Providing End-to-End Statistical Delay Guarantees in cascades of Earliest Deadline First schedulers

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***Abstract* — This paper proposes an analytical method to evaluate the delay violation probability of traffic flows with statistical Quality-of-Service (QoS) guarantees in an Earliest Deadline First (EDF) scheduler. The statistical QoS targets for each service class are expressed in terms of a delay threshold and delay violation probability. We study both cases isolated nodes and end-to-end paths comprising multiple schedulers. Moreover, we assume that traffic admits a linear variance envelope, therefore, we account for Leaky-Bucket-regulated traffic, for general Markov-Modulated Poisson Process sources and Markov-Modulated Fluid Process sources and, more in general, to the wide class of sources for which the variance of the cumulative generated traffic can be upper bounded by a linear function of time. Under these assumptions, we are able to derive an approximation on delay distributions for each class of the EDF scheduler. Moreover, by exploiting a novel framework for the calculation of statistical end-to-end delay bounds (the bounded variance network calculus) we iterate our formulas, derived for the isolated node, to multi-node paths and, in turn, we provide analytical forms for the end-to-end delay. Numerical investigation shows that our approximations are very close to the simulated values.**

***Index Terms* — End-to-End delay, Markov Modulated Poisson Process, statistical guarantees, network calculus, Earliest Deadline First.**

I. Introduction

The provisioning of Quality-of-Service (QoS) guarantees has become an important and challenging topic in the design of differentiated and high-speed packet networks. One way to accomplish QoS differentiation is to define a number of QoS classes and use a scheduling mechanism to treat these classes differently. A popular scheduling paradigm is Earliest Deadline First (EDF) [1, 2].

An EDF scheduler assigns deadlines to packets arriving at the scheduler and then serves the packets in an increasing order of their assigned deadlines.

In addition, characterization of aggregate multimedia traffic also plays a potential role in controlling of the network performance. In literature, both deterministic and statistical models have been extensively treated [3]. While deterministic services [1] provide a simple model for resource allocation, they are largely conservative [4, 5] and network resources are used inefficiently [6]. On the other hand, statistical services [1] allow to a given fraction of traffic to exceed a fixed delay threshold. With statistical services it is possible to significantly increase the utilization of network resources. In the literature, it is well known that statistical services can achieve much higher efficiency than deterministic services [7][8].

The literature on the provisioning of statistical services [9, 10] is wide, both in single-node and multi-node scenarios and with different service disciplines.

In this paper, we analyze and propose a novel solution for the provisioning of statistical services in single-node and multi-node scenarios, for EDF schedulers, as the optimal solution of this problem is not provided in the literature.

EDF is known to be the optimal scheduling policy for a deterministic service at a single switch, in the sense that it has the largest schedulable region among all scheduling policies [11, 12, 13]. Even though such an optimality of the EDF scheduling has not been proven in terms of statistical QoS guarantees, it has been shown in [14] that an EDF scheduler has a better performance than Generalized Processor Sharing (GPS) scheduler even for a statistical service.

First, we address the single EDF scheduler where flows of the service class i request a statistical QoS are expressed in terms of delay threshold, d_i and delay violation probability, p_i. We calculate the expression of the complementary distribution function of the scheduling delay for all service classes given the statistical description of the input traffic and the output link capacity, C. The second novel contribution of this paper is the extension of our analysis to the cascade of EDF schedulers.

The performance of EDF single-node scheduler have been extensively investigated, in [15] the author derives techniques for bounding the probability of delay violations when the session injections are independent but no closed-form formulas are given. The extension to multi nodes is obtained by inserting appropriate rate controllers between each hop. In [16] and [17] the extension to the multi-node case is possible using reshaping at each node and coordinated schedulers, respectively.

Recently Liebeherr et alii in [18] have provided a new framework, using the network calculus, for the calculation of end-to-end delay in multi-node paths, which provides better results than [15].

In [18] the multi-node analysis is carried out by exploiting the min-plus algebra, a powerful tool of the stochastic network calculus to calculate end-to-end service curves and, in turn, to determine a statistical approximation for the end-to-end delay. Our approach is alternative, as we construct our method starting from the two-moment analysis of isolated nodes, as developed and formalized by Choe and Shroff [20] and, in particular, we use the Maximum Variance Asymptotic upper bound to calculate the distribution of delay at a single node. We iterate our single-node formulas by adopting the methods developed in the Bounded Variance Network Calculus [19]. In this way, starting from the single-node expression of delay, we are able to calculate an approximated analytical expression of the end-to-end delay. With reference to [18], we provide an approximation of end-to-end delay rather than a statistical upper bound. The numerical analysis will show that our approximations of end-to-end delay are closer to the actual network performance than the statistical bounds of [18].

