a generic topology.

The original NIST Zigbee model did not take into account signal attenuation [18]. Therefore, in our simulations we have introduced the channel attenuation according to the Friis propagation model. In particular, the Friis formula is given by equation (1) and, in this paper, we assume $G_{\mathrm{r}} = G_{\mathrm{t}} = 1$ (omnidirectional antennas), $\lambda = 0.125$ m ($f_{\mathrm{c}} = 2.4$ GHz), and $\alpha = 2.1$. In all cases, r is shorter than 100 m, which is the maximum transmission range allowed by the Zigbee standard. If the received power is higher than a pre-defined threshold, fixed to -90 dBm, the nodes can exchange packets.

For each of the considered topologies, the distance between the nodes and, consequently, the power attenuation, is computed offline, on the basis of the coordinates of the nodes. These values are then used to fill the adjacency matrix. In particular, consider a pair of nodes (s_i, s_j) with $i \neq j$: if s_i is sufficiently close to transmit to s_j, we insert a "1" in the corresponding entry of

[2]The width of the side of surface will become meaningful for the typical values of the transmit power considered in the following. Moreover, the maximum transmission range allowed by the Zigbee standard is 100 m.



(a) (b) (c) (d)

Fig. 1. Considered network topologies with $N = 20$ nodes.

Fundamental time unit	4 μs
LIFS	640 μs
CCA time	128 μs
ACK window duration	864 μs
T_{TAT}	192 μs
T_{B}	320 μs
L (packet length)	512 (payload) + 120 (header) bits

TABLE I
PARAMETERS OF THE ZIGBEE STANDARD.

the adjacency matrix (i.e., i-th row and j-th column); otherwise, we mark the absence of communication with a "0." We remark that the communication links may be asymmetric: in other words, even if s_i can communicate with s_j, the opposite may not hold. The distances between the nodes are also used to determine (i) the minimum (per-node) transmit power which allows each node to reach the AP and (ii) the maximum transmit power which guarantees that each node can reach any other node in the network.

The Zigbee standard provides indications about the values of the main network parameters introduced in Section III-A. The values of the relevant parameters for our simulations are shown in Table I. The simulations have been repeated several times with different seed initialization parameters, in order to guarantee that possible statistical fluctuations are avoided.

We remark that the simplified theoretical model presented in Section II is compliant with the simulation model just described.

IV. Performance Analysis

In this section, we present the performance results in the presence of transmit power control. In particular, we focus on the following key performance indicators: the PER and the delay D (dimension: [s]). The delay is defined as the average time interval between transmission and correct reception instants of a data packet.

The simulations have been carried out referring to star topologies, i.e., all nodes transmit directly to the AP, and using different values of the overall network transmit power and, consequently, different values of the transmit powers allocated to the sensors. In particular, we have considered two possible transmit power allocation strategies: (i) each node has the same transmit power and (ii) the transmit power varies from node to node and is allocated using the strategy presented in Subsection II-B. In all cases, the obtained simulation results are directly compared with the results predicted by the theoretical model. In fact, referring to the scenarios shown in Fig. 1, we have set the same transmit power at each node in order to allow each node to reach at most the AP (the per-node transmit power is denoted as $P_{\mathrm{t}}^{\mathrm{min}}$). In the following, we will denote as $\{P_{\mathrm{i}}\}$, $i = 1, \ldots, N$, the transmit powers assigned to the nodes using the proposed power allocation strategies, whereas we will denote the overall available power as P_{tot}. In particular, we will denote as $P_{\mathrm{tot}}^{\mathrm{min}}$ the overall power that guarantees that each node, using a transmit power equal to $P_{\mathrm{t}}^{\mathrm{min}} = P_{\mathrm{tot}}^{\mathrm{min}}/N$, can reach at most the AP.

