# Modelling Network Selection and Resource Allocation in Wireless Access Networks with Non-Cooperative Games

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***Abstract*—In future generation wireless access networks, the users will have the chance of choosing among multiple connectivity opportunities provided by different access networks (*network selection problem*). Moreover, the network operators themselves will have to implement effective *resource allocation strategies* taking *wise* decisions on the used technologies, frequencies, power levels, etc.**

**This paper proposes a game-theoretic framework to model the problems of network selection and resource allocation, capturing the interdependencies of decisions taken by different players (users vs networks). Namely, we cast the problem as a non-cooperative game where users and access networks act selfishly according their specific objectives: maximization of the perceived quality of service for the end users, maximization of the number of customers for the access networks. We characterize the equilibria of the game by resorting to mathematical programming, and we derive numerical results to assess the "quality" of the equilibria.**

## I. Introduction

The recent achievements in the field of wireless communications have boosted development and diffusion of many different wireless networking systems. Cellular second-generation (2G) systems, like GSM, are nowadays widely deployed aside with third-generation ones (UMTS, cdma2000) providing almost worldwide connectivity to voice and internet services. Moreover, other wireless access technologies have grown in parallel; for example, Wireless Local Area Networks (WLAN) based on the IEEE 802.11 are massively deployed to provide hot-spot coverage in indoor scenarios, like cafes, restaurants, conference rooms, hospitals, airports, office buildings etc. Finally, Wireless Mesh Networking (WMN) solutions have gained popularity in the last years as promising and cost effective infrastructures for providing wireless connectivity to mobile users on geographical scale.

The proliferation of wireless access technologies, and the evolution of the end-user terminals (smart phones, PDAs, etc…) are leading fast towards a ubiquitous, pervasive and rich connectivity offer, such that the end users won't be only always covered/connected, but also always connected/covered by multiple access networks/technologies [1]. Such process will be further sped up by the diffusion of spectrum agile, cognitive radios devices[1], able to dynamically adapt transmission parameters (frequency, modulation, power, etc.…) in an opportunistic way depending on the quality of the current connectivity opportunities [2].

This scenario, at the same time, creates new opportunities and poses novel challenges at different networking levels. On the network's side, there exists a *resource allocation* problem dealing with the development of effective strategies to allocate and dynamically manage the radio resources when different networks operated by different and potentially competing actors coexist; moreover, effective network solutions to handle vertical-handover of the end users must be introduced. On the end-user's side, the main challenge deals with *network selection*, that is, the development of strategies to automatically select the "best" connectivity opportunity to match the user Quality Of Service (QoS) constraints.

A common approach in the published literature is to handle the problems of network resource allocation and user network selection separately. Since however the two problems are strictly related, in this paper we consider them jointly and propose a game-theoretic framework to assess the performance of strategies for network selection (user's side) and resource allocation (network's side). We model the problem as a non-cooperative game where end users and access networks act selfishly according their specific objectives: maximization of the perceived quality of service for the end users, and maximization of the number of customers for the access network operators. We characterize the equilibria of the game by resorting to mathematical programming models, and we thoroughly comment on the quality of the game equilibria through numerical results.

The paper is organized as follows: in Section II, we overview previously published work in the fields of resource allocation and network selection, highlighting the novelty of the proposed contribution. Section III defines the reference scenario and qualitatively describes the problems tackled throughout the paper. In Section IV, we introduce the Interference-based Network Selection Game (INSG), comment on its properties and propose a method to determine and characterize its Nash equilibria. Section V formalizes the joint Network Selection and Resource Allocation Game (NSRAG) and introduces an effective algorithm to obtain the equilibria

[1] Both in end user and network devices

of the game for network instances of reasonable size. Concluding remarks and comments on ongoing related activities are reported in Section VI.

## II. Related Work

Resource allocation in wireless access networks and network selection are classical problems widely addressed by the research community with different goals and methodologies.

Generally speaking, the problem of allocating resources in wireless access networks deals with the configuration of several parameters of the specific radio technology used for providing access. Referring to homogeneous access scenario, 2G systems require effective strategies to plan the use of different carrier frequencies throughout different cells in order to minimize inter-cell interference; a similar frequency allocation problem (with different requirements and solutions) does exist in WiFi-based WLANs [3] and Wireless Mesh Networks [4], whereas 3G systems call for code assignment and power control functionalities [5]. The very same resource allocation problem is present in heterogeneous wireless access networks, where the resource allocation strategies within different coexisting technologies may be strictly dependent and correlated [6].