The rest of the paper is organized as follows. In Section II we provide a brief summary of the two-moment analysis. In Section III we develop the analysis of isolated EDF schedulers and the multi-node case is dealt with in Section IV. Section V presents numerical analyses and comparisons. Finally, the paper is concluded in Section VI.

II. Two-Moment Analysis: single and multi nodes

A. *Two-Moment Analysis of Network Delay*

In this work, the Choe's and Shroff's Maximum Variance Asymptotic upper bound [20] is used for the analysis of delay in isolated schedulers. Thus, the analysis falls in the category of the two-moment analyses of network delay, based on an approximated Gaussian model of traffic, driven by the convergence rules of the Central Limit Theorem [21]. Moreover, the analysis uses widely the concepts of statistical traffic and service envelopes, as defined by Qiu and Knightly [22]. The cumulative traffic $X(t)$ generated by a flow in a time interval with duration t is described by its statistical traffic envelope, $B(t)$, which is defined as a random process such that the traffic generated by the flow in a time interval with duration equal to t satisfies the condition: $\forall z,t: \Pr\left(X(t)>z\right)\le \Pr\left(B(t)>z\right)$.

This inequality is commonly referred to as a *statistical inequality* and it is represented alternatively as $X(t)\overset{ST}{\le} B(t)$. The *statistical service envelope* is defined as a probabilistic description of the service that can be offered to a traffic flow at a given node [22]. In order to define the statistical service envelope, the notion of *available service* is needed. Given a multi-class scheduler and the input traffic of class i, $X_i^{\text{in}}(t)$, in a reference time interval [0, t], the available service for $X_i^{\text{in}}(t)$ is referred to as $X_{\text{av},i}^{\text{out}}(t)$ and it is defined as the output of class i, when class i is *minimally backlogged*. At a given time instant, the *backlog* of a service class in a scheduler is the total amount of traffic stored in the corresponding buffer, waiting for transmission. A service class is minimally backlogged in [0, t] if (i) it is continuously backlogged in [0, t] and (ii) the input of class i in [0, t] is the minimal input traffic such that condition (i) holds (this input traffic is referred to as the *minimally backlogged input traffic* and it is denoted as $\tilde{X}_i^{\text{in}}(t)$). The statistical service envelope of class i is the random process $S_i(t)$ such that $S_i(t)\overset{ST}{\le} X_{\text{av},i}^{\text{out}}(t)$.

By using statistical traffic and service envelopes it is possible to exploit statistical multiplexing in order to obtain statistical service guarantees. A commonly used statistical performance target is the maximum probability p of exceeding the delay threshold d:

$$\Pr\left(D>d\right)\le p, \tag{1}$$

where D is the actual delay experienced by traffic in the scheduler. The probability of violating a given delay threshold, d, has the following statistical upper bound [23]:

$$\Pr\left(D>d\right)\le \Pr\left(\max_{t\ge 0}\left(B(t)-S(t+d)\right)>0\right), \tag{2}$$

In general, the calculation of (2) is quite complex. Therefore, the Maximum–Variance Asymptotic upper bound (MVA) is commonly adopted, under the assumption that both $B(t)$ and $S(t)$ are Gaussian. By considering a traffic flow with statistical traffic envelope $B(t)$, statistical service envelope $S(t)$, and a delay threshold d, the MVA defines:

$$\sigma^2(t)=Var\left(B(t)-S(t+d)\right), \tag{3}$$

$$\alpha(t)=\frac{0-E\left(B(t)-S(t+d)\right)}{\sigma(t)}. \tag{4}$$

It can be shown that

$$\Pr\left(D>d\right)\le \Pr\left(\max_{t\ge 0}\left(B(t)-S(t+d)\right)>0\right)\le e^{\frac{-\alpha_{\min}^2}{2}} \tag{5}$$

with $\alpha_{\min}=\min_{t\ge 0}\left(\alpha(t)\right)$.

Since the service envelopes of many important schedulers are known, with (5) it is possible to calculate an upper bound of the probability of violating a given delay threshold, if traffic is Gaussian, or an approximation of end-to-end delay if traffic is non-Gaussian. The quality of the approximation improves as the number of flows grows, under the rules of the Central Limit Theorem, as discussed in [20].

B. *The Bounded Variance Network Calculus*

In order to extend the two-moment analysis to end-to-end paths it is necessary to introduce the concept of bounded variance network calculus, firstly discussed in [19]. One of the hardest problems to be faced by multi-node network analysis is the characterization of the output traffic of a scheduler, an unavoidable task, as the traffic output by a scheduler is the input traffic of the next scheduler.