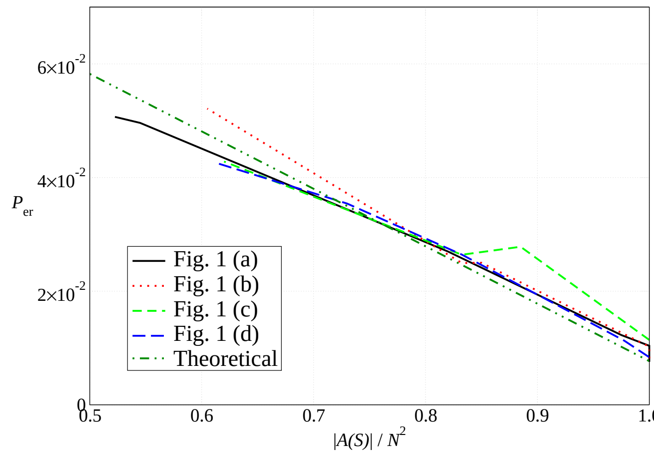


Fig. 2. Packet error rate as a function of the sparsity index of the adjacency matrix. In all considered scenarios, the packet generation rate is set to $g = 1$ pck/s and the number of nodes in the network is $N = 20$.

A. Impact of the Adjacency Matrix

According to the analytical results in Section II, the performance, in terms of PER, depends only on the number of ones in the adjacency matrix, regardless of their specific positions. This emerges clearly from the results shown in Fig. 2, where the PER is shown as a function of $|A(\mathcal{S})|/N^2$, i.e., the sparsity index of the adjacency matrix. In this figure, the performance with the network topologies in Fig. 1 is evaluated. For all scenarios, the packet generation rate is set to $g = 1$ pck/s. In the same figure, the PER predicted by the analytical model, given by the expression in (7), is also shown. As one can see, the simulation curves are very close to the corresponding analytical analytical curve and this is more pronounced for values of $|A(\mathcal{S})|/N^2$ in the proximity of 1. In fact, when the value of $|A(\mathcal{S})|/N^2$ is close to 1, the network is strongly connected and a node can sense any other node. Observing the Opnet log files (not reported in this paper for lack of space) stored by the nodes during the simulations, when $|A(\mathcal{S})|/N^2$ approaches 1 (i.e., the network is fully connected and during a CCA operation each sensor can detect the transmissions of all other sensors), the packets are dropped only during the TAT, when a node cannot sense other active nodes in the network during the transmission of its packets. On the other hand, when the value of $|A(\mathcal{S})|/N^2$

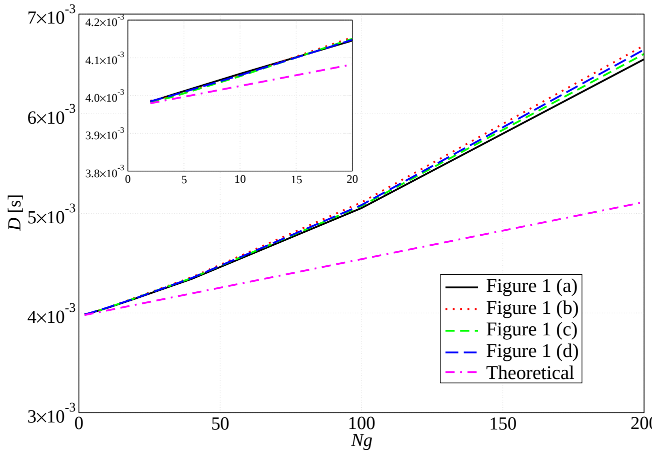


Fig. 3. Delay as a function of the aggregated offered traffic load Ng. Various topologies with $N = 20$ nodes are considered. The sparsity index is set at 0.87.