In such heterogeneous environments, the resource allocation on the network side is strictly coupled with the network selection problem at the user's side, which arises in those cases where the end users can choose among multiple connectivity opportunities provided by the same access network (operator) or by different ones. Such selection opportunity has impacts both on the end user equipment, which may be geared with some tools for automatically optimizing the selection, and on the networks which must be able to support the user mobility between different technologies. The mobility of the users within or among different types of networks is often referred to as vertical handoff [14].

The two scenarios/problems highlighted above are often tackled separately. In the field of network selection, references [11] and [12] study different metrics to measure the QoS level of the end users, which can be consequently used to drive the selection phase. Song and Jamalipour [10] address the problem of network selection resorting to mathematical modelling and computing techniques. Namely, the authors propose to use Grey Relational Analysis and Analytic Hierarchy Processing to determine the utility related to different selection choices.

Besides the general pieces of work on the definition of users utility, there has been a good deal of research on the development of practical protocols for network selection for specific access technologies. Lee and Miller address in [16] the problem of selecting among several 802.11-based access points, by proposing an effective solution to distribute roaming information to the end users, which can be used to discriminate in the selection phase. The same scenario with multiple 802.11 access points is considered in [13], where the authors study the load balancing among the different access points by steering the end user decisions while accounting both for user preferences and network context. Bernaschi *et al.* [15] propose a vertical handover protocol to handle the user mobility between WLAN and cellular systems, and the proposed protocol is thoroughly evaluated through simulation.

In general, all the aforementioned pieces of work tackle the problems of network selection and resource allocation by proposing protocols and/or algorithms for specific network scenarios (technologies, systems, etc.…), and resorting to simulation and prototyping in the performance evaluation phase. On the other hand, the aim of this paper is to gather general insights of the joint problem of network selection and resource allocation through a game theoretic approach. Game theory has been widely used to model resource allocation problems in wireless networks, since it provides powerful modelling tools to capture the dynamics and the equilibria of multi-agents situations [2].

In this field, Niyato and Hossain propose in [7] a game-theoretic approach for studying bandwidth allocation in heterogeneous wireless networks. Different from our work, the focus is on resource allocation only, and the problem is casted as a bankruptcy game where different networks form a coalition to provide bandwidth to the end users. Game theoretic tools (the concept of game core and Shapely value) are used to determine the quality of the bandwidth allocation. A non-cooperative game model is used by the same authors in [9] and [8]. In [9] the focus is on the problem of bandwidth allocation in 802.16-like networks, whereas, reference [8] introduces a non-cooperative game to model the interactions of different access networks (WLAN, cellular systems and WMAN). In this valuable piece of work, the authors derive both long-term and short-term criteria to allocate bandwidth within different technologies to incoming users. Different from our work, the aforementioned manuscripts focus on the resource allocation, only. A similar non-cooperative scenario in the field of resource allocation is addressed in [17] and [18].

## III. Reference Scenario

The reference network scenario considered in this work is composed of a set of wireless Access Networks (ANs) $\mathscr{N} = \{1,\dots,n\}$, and a set of end users $\mathscr{U} = \{1,\dots,m\}$. As a first step of analysis we further assume an homogeneous network scenario where all the access networks feature the same nominal offered bandwidth. This may well represent the case when multiple WiFi hot spots geared with the same technology cover a given area (e.g., in airports, railway stations, community neighborhood, etc…). Each access network is assigned specific radio resources[3] chosen within a given set $\mathscr{F} = \{1,\dots,Q\}$. Without loss of generality, we assume hereafter that these radio resources are non overlapping frequency channels. Each user can select among different covering access networks.