The bounded-variance network calculus addresses this problem by providing an approximation of the variance of the scheduler's output traffic, in order to be able to iterate the MVA equations (3), (4), (5).

This approximation is based on the novel inequality, proved in [24], on the variance of the minimum of two bivariate Gaussian random variables. This inequality state that, given two bivariate Gaussian random variables x and y, the variance of z=min(x,y) is bounded as

$$\text{var}(z)\le \max\left(\text{var}(x),\text{var}(y)\right). \tag{6}$$

This inequality allows us to bind the variance of the output traffic of a node as a function of the node's input traffic.

In fact, let us consider a tagged flow crossing a sequence of H schedulers, where each scheduler provides Q_h service classes and, in the hth scheduler, the flow is served in service class q_h. By denoting the input and output traffic envelopes of the tagged traffic flow at the hth scheduler of the end-to-end path as $B_{q_h}^{\text{in}}(t)$ and $B_{q_h}^{\text{out}}(t)$, respectively, and by referring to the service envelope of the tagged flow at the hth scheduler as $S_{q_h}(t)$, an approximation of the output traffic envelope of the tagged flow at the hth scheduler is given by:

$$B_{q_h}^{\text{out}}(t) \approx \min\left(B_{q_h}^{\text{in}}(t), S_{q_h}(t)\right). \qquad (7)$$

In fact, if the tagged flow offers more traffic than the amount of service that the scheduler can provide, then the output traffic is equal to the service capacity, otherwise equals the offered traffic. By applying inequality (6) to (7) we find an upper bound for the variance of the output traffic of node h:

$$\text{var}\left(B_{q_h}^{\text{out}}(t)\right) \le \max\left(\text{var}\left(B_{q_h}^{\text{in}}(t)\right), \text{var}\left(S_{q_h}(t)\right)\right). \qquad (8)$$

If traffic is not Gaussian, the proposed variance (8) of output traffic is an approximation, as discussed in [25].

Given a sequence of H nodes crossed by a tagged traffic flow, it is possible to proceed iteratively starting from node 1 and, since $B_{q_{h+1}}^{\text{in}}(t) = B_{q_h}^{\text{out}}(t)$, a statistical characterization of the input traffic of each node can be calculated with (8). Therefore, by applying the MVA, the probability density $f_{d_h}(t)$ of the delay d_h at the hth node is easily derived. Finally, the end-to-end delay, d_{e2e} is equal to $d_1+d_2+\ldots+d_H$, the density of the end-to-end delay is calculated as $f_{d_{e2e}}(t) = f_{d_1}(t) * f_{d_2}(t) * \cdots * f_{d_H}(t)$, by assuming statistical independence of the delay accumulated in different nodes (this assumption is discussed in [25]). The probability that the end-to-end delay of the tagged flow exceeds a threshold d is then calculated as:

$$\Pr\left(d_{e2e} > d\right) = \int_d^\infty f_{d_{e2e}}(\tau)\, d\tau \qquad (9)$$

III. Single EDF Analysis

In this section, we develop the analytical model to evaluate the distribution of delay and to treat the resource allocation and the admission control problems for a single EDF scheduler.

An EDF scheduler serves n queues, where the ith queue has service class i, an associated delay threshold d_i, and delay violation probability p_i. The delay bound, d_i refers to the time elapsing from the instant when traffic enters its queue in the scheduler to the instant when the traffic is transmitted on the output link. The output link has a transmission capacity of C [bit/s].

In an EDF scheduler, every class i is associated with a delay bound δ_i. A class i packet arriving at t is assigned deadline $t+\delta_i$, and the EDF service discipline always selects the packet with the smallest deadline for service.

A. Traffic models

Our analysis is carried out with the hypothesis that traffic has a linear variance envelope. Given a traffic flow with $X(t)$ cumulative arrivals, its two-moment statistical envelope is:

$$E\left(X(t)\right) = rt; \quad \text{var}\left(X(t)\right) \le rbt, \qquad (10)$$

where r is the average rate of the traffic flow and b is a positive real constant. This type of envelope accounts for token-bucket-regulated traffic [26, 27], by setting r equal to the token rate and b equal to the bucket depth. The envelope (10) applies also to the universally known Markov Modulated Poisson Process and Markov Modulated Fluid Process sources and in [28] it has been recently provided a method for the calculation of the r and b parameters of a generic Markov-Modulated source.

Our analytical formulas apply for traffic with a linear variance envelope. However, the basic principles of the Maximum Variance Asymptotic upper bound and of the bounded variance network calculus hold in general, for any arbitrary form of var($X(t)$). For a non linear var($X(t)$) our method holds, but solutions can be very complex.