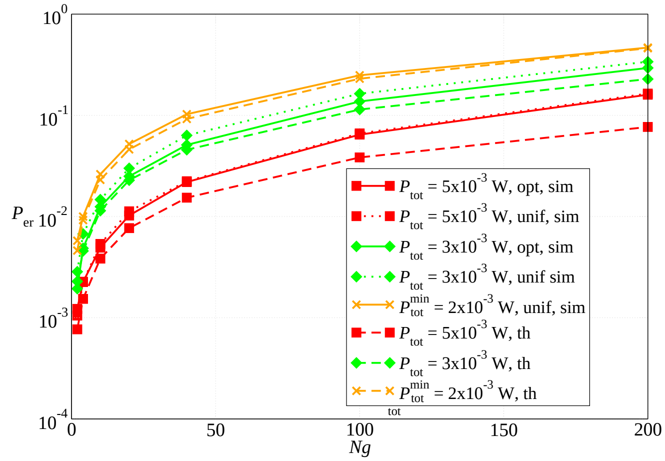


Fig. 4. Packet error rate as a function of the aggregated offered traffic load Ng. Different transmit power allocation strategies are considered for the topology with $N = 20$ nodes presented in Fig. 1 (b): (i) fixed per-node transmit power (unif) and (ii) optimized transmit power (opt). Both simulation (solid lines) and analytical (dashed lines) results are presented. The dotted lines refer to scenarios where no optimized power allocation strategy is used.

decreases, some nodes may become isolated from the other nodes (except for the AP) and may no longer be able to sense them, so that those packets may collide at the AP, leading to an increase of the PER.

Referring to Fig. 2, one can see that the topology of the nodes in the network has a very limited impact on the PER when $|\bar{A}(\mathcal{S})|/N^2$ is close to 1 (the curves basically overlap). When $|\bar{A}(\mathcal{S})|/N^2$ becomes lower, instead, the PER is higher in the scenarios relative to the topologies in Fig. 1 (b). For instance, considering node 12 in Fig. 1 (b), when the transmit power is set to the minimum allowed common value $P_{\mathrm{t}}^{\min}$ required to reach the AP, this node is isolated from most of the remaining nodes in the network. The packets transmitted by this node are likely to collide with those transmitted by nodes which are out of its transmission range, thus degrading the performance in terms of the PER.

The results in Fig. 2 also underline that the performance does not depend on the considered topology. In fact, for $Ng = 20$ pck/s, the good overlap of the curves is due to the proposed transmit power allocation strategy, which minimizes the numbers of collisions between packets transmitted by the nodes. Since the number of backoffs experienced by a node before transmitting is small in low traffic scenarios, there is a good agreement between the analytical and the simulation results. For larger values of Ng, the corresponding results of which are not presented in this paper for lack of space, the agreement is slightly worse. In this case, in fact, it is likely that a node incurs in more than one backoff period before transmitting a packet.

B. Impact of Traffic

As shown in Section II, the performance of a Zigbee WSN, under the assumption of low traffic load, depends only on the number of ones in the adjacency matrix. In Fig. 3, where the delay is shown as a function of the aggregated offered traffic load Ng, the sparsity index $|\bar{A}(\mathcal{S})|/N^2$ is set to 0.87. Considering Fig. 3, there is a good overlap between the simulation curves relative to the topologies presented in Fig. 1. In particular, the number of ones in the adjacency matrix, i.e., the number of active connections, is the only characteristic that affects, for small values of the aggregated offered traffic load, the network performance. When the traffic load increases, however, the assumptions made in Section II do not hold anymore. In this figure, the analytical curve given by the expression in (8) is also shown. In this case as well, the delay predicted by the analytical model is lower than that obtained with simulations. As in the previous case, the impact of the backoff procedure, due to the packet retransmission, has not been taken into account, so that the average delay predicted by our analytical model is lower.

C. Impact of the Power Allocation Strategy

In this section, we present the impact of the proposed transmit power allocation strategy on the performance of Zigbee WSNs. In particular, we consider strategies such that the same transmit power is allocated to each sensor or different transmit powers are associated to the various sensors. In the former case, the transmit power is set in order to allow each sensor either to communicate with any other sensor in the network or to communicate at most with the AP. In the latter case, the transmit power is different at each sensor and is set according to optimization strategy presented in Section II, where the total amount of available transmit power is assigned to each sensor in order to minimize the PER at the AP. This strategy leads to allocate low transmit powers to nodes which are isolated and high transmit powers to nodes which can be connected with a large number of remote nodes. In this way, it is possible to minimize the number of collisions at the AP. Of course, there will still be nodes which cannot sense each other (leading to possible collisions), but this is due to the limited amount of overall network transmit power.