Coverage is defined in the following way: let $P^i_{tx}$ be the power transmitted by AN $i$; user $j$ is covered by AN $i$ if the following inequality is verified:

$$P^i_j = \alpha P^i_{tx} d^{\eta}_{ij} 10^{\frac{\varepsilon}{10}} \geq P_{th}, \tag{1}$$

[2]Competing/collaborating network operators, etc…

[3]E.g., frequency channels

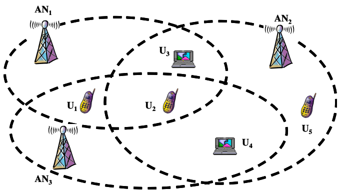

Fig. 1. *Reference Network Scenario.*

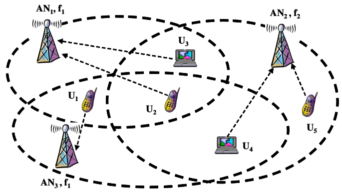

Fig. 2. *Reference Network Scenario.*

where $d_{ij}$ is the distance between position $i$ and position $j$, $\eta$ the attenuation factor and $10^{\frac{\varepsilon}{10}}$ accounts for the loss due to slow shadowing, being $\varepsilon$ a normal variate with zero mean and $\sigma^2$ variance. $P_{th}$ is a threshold value for the required received power.

In order to describe the problem formally we introduce the following notation:

- $U_j \subseteq \mathscr{U}$ denotes the set of end users covered by access network $j$ ($j \in \mathscr{N}$).
- $N_i \subseteq \mathscr{N}$ denotes the set of access networks covering a given end user $i$ ($i \in \mathscr{U}$).

The subsets $U_j$ can be obtained through an accurate site survey by evaluating the power received by each end user from all the access networks through Eq. (1), while subsets $N_i$ can then be derived from the simple rule $i \in U_j \leftrightarrow j \in N_i$.

Referring to Figure 1, we can now cast two different problems:

**Network Selection, NS** The first problem we tackle is pure network selection, where radio resources are statically assigned to the access networks. In such scenarios, the end-users can potentially take wise decisions on which access network to connect to on the basis of several merit functions including the current load of the network and the cost-for-connectivity. In this case, the user decision problem can be modelled as a non-cooperative game in which the players are the end users, the feasible strategies are the available access networks, and the pay-off is the aforementioned merit factor (load and/or cost). The game is non-cooperative since it is reasonable to assume that each end user adopts a strategy which maximizes her own pay-off, regardless of the status of the other users. In Section IV, we formalize and solve the network selection game where the payoff of each user depends on the number of interfering stations in the chosen access network.

**Network Selection and Resource Allocation, NSRA** The radio resource allocation is not pre-determined, as in the problem of pure network selection, but the access networks can decide a specific radio resource allocation strategy to maximize their own utility. Such utility depends, in turn, on the specific decision of the end users on where to associate. Therefore, in Section V, we model this scenario as a bi-level stage game where the payoffs for the resource allocation game played by the access networks depend on the outcome of a nested Network Selection Game played at the lower level by the end users.

## IV. Interference Based Network Selection Game

To formally define the Interference based Network Selection Game (INSG) we introduce the following definitions:

**Definition** The *strategy set* of user $i$, $\mathscr{S}_u^i$, (with $i \in \mathscr{U}$) is the set of access networks which can be selected by user $i$.

Referring to Figure 1, the *strategy set* of the users are defined as:

$$\mathscr{S}_u^1 = \{AN_1, AN_3\} \quad \mathscr{S}_u^2 = \{AN_1, AN_2, AN_3\}$$
$$\mathscr{S}_u^3 = \{AN_1, AN_2\} \quad \mathscr{S}_u^4 = \{AN_2, AN_3\} \; \mathscr{S}_u^5 = \{AN_2\}$$

**Definition** The *strategy space* $\mathscr{S}_u$ is the set of all the possible combinations of strategies played by the users, that is

$$\mathscr{S}_u = \mathscr{S}_u^1 x \mathscr{S}_u^2 x \dots x \mathscr{S}_u^m$$

**Definition** An element $S_u \in \mathscr{S}_u$, $S_u$=$(S_u^1, S_u^2, \dots, S_u^m)$ with $S_u^1 \in \mathscr{S}_u^1, \dots, S_u^m \in \mathscr{S}_u^m$, is a *strategy profile* of the game.

Each strategy profile induces a cost associated to the end users. In particular, we assume here that such cost is proportional to the interference perceived by the end user in the access phase.