In the numerical analysis of Section V we will consider the Markov-Modulated On-Off sources selected in [18]. Traffic is a continuous time process driven by a homogeneous two-state Markov chain with transition rate λ [s^{-1}] from the Off state to the On state, while the rate of the inverse transitions is equal to μ [s^{-1}]. In the On state, an individual source transmits at a constant rate P [bit/s]. No arrivals are generated in the Off state.

We denote as $a(t)$ the traffic envelope relative to the fresh arrivals of an individual source. The upper bound of the variance of $a(t)$ is equal to (see [28] to compute easily the variance of the traffic envelope of Markov-Modulated sources)

$$\text{var}\left(a(t)\right) \le 2\frac{\lambda\mu}{(\lambda+\mu)^3} P^2 t. \qquad (11)$$

The average value of $a(t)$ is easily calculated:

$$E\left(a(t)\right) = \frac{\lambda}{(\lambda+\mu)} Pt. \qquad (12)$$

Therefore, the two-moment envelope of $a(t)$ can be written as (10) with r and b given by:

$$r = \frac{\lambda}{(\lambda+\mu)} P; \quad b = 2\frac{\mu}{(\lambda+\mu)^2} P. \qquad (13)$$

B. Resource Allocation in EDF scheduler

We analytically calculate the output link capacity C needed to satisfy the performance constraint, expressed by equation (1), in an EDF scheduler with n priority levels. The service class i transports a traffic aggregate composed of N_i individual flows, each of these traffic flows has a two-moment envelope as specified in (10). For the individual flow of the service class i the r and b parameters are r_i and b_i, respectively. All individual flows are assumed to be mutually independent, therefore, the two-moment envelope of the traffic aggregate $B_i(t)$ feeding the priority i queue has average value and variance given by

$$E\left(B_i(t)\right) = N_i r_i t; \quad \text{var}\left(B_i(t)\right) \le N_i r_i b_i t \qquad (14)$$

In an EDF scheduler the statistical service of the priority i

traffic is given by [22]

$$S_i(t) = \max\left(0, Ct - \sum_{n \neq i} B_n\left(t - \max\left(0, \delta_n - \delta_i\right)\right)\right) \quad (15)$$

Therefore, the average value of the statistical service envelope available for service class *i* can be evaluated as

$$E\left(S_i(t)\right) \approx Ct - \sum_{n \neq i} N_n r_n \left(t - \max\left(0, \delta_n - \delta_i\right)\right) \quad (16)$$

and the variance is equal to:

$$\mathrm{var}\left(S_i(t)\right) \approx \sum_{n \neq i} N_n r_n b_n \left(t - \max\left(0, \delta_n - \delta_i\right)\right) \quad (17)$$

By substituting (14), (16) and (17) in (4) we obtain the expression of the function $\alpha_i(t)$:

$$\alpha_i(t) = \frac{-N_i r_i t + E\left(S_i\left(t + d_i\right)\right)}{\sqrt{N_i r_i b_i t + \mathrm{var}\left(S_i\left(t + d_i\right)\right)}}$$

whose absolute minimum is equal to:

$$\alpha_{i,\min} = \frac{-2\left(\left(\max\left(0; A_i d_i - E_i\right) - C d_i\right)\left(B_i + N_i r_i b_i\right) - \max\left(0; B_i d_i - F_i\right)\left(A_i - C + N_i r_i\right)\right)}{\left(B_i + N_i r_i b_i\right)\sqrt{-\max\left(0; B_i d_i - F_i\right) + \frac{\left(\max\left(0; A_i d_i - E_i\right) - C d_i\right)\left(B_i + N_i r_i b_i\right)}{A_i - C + N_i r_i}}} \quad (18)$$

in (18), the symbolic parameters are defined as follows:

$$A_i = \sum_{n \neq i} N_n r_n \qquad E_i = \sum_{n \neq i} N_n r_n \max\left(0, \delta_n - \delta_i\right)$$

$$B_i = \sum_{n \neq i} N_n r_n b_n \qquad F_i = \sum_{n \neq i} N_n r_n b_n \max\left(0, \delta_n - \delta_i\right)$$

Finally, by substituting (18) in (5) we obtain the delay violation probability of the traffic aggregate with service class *i*:

$$\Pr\left(D_i > d_i\right) \leq \exp\left(\begin{array}{l} -2\left(N_i r_i d_i\left(b_i\left(C - A_i\right) + B_i\right) + E_i\left(B_i + N_i r_i b_i\right) + F_i\left(C - N_i r_i - A_i\right)\right) \cdot \\ \cdot \dfrac{\left(C - N_i r_i - A_i\right)}{\left(B_i + N_i r_i b_i\right)^2} \end{array}\right) \quad (19)$$