In Fig. 4, the PER is shown as a function of the offered traffic load Ng in the network, for the topology presented in Fig. 1 (b). Different values of overall network transmit power, with the corresponding sparsity indexes of the adjacency matrix shown in Table II, are considered. Under the transmit power allocation strategy presented in Section II, in Fig. 4 a performance comparison between scenarios with and without the use of the

Available power ($P_{\rm tot}$)	Sparsity index
$5 \cdot 10^{-3}$ W	1
$3 \cdot 10^{-3}$ W	0.85
$2 \cdot 10^{-3}$ W ($P_{\rm tot}^{\rm min}$)	0.605

TABLE II
SPARSITY INDEXES OF THE ADJACENCY MATRIX FOR THE SCENARIO PRESENTED IN FIG. 1 (B). DIFFERENT VALUES OF OVERALL NETWORK TRANSMIT POWER ARE CONSIDERED.

proposed transmit power allocation strategy is presented. The overall transmit power available in the network is allocated either assigning a common transmit power to all nodes or using the proposed transmit power allocation strategy. Of course, in the latter case, the sparsity index of the adjacency matrix is maximized according to the available power and, referring to the results presented in Fig. 2, the higher is the sparsity index, the lower is the PER. Comparing the curves referring to $P_{\rm tot} = 5 \cdot 10^{-5}$ W with those referring to $P_{\rm tot} = 3 \cdot 10^{-5}$, the higher number of ones introduced in the adjacency matrix by the proposed transmit power allocation strategy allows to significantly reduce the PER. This confirms that the PER performance depends only on the number of ones in the adjacency matrix. In the other cases, when the number of connections between the nodes decreases, the probability of collisions at the AP increases, since it is likely that one transmitting node cannot sense another transmitting node out of its transmission range. However, the curves have the same trend for all values of offered traffic load. In particular, when Ng is low, it is likely that the number of collisions at the AP is low. Instead, when the traffic load is larger, the probability that two nodes transmit at the same time increases and, subsequently, the PER increases as well. In Fig. 4, the analytical results are shown as dashed lines. These curves are close to those associated with simulation results, especially for scenarios in which the sparsity index is small. Once more, the good agreement between the analytical results and the simulation results is confirmed, thus further validating the analytical model. For the sake of comparison, in Fig. 4 the PER in scenarios where no transmit power allocation strategy is used (dotted lines) is also shown. In these cases, the performance is worse than in the case with the optimized transmit power allocation strategy. In fact, given a value of overall network available power, the proposed power allocation strategy allows to maximize the sparsity index of the adjacency matrix and, therefore, reduce the PER.

V. CONCLUDING REMARKS

In this paper, we have presented an optimized transmit power allocation strategy which allows to minimize the PER at the AP of a WSN. First of all, we have derived a simplified analytical model which describes the performance of a Zigbee WSN, in terms of PER and delay, under the assumption of low offered traffic load. Then, we have presented the proposed optimization-theoretic transmit power control approach, developed under the assumption of finite overall network transmit power. In particular, we have shown that the performance basically depends on the number of ones in the adjacency matrix: this number represents the active connections between the nodes and is thus an index of network connectedness. Our analytical model has been validated through the use of the Opnet simulator, underlying the impact, on relevant network performance indicators (PER and delay), of the sparsity index of the adjacency matrix, the offered traffic load, and the transmit power allocation strategy. In particular, we have verified that the proposed transmit power control approach, by maximizing the sparsity index of the adjacency matrix, allows to minimize the PER for a given total network transmit power. In [19] we extend the analytical framework introduced here and we clearly show the advantages, not presented here due to lack of space, of the proposed power allocation scheme over uniform power allocation and RSSI-based power allocation scheme presented in [20]. In addition, in [19], we take into account the characterization of a WSN in terms of network lifetime.

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