**Definition** The *interference level* of user $i$ in the strategy profile $S_u$, is the total number of end users covered by the same ANs covering $i$, and using the same radio resources, that is,

$$n_i(S_u) = \left| U_{\bar{N}_i} \right|, \tag{2}$$

being $U_{\bar{N}_i}$ the set of users covered by all the ANs covering user $i$ and using the frequency user $i$ is using.

If the association pattern is the one of Figure 2, the number of interferers per users is the following: $n_{U_1}(S_u) = n_{U_2}(S_u) = n_{U_3}(S_u) = 2$, and $n_{U_4}(S_u) = n_{U_5}(S_u) = 1$. Note that $U_1$ interferes with $U_2$ and $U_3$ even if it is associated to a different AN because it uses the same radio resources (f1) as $U_2$ and $U_3$.

With this interference level definition we are assuming the user quality of service to be a function of interference perceived by the user itself. This is quite common when modelling problem of wireless access, and has the advantage to provide tractable but consistent expression of the user perceived saturation throughput [3].

The INSG can now be formally defined as:

$$INSG = \langle \mathscr{U}, \mathscr{N}, \mathscr{S}_u, \{n_i(S_u)\}_{i \in \mathscr{U}} \rangle. \tag{3}$$

By definition, the generic end user $i$ selfishly plays the strategy $\bar{S_u^i}$ which minimizes her experienced interference level, that is:

$$\bar{S_u^i} = argmin_{S_u^i} n_i(S_u) \qquad S_u^i \in \mathscr{S}_u^i \tag{4}$$

It is easy to see that the INSG belongs to the class of *congestion games* [20], [21]. Congestion games are typical of those situations where a finite number of resources (routing paths, servers, processing utilities, etc. . . ) can be shared among multiple users, the strategy of each user being the set of resources the user can choose. The payoff/cost of each user depends on the number of the other users choosing the very same resource (congestion situation). In the INSG, the common resources are the ANs and the users' payoffs are the interference levels which depend on the *congestion* level of the chosen access network[4]. It can be further observed that INSG is a particular type of congestion game known as *crowding game* [22]. Crowding games are single-choice congestion games where the payoff is player-specific. From the consideration above, we can state the following:

*Theorem 4.1:* The INSG admits at least one pure-strategy *Nash* Equilibrium (NE).

*Proof:* Proof comes directly from the consideration that every congestion game with player specific pay-off function always admits a pure-strategy NE (see Theorem (1) in [23]). ■

The concept of Nash equilibrium is widely adopted in game theory to characterize the stability of games. Indeed, a Nash equilibrium is a status of the game where each player has not any incentive in deviating from the played strategy unilaterally [19]. However, provided that a game admits Nash equilibria, it becomes fundamental to find such equilibria and characterize them. In fact, a game can possess several equilibria of different "quality". In the next section we comment on the "quality" of the Nash equilibria of the INSG, and we provide an operational method to find them.

### A. Characterizing INSG Equilibria

We derive hereafter a mathematical programming formulation of the INSG, which can be used to find and characterize different Nash equilibria of the game. Namely, we design a Integer Linear Programming (ILP) formulation of an admissibility problem whose constraints enforce equilibria situations.

To this end, we introduce the following parameters to represent coverage and interference among users:

$$c_{im} = \begin{cases} 1 & \text{if user } i \text{ can choose AN } m \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$c_{im}$ is equal to 1 if Eq. (1) holds true for user $i$ and access network $m$.

The interference perceived by the users depends on the specific resource allocation among the access networks. Thus, we define such resource allocation through the following parameters:

$$d_{mk} = \begin{cases} 1 & \text{if AN } m \text{ works on frequency } k \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$a_{ik} = \begin{cases} 1 & \text{if user } i \text{ can associate to frequency } k \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

$$b_{ijk} = \begin{cases} 1 & \text{if users } i \text{ and } j \text{ may interfere on frequency } k \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Binary decision variables are used to define the association of the users to specific ANs, and to specific frequency channels, consequently. Namely, we introduce:

$$y_{im} = \begin{cases} 1 & \text{if user } i \text{ chooses AN } m \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

$$x_{ik} = \begin{cases} 1 & \text{if user } i \text{ is associated to frequency } k \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