In order to satisfy concurrently the QoS targets of all traffic aggregates, (19) must satisfy (1), for all service classes. For example, by applying the performance constraint expressed by (1) to (19), we obtain the capacity C_i, needed to serve N_i traffic flows of served in *i* class by satisfying the QoS requirements expressed by d_i and p_i:

$$C_i \geq \frac{\left(\max\left(0; A_i d_i - E_i\right) + \sqrt{\left(E_i + N_i r_i d_i\right)^2 - 2 \ln p_i \left(F_i + N_i r_i b_i d_i\right)}\right)}{2\left(F_i + N_i r_i b_i d_i\right)} \cdot \left(B_i + N_i r_i b_i\right) + \frac{1}{2}\left(A_i + N_i r_i\right) \quad (20)$$

C. Admission Control in EDF scheduler

By solving (19) for N_i, we determine the number of flows of *i* priority, for each service class, that it is possible to accept on a link of capacity *C*, guaranteeing the QoS targets expressed in terms of delay and delay violation probability (d_i and p_i). The calculation requires a long algebraic manipulation, but it is possible to obtain the closed-form expression of equation (21) for the maximum number of flows in service class *i*, as a function of the number of flows in other service classes $n \neq i$:

$$N_i \leq \frac{1}{r_i\left(2\left(b_i\left(C d_i - \max\left(0; A_i d_i - E_i\right)\right) + \max\left(0; B_i d_i - F\right)_i\right) - b_i^2 \ln p_i\right)} \cdot \left(\begin{array}{l} \left(A_i b_i + B_i - C b_i\right)\left(\max\left(0; A_i d_i - E_i\right) - C d_i\right) + \\ -2\left(\max\left(0; B_i d_i - F_i\right)\right)\left(A_i - C\right) + b_i B_i \ln p_i + \\ + \sqrt{\left(-A_i b_i + B_i + C b_i\right)^2 \left(\begin{array}{l} \left(\max\left(0; A_i d_i - E_i\right) - C d_i\right)^2 \\ +2 \max\left(0; B_i d_i - F_i\right) \ln p_i \end{array}\right)} \end{array}\right) \quad (21)$$

IV. Multi-Node Analysis

In this section, we provide the delay violation probability in cascade of EDF schedulers. We adopt the reference scenario used in [18] with a series of *H* nodes and two traffic classes, as shown in Fig. 1. In this scenario, an aggregate end-to-end flow, composed by multiple individual flows, passes through all schedulers from 1 to *H*. These end-to-end flows are referred to as *through* flows.

Each scheduler serves another aggregate flow, referred to as *cross* flows. Each cross flow traverses one node, interferes with through flows, and then it exits the network. Cross flows offered to different schedulers are distinct and statistically independent. The network is composed by a series of *H* nodes, and each node receives and forwards to the next node in the chain the through traffic flows. Each node acts as an EDF scheduler with two traffic classes 1 and 2. Every class *i* (with *i*=1, 2) packet arriving at *t* is assigned deadline $t + \delta_i$, and the EDF service discipline always selects the packet with the smallest deadline for service.

The cross and through traffic flows are served in 1 and 2 service class, respectively.

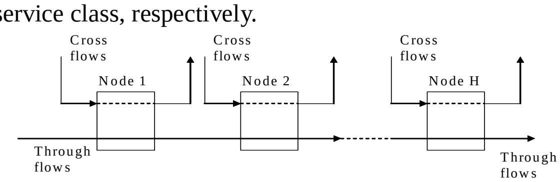

Fig. 1: Network with cross traffic.

We specialize our calculations for the scenario studied in [18], where both through and cross sources are modeled as the Markov-Modulated On-Off sources described in Section III.A. The total number of through flows is equal to N_2 and the number of cross flows that traverse each node is equal to N_1. The parameters of the Markov model of cross and through flows are (λ_1, μ_1, P_1) and (λ_2, μ_2, P_2), respectively.

In order to calculate end-to-end delay of through flow, the service envelope at the generic node *h* is needed. The result is obtained by applying the bounded variance network calculus [19] whose basic principles have been outlined in Section II.B. The traffic envelope of the aggregate input cross flows at the *h*th scheduler is referred to as $B_{1,h}^{in}(t)$ and it evaluates to

$$E\left(B_{1,h}^{in}(t)\right)=N_1 r_1 t;\quad \mathrm{var}\left(B_{1,h}^{in}(t)\right)\le N_1 r_1 b_1 t,$$

where the r_1 and b_1 parameters of the linear traffic envelope of the individual cross flow is calculated with (13). The service envelope of the through flows at the *h*th scheduler, referred to as $S_{2,h}(t)$, is calculated with (16) and (17):

$$E\left(S_{2,h}(t)\right)\approx Ct-N_1 r_1\left(t-\max\left(0,\delta_1-\delta_2\right)\right)$$
$$\mathrm{var}\left(S_{2,h}(t)\right)\approx N_1 r_1 b_1\left(t-\max\left(0,\delta_1-\delta_2\right)\right)$$

We note that if $\delta_1 < \delta_2$ the service envelope of the EDF scheduler becomes equal to that of the SP scheduler, but for $\delta_1 > \delta_2$ it is different. In particular, in this paper we assume $\delta_1 > \delta_2$ and then we are able to substitute the max(0, δ_1 - δ_2) with δ_1 - δ_2. The fresh input through flows at the first node have the traffic envelope

$$E\left(B_{2,1}^{in}(t)\right)=N_2 r_2 t;\quad \mathrm{var}\left(B_{2,1}^{in}(t)\right)\le N_2 r_2 b_2 t,$$

and we proceed with the following conservative approximation

$$E\left(B_{2,1}^{in}(t)\right)=N_2 r_2 t$$
$$\mathrm{var}\left(B_{2,1}^{in}(t)\right)\le \max\left(N_2 r_2 b_2 t,\ N_1 r_1 b_1\left(t-\left(\delta_1-\delta_2\right)\right)\right)$$

that will allow us to derive simple expression of the end-to-end delay. By applying (8), the variance of the traffic envelope of through flows at the input of node 2 is computed as:

$$\begin{aligned}&\mathrm{var}\left(B_{2,2}^{in}(t)\right)=\mathrm{var}\left(B_{2,1}^{out}(t)\right)\le\\ &\max\begin{pmatrix}\max\left(N_2 r_2 b_2 t,\ N_1 r_1 b_1\left(t-\left(\delta_1-\delta_2\right)\right)\right),\\ N_1 r_1 b_1\left(t-\left(\delta_1-\delta_2\right)\right)\end{pmatrix}=\\ &\max\left(N_2 r_2 b_2 t, N_1 r_1 b_1\left(t-\left(\delta_1-\delta_2\right)\right)\right)\end{aligned}\tag{22}$$

By iterating this procedure for the downstream nodes we obtain the bound for the variance of the through traffic at the input of all the EDF schedulers:

$$\forall h\in[1,H]:\mathrm{var}\left(B_{2,h}^{in}(t)\right)\le\max\left(\mathrm{var}\left(B_{2,h-1}^{in}(t)\right),\mathrm{var}\left(S_{2,h-1}(t)\right)\right)$$

Therefore, we can apply (19) to calculate the delay violation probability at the *h*th scheduler:

$$\begin{aligned}&\Pr\left(D_2>d_2\right)\le\\ &\exp\left(-2\frac{\left(C-N_2r_2-A_2\right)}{\left(B_2+N_2r_2b_2\right)^2}\cdot\left(E_2\left(B_2+N_2r_2b_2\right)+F_2\left(C-N_2r_2-A_2\right)\right)\right)\cdot\\ &\cdot\exp\left(-\frac{2N_2r_2\left(C-N_2r_2-A_2\right)\left(b_2\left(C-A_2\right)+B_2\right)}{\left(B_2+N_2r_2b_2\right)^2}\cdot d_2\right)\end{aligned}\tag{23}$$

where

$$A_2=N_1r_1\qquad E_2=N_1r_1\left(\delta_1-\delta_2\right)$$
$$B_2=N_1r_1b_1\qquad F_2=N_1r_1b_1\left(\delta_1-\delta_2\right)$$

To simplify the notation we write (23) as $P\left(D_2>d_2\right)\le\chi_2\exp\left(-\kappa_2\,d_2\right)$ where κ_2 is the coefficient multiplying d_2 in (23) and χ_2 a multiplicative constant. The probability density of delay for the through (class 2) flows at node *h* is equal to $f_{d_{i,h}}(t)=\chi_2\cdot\kappa_2\exp\left(-\kappa_2\,t\right)$ and we find the density of the end-to-end delay as:

$$f_{d_{i,e2e}}(t)=\frac{\left(\chi_2\cdot\kappa_2\right)^H t^{H-1}}{(H-1)!}\exp\left(-\kappa_2\,t\right)\ .\tag{24}$$

The explicit expression of the end-to-end delay bound violation probability is calculated with (9):

$$\Pr\left(d_{i,e2e}>t\right)=\int_t^{\infty}f_{d_{i,e2e}}(\tau)\,d\tau=\chi_2^H\cdot e^{-\kappa_i t}\sum_{j=0}^{H-1}\frac{\left(\kappa_i t\right)^j}{j!}\ .\tag{25}$$

V. Simulation Results

In this section the quality of the approximations of delay obtained with our method are compared with the bounds of the min-plus algebra. Moreover, both results are compared to the actual network performance obtained by simulation.