We can now state the admissibility problem $P$ whose constraints are:

$$\sum_{k \in \mathscr{F}} x_{ik} = 1 \qquad \forall\ i \in \mathscr{U} \tag{11}$$

$$\sum_{m \in \mathscr{N}} y_{im} = 1 \qquad \forall\ i \in \mathscr{U} \tag{12}$$

$$x_{ik} \leq a_{ik} \qquad \forall\ i \in \mathscr{U}, k \in \mathscr{F} \tag{13}$$

$$y_{im} \leq c_{im} \qquad \forall\ i \in \mathscr{U}, m \in \mathscr{N} \tag{14}$$

$$y_{im} d_{mk} \leq x_{ik} \qquad \forall\ i \in \mathscr{U}, m \in \mathscr{N}, k \in \mathscr{F} \tag{15}$$

$$M(x_{ik} + a_{il} - 2) + \sum_{j \in \mathscr{U}, j \neq i} x_{jk} b_{ijk} \leq \sum_{j \in \mathscr{U}, j \neq i} x_{jl} b_{ijl} \tag{16}$$
$$\forall\ i \in \mathscr{U},\ k \in \mathscr{F},\ l \in \mathscr{F}$$

Constraints (11) and (12) ensure that each user chooses only one frequency, and one access network, respectively. Constraints (13), (14), and (15) guarantee the feasibility of the association. Constraints (16) force each user to choose the strategy (access network) which leads to the minimum interference condition, that is, they ensure that if the single user unilaterally changes her strategy, the change does not improve her own payoff[5], which is the definition of Nash equilibrium. In fact, let us consider the following expression:

$$\overbrace{\sum_{j \in \mathscr{U}, j \neq i} x_{jk} b_{ijk}}^{(1)} \leq \overbrace{\sum_{j \in \mathscr{U}, j \neq i} x_{jl} b_{ijl}}^{(2)} \tag{17}$$
$$\forall\ i \in \mathscr{U}, k \in \mathscr{F}, l \in \mathscr{F} \qquad \text{if } x_{ik} = 1 \text{ and } a_{il} = 1$$

where (1) represents the number of users that interfere with $i$ when she chooses frequency $k$, while (2) represents the number of users that interfere with $i$ when she chooses frequency $l$. User (player) $i$ chooses frequency (strategy) $k$, $x_{ik} = 1$, if and only if there is not any other feasible frequency $l$, $a_{il} = 1$, that provides lower interference. The factor $M(x_{ik} + a_{il} - 2)$ on the left hand side of the inequality is used to activate the constraint (16) only when $x_{ik} = 1$ and $a_{il} = 1$.

We can now state the following:

*Proposition 4.2:* Any feasible solution of the admissibility problem $P$ is a Nash Equilibrium (NE) for INSG.

*Proof:* The proof holds by construction of the problem $P$. ■ ■

As previously stated, Nash equilibria can be multiple and of different quality. One of the most used concepts in game theory to classify equilibria is Pareto optimality. An equilibrium identified by a given strategy profile is said to be Pareto optimal if does not exist any other strategy profile which leads to an enhancement in the payoffs of all the users [19].

To find among the different NE those which are Pareto optimal, we introduce the concept of equilibrium *efficiency* defined as the average number of interferers per user. Then,

[4]Number of users choosing the same or interfering ANs

[5]the interference level is non-decreasing

we introduce an optimization version $P_1$ of the admissibility problem $P$ which features the following objective function:

$$f_{opt}(X) = min \sum_{i \in \mathscr{U}} \sum_{k \in \mathscr{F}} \sum_{j \in \mathscr{U}, j \neq i} x_{ik} b_{ijk} x_{jk} \quad (18)$$

The non-linear objective function in Eq. (18) can be easily linearized with standard techniques to get back to a linear formulation of $P_1$. Details on how to linearize Eq. (18) are skipped for the sake of brevity.

The following result holds.

*Theorem 4.3:* Any solution of the optimization problem $P_1$ is a Pareto-optimal Nash Equilibrium for INSG.