The computer simulation model of the activity of the EDF scheduler with two queues is simple: then, a cross traffic packet arriving to the scheduler at time *t* is stamped with a deadline $t+\delta_1$, whereas, a though traffic packet is stamped with a deadline $t+\delta_2$. All packets in the scheduler are served by increasing order of their deadline. When a through packet is served, it is passed as input through traffic to the next scheduler, while served cross packets are dropped (see Fig. 1).

In Section V.A, we analyze a multi-node scenario and we compare our approximations of the end-to-end delay with the probabilistic end-to-end delay bound of the min-plus algebra [18] and with the delay measured with simulations. Simulation results are reported together with the width of the 95% confidence intervals. In Section V.B, we analyze and compare the schedulability regions of our approach, of the min-plus algebra and the real region obtained with computer simulation. The parameters of the traffic sources are given in Table 1. The capacity of the output links of all schedulers is equal to 100 Mbit/s. The local deadlines assigned to the service classes 1 and 2 are equal to δ_1 = 0.02 and δ_2 = 0.01 seconds, respectively.

Table 1: Source Traffic Parameters.

Priority	Class i	λ (s^{-1})	μ (s^{-1})	*P* (Mbit/s)	Traffic parameters calculated with (13)	
					r (Mbit/s)	*b* (kbit)
High	1	40	960	5	0.2	9.6
Low	2	80	920	2.5	0.2	4.6

A. Multiple Nodes with Cross Traffic

With reference to Fig. 2, we consider a sequence of 1 and 10 EDF schedulers and we show the delay exceeded with probability 10^{-1}, 10^{-3}, and 10^{-6} in the subplots (a), (b), and (c), respectively. For each value of *H* the Figure shows results obtained with min-plus algebra, our analysis, and simulation.

The comparison of the numerical values of delays shows that the bounded-variance network calculus [19] (in all cases (a), (b), and (c)) provides a significantly better estimation of the end-to-end delay than the min-plus algebra. For example, in (b) with *H*=10 and a total of 450 flows per node (N_1=100 and N_2=350), the simulation results provide a measure of the end-to-end delay in the range 3 ms +/- 0.42 ms, the min-plus algebra provides a delay threshold of 49.83 ms (with a percentage deviation of +1,561% from the simulation result), while our delay approximation with our analysis is equal to 3.73 ms (with a percentage deviation of +24.33% from the simulation result).

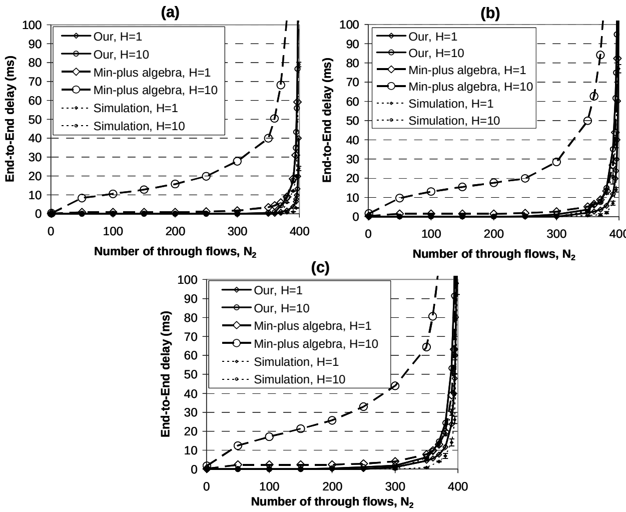

Fig. 2: End-to-end delay exceeded with probability 10^{-1} (**a**), 10^{-3} (**b**), and 10^{-6} (**c**) as a function of the number of though flows per node (N_2) with N_1=100 flows, and 1 and 10 EDF schedulers (δ_1=0.02 s, δ_2=0.01 s).

B. *Single Node scenario: Number of Admission Flows*

We consider an isolated EDF and fix for service class 2 a delay threshold d_2 = 5 ms and a delay violation probability lower than or equal to 10^{-6}. The method to determine the schedulability region of the EDF scheduler, provided in [18], fixes a statistical delay requirement (d_2, p_2) for the flows in service class 2, but no delay targets are established for class-1 flows and their number is let range from 0 up to the theoretical maximum value C/r_1, corresponding to the policy of allocating the average rate and causing an infinite delay in the scheduler. In order to compare our method with the method of [18], we fix a statistical delay target for class-2 flows (d_2, p_2), while we set $d_1 = \infty$ as it is done in [18].