*Proof:* We prove the theorem by contradiction and start off from the following proposition: $X^*$ is a solution to $P_1$, and the NE identified by $X^*$ is not Pareto-optimal. If such proposition is true, it must exist another solution to $P_1$, $\bar{X}$, which gives lower interference for at least one player but at most the same interference for every other player, that is:

$$n_i(\bar{X}) < n_i(X^*) \qquad \text{for at least one i} \in \mathscr{U}$$

$$n_j(\bar{X}) \leq n_j(X^*) \qquad \text{for} j \in \mathscr{U} \text{and j} \neq i$$

. Then it follows that:

$$\sum_{i \in \mathscr{U}} \sum_{k \in \mathscr{F}} \sum_{j \in \mathscr{U}, j \neq i} \bar{x_{ik}} b_{ijk} \bar{x_{jk}} < \sum_{i \in \mathscr{U}} \sum_{k \in \mathscr{F}} \sum_{j \in \mathscr{U}, j \neq i} x^*_{ik} b_{ijk} x^*_{jk},$$

which means that $X^*$ is not a solution to $P_1$, contradicting the starting proposition. ■ ■

Besides determining the optimal equilibria, it is interesting to derive and characterize the worst equilibria, also. To this end, we define a further optimization model with the same constraints as $P$, but with the following objective function aiming at maximizing the total (average) number of interferers.

$$f_{worst}(X) = max \sum_{i \in \mathscr{U}} \sum_{k \in \mathscr{F}} \sum_{j \in \mathscr{U}, j \neq i} x_{ik} b_{ijk} x_{jk}, \quad (19)$$

Let $P_2$ be this new optimization problem. Any solution of $P2$ is a NE (see Proposition 4.2) in which the average number of interferers per end users is maximized. The ratio:

$$\gamma = \frac{f_{opt}}{f_{worst}} \quad (20)$$

provides a quality measure of the equilibria space of INSG. Roughly speaking, the closer to 1 $\gamma$ is, the more "similar" [6] are the Nash equilibria.

### B. Numerical Results

In order to test the quality of INSG equilibria, we have implemented an instance generator able to create synthetic instances representing multi-access network scenarios. The software takes as input the following parameters: the edge of the square area to be simulated ($L$), the number of end users ($m$), the number of access networks ($n$), and the coverage range of each access network, expressed in meters ($r$). In a basic set of instances, each network is assumed to have a circular coverage region with radius $r$.

[6] with respect to the defined efficiency

TABLE I
*Quality of INSG Nash equilibria when varying the number of end users (m) compared to random and best-received association policies. Uniform network scenario with $n$ =10 access networks, $L$ =500m, and $r$ =50m.*

| | INSG NE (Best/Worst) | RANDOM | BEST RECEIVED | $\gamma$ |
|---|---|---|---|---|
| **m=100** | 9.78-9.786 | 9.948 | 10.022 | 0.9993 |
| **m=120** | 11.813-11.823 | 12.02 | 12.025 | 0.9991 |
| **m=140** | 13.903-13.911 | 14.125 | 14.118 | 0.9993 |
| **m=160** | 16.068-16.072 | 16.313 | 16.28 | 0.9997 |
| **m=180** | 17.94-17.944 | 18.189 | 18.141 | 0.9997 |
| **m=200** | 20.053-20.056 | 20.3 | 20.3 | 0.9998 |

TABLE II
*Quality of INSG Nash equilibria when varying the number of access networks (n) compared to random and best-received association policies. Uniform network scenario with $m$ =100 users, $L$ =500m, and $r$ =50m.*

| | INSG NE (Best/Worst) | RANDOM | BEST RECEIVED | $\gamma$ |
|---|---|---|---|---|
| **n=10** | 10.432/10.436 | 10.691 | 10.74 | 0.99962 |
| **n=15** | 7.918/7.93 | 8.732 | 8.74000 | 0.99849 |
| **n=20** | 5.754/5.786 | 7.1592 | 7.328 | 0.99447 |

According to the above parameters, the generating tool randomly draws the positions of the $n$ access networks and of the $m$ end users. In order to generate feasible instances only (i.e., for which all the end users can be covered), the generator does the following: firstly, the positions of the access networks are randomly generated within the simulated area, and secondly, end users are randomly located in the area covered by at least one AN.

All the results reported in the remainder have been obtained formalizing the ILP problems $P$, $P_1$, and $P_2$ in AMPL [24] and solving them with CPLEX commercial solver [25]. Unless differently specified, the reported results are average values on 100 randomly generated instances.

Table I reports the results obtained on a uniform topology with $n$ =10 randomly deployed access networks, when varying the number of end users ($m$), in case $L$ =500 meters, and $r = 50$ meters. The average number of interferers is reported for Pareto optimal and non optimal Nash equilibria (NE) of the INSG, compared to those cases where the end users choose randomly the access network to associate to, and where the end users choose the closest[7] network.

The main result coming from this analysis is that the NE of the INSG are almost equivalent from the efficiency point of view. In fact, the ratio $\gamma$ between the average number of interferers in the best NE and the one in the worst NE is very close to one. Moreover, it is also worth noting that there is a slight difference in terms of efficiency between the three network selection policies. Namely, random network selection and best received network selection lead to higher number of interferers per user on average.

Table II shows similar results as those in Table I when varying the number of access networks available in the area. The ratio between best and worst equilibria remains close to unity, even if a slight decrease can be observed as the number of access networks increases, i.e., the strategy space gets bigger. Furthermore, it is interesting to note that as the number of access networks increases, the difference between the three assignment policies becomes more relevant. Figure

[7] Highest received power on the beacon messages

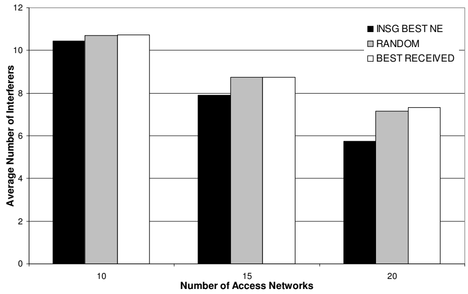


Fig. 3. *Average number of interferers when varying the number of available access networks under different network selection policies. Specific network scenario with m =100 users, L =500m, and r =50m.*

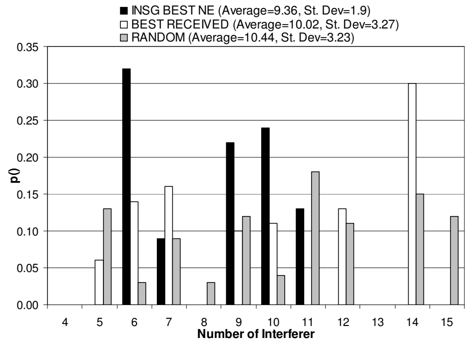


Fig. 4. *Probability Density Function of the number of per-user interferers, when varying the network selection policy. (n =10 access networks, m =100 users, L =500m, and r =50m.)*

3 zooms on this effect by showing the average number of interferers at the equilibria of the INSG and when applying random and best-received assignment policies.

Besides the difference in the average values of interferers, it is also worth getting some insights in the distribution of per users interferers, under the three network selection policies. To this end, Figure 4 reports the p.d.f. of the number of interferers for a specific network scenario with $n = 10$ access networks and $m = 100$, in case $L = 500$ meters, and $r = 50$ meters. The average number of interferers, as well as the corresponding standard deviation are also reported in the legend. As clear from the figure, the Nash equilibrium of the INSG is characterized by a more compact p.d.f. of the number of interferers (lower standard deviation), which means that the selfish behavior of the users trying to minimize their experienced interference leads globally to a fair situation, where the interference status of the single users is similar.

To conclude the analysis on the INSG, we report in Table III the number of strategies, the number of Nash equilibria (total and optimum) and the solution time in case of $n = 10$ access networks and different number of users. We also report the time needed to enumerate all the equilibria. Obviously, the strategy space gets bigger as the number of users increases, as well as the solution time to get a single equilibrium. Moreover, each instance is characterized by a large number of equivalent equilibria.

## V. Network Selection and Resource Allocation

The focus of previous section was on the users' side. Hereafter, we drop the assumption of a fixed frequency assignment and let the single access network take part in the game. Namely, whilst the end users keep playing to minimize their experience congestion (that is, interference), each access network plays the frequency assignment strategy which maximizes its own revenues. Referring back to the definitions introduced in Section IV, we can extend the concepts of *strategy set*, *strategy space* and *strategy profile* to the access networks.

TABLE III

*Cardinality of the strategy space, number of equilibria and solution time in case of a sample network scenario; (n =10 ANs, L =500m, and r =50m).*

| | # Strategies | # Equilibria | # Optimal Equilibria | Time [s] | Time per Equilibrium [s] |
|---|---|---|---|---|---|
| **m=100** | 8192 | 60 | 60 | 75 | 1.25 |
| **m=120** | 131072 | 630 | 350 | 1697 | 4.8486 |
| **m=140** | 524288 | 462 | 252 | 1547 | 6.1389 |

**Definition** The *strategy set* of access network $j$, $\mathscr{S}_n^j$, (with $j \in \mathscr{N}$) is the set of frequencies which can be chosen by access network $j$.