Fig. 3 shows the schedulability regions of the EDF schedulers, i.e., the region of the (N_1, N_2) pairs satisfying the statistical QoS constraint $\Pr(D_2 > d_2) \leq 10^{-6}$. In particular, we plot the schedulability regions with $\delta_2 = d_2$= 5 ms, and deadline service class 1 δ_1 equal to 10, 20 ms, and 50 ms in (a), (b), and (c), respectively. We plot the regions obtained with our method (by applying (21)), with the min-plus algebra and with simulation. We note that increasing progressively the number of class-1 flows, the maximum number of class-2 flows accepted decrease. Moreover, the number of class-2 accepted flows depends by the local deadline of the class-1 flows δ_1. In particular, by increasing the local deadline δ_1, the schedulable region increases, as shown in Fig. 3.

Fig. 3 shows two additional curves, i.e., the average allocation curve and the peak allocation curve. The average allocation curve corresponds to the policy of allocating the average rate to each traffic flow, it is represented by the triangle $N_1 r_1 + N_2 r_2 \leq C$ and, given a pair (N_1, N_2) in the region delimited by the average allocation rule, the QoS constraint $\Pr(D_2 > d_2) \leq 10^{-6}$ is not guaranteed. The Figure also plots the region obtained with the very conservative peak allocation rule, identified by the triangle $N_1 P_1 + N_2 P_2 \leq C$. All the (N_1, N_2) pairs in this region guarantee the QoS target but the link capacity is severely underutilized. From the analysis of the schedulability regions we observe that the match of our analytical curve and simulation is satisfactory. The difference between our method and the min-plus algebra is smaller with one isolated node and it increases as the number of nodes grows.

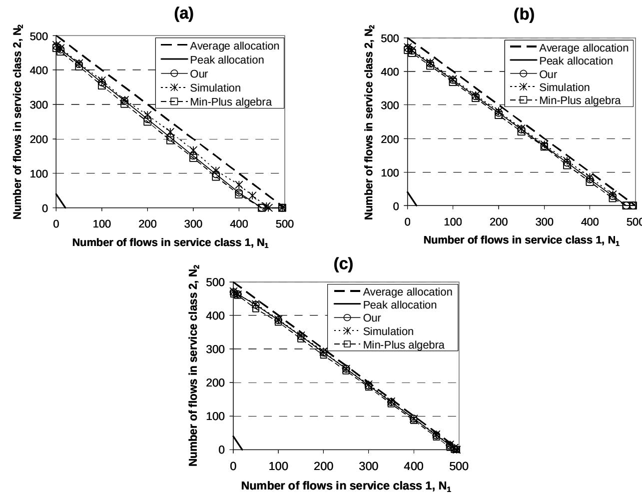

Fig. 3: Isolated EDF scheduler (H=1) with local deadline δ_2=5 ms, delay threshold d_2=5 ms. Number of admissible through flows as a function of the number of cross flows with p_2=10^{-6} and local deadline δ_1 equal to 10 ms (a), 20 ms (b), and 50 ms (c).

VI. Conclusions

In this paper we have provided two main contributions. First, given an isolated EDF scheduler with n service classes, where each class is associated to a differentiated statistical QoS target expressed as a delay threshold d and a delay violation probability p, we have developed a method to calculate in closed-form (a) the complementary distribution function of the scheduling delay, (b) the capacity needed to fulfill jointly all QoS targets given the traffic profiles and number of sources and (c) the solution of the admission control problem, i.e. the identification of the maximum number of flows that it is possible to accommodate in each service class satisfying QoS targets. We have carried out the analysis by assuming that input cumulative traffic has a linear variance envelope. This assumption accounts for token-bucket-regulated traffic, for Markov Modulated Poisson Process sources, and for Markov-Modulated Fluid Process sources.

Secondly we have provided a method (in the framework of our new bounded-variance network calculus) to iterate our analytical formulas in a multi-node scenario, in such a way to calculate analytically the closed-form expression of the end-to-end delay of traffic flows crossing a cascade of EDF schedulers.

We have compared our results both with simulation and with the min-plus algebra, the most recent method to calculate end-to-end delay bounds in both single-node and multi-node paths. As far as multi-node paths are concerned, we have shown that our analytical evaluation of end-to-end delay is much closer to simulation results than the min-plus algebra. Remarkably, our results are expressed as analytical closed forms while numerical optimizations are required by the min–plus algebra.

We have also compared our methods with the min-plus algebra in a single-node scenario, by solving the admission control problem with both methods. The result of the comparison is that our method provides results closer to simulative measures than the min-plus algebra. However, the difference between the two methods is smaller than in the multi-node case.

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