In the case of Figure 2, if three frequency channels (f1, f2, f3) are available to all the access networks, the strategy sets are: $\mathscr{S}_n^1 = \mathscr{S}_n^2 = \mathscr{S}_n^3 = \{f1, f2, f3\}$.

**Definition** The *strategy space* $\mathscr{S}_n$ is the set of all the possible combinations of strategies played by the access networks, that is

$$\mathscr{S}_n = \mathscr{S}_n^1 x \mathscr{S}_n^2 x \ldots x \mathscr{S}_n^n$$

**Definition** An element $S_n \in \mathscr{S}_n$, $S_n = (S_n^1, S_n^2, \ldots, S_n^n)$ with $S_n^j \in \mathscr{S}_n^j$, is a *strategy profile*.

The payoff for the generic access network $j$ associated to the access network *strategy profile* $S_n$, $p_j(S_n, INSG)$, can be defined as the number of end users which decide to associate to the access network $j$ under the access network's strategy profile $S_n$. Consequently, the payoffs of the resource allocation game among access networks depend on the underlying game of network selection played by the end users. If we decouple the decision time of end users and access networks and assume the access networks and end users play their strategies in different steps, this leads to the definition of a bi-level stage game; the lower level game leads to Nash equilibria of the network selection with the frequency assignment as a parameter, whereas the upper-level game seeks to allocate the frequencies among access networks given the responses of the end users to these assignments.

Formally, the bi-level game of Network Selection and Resource Allocation Game (NSRAG) can be defined as:

$$NSRAG = \langle \mathscr{N}, \mathscr{S}_n, \{p_j(S_n, INSG)\}_{j \in \mathscr{N}} \rangle \quad (21)$$

where $\mathscr{N} = \{1, \ldots, n\}$ is the set of access networks, and $p_j(S_n, INSG)$ is the payoff associated to access network $j$ playing the strategy profile $S_n$, which depends on the nested INSG.

The following property holds for the NSRAG:

*Theorem 5.1:* Any NE for the NSRAG is Pareto-Optimal from the access networks point of view.

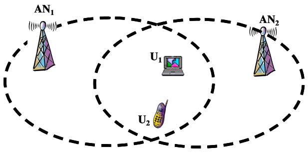


Fig. 5. *Sample Network Scenario, with two access networks (AN1, AN2) covering two users (U1, U2).*

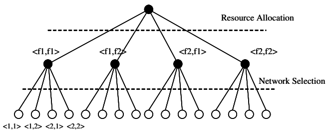


Fig. 6. *Extended form of NSRAG for the network topology of Figure 5.*

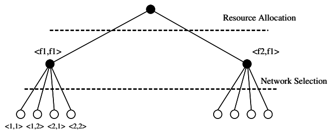


Fig. 7. *Reduced form of NSRAG for the network topology of Figure 5.*

*Proof:* The proof comes from the observation that the payoff of the access networks sums up to a given value, which is the number of users. Consequently, given a NE point for NSRAG, it cannot exist another strategy profile of NSRAG where the payoffs of the networks improve. By contradiction, if such strategy profile existed, the access networks would all get a higher number of assigned users playing it, leading to a greater sum of payoffs, which contradicts the starting assumption. ■

### A. Characterizing NSRAG Equilibria

To characterize the equilibria of the NSRAG, we develop hereafter a solution method based on a steered enumeration of the access networks strategy space, $\mathscr{S}_n$. The solution procedure at first identifies strategy profiles of the access network strategy space which are *equivalent* from the end users and access networks point of view.

**Definition** Two strategy profiles $S_n(1)$ and $S_n(2)$ of the access networks are equivalent if they lead to equivalent NE in the NSRAG.

Strategy profiles equivalent to a given strategy profile can be pruned from the strategy space of the access networks, thus reducing the dimension of the strategy space itself.

As an example, for the network reported in Figure 5, if the available set of frequencies is {f1, f2}, the strategy profiles $S_n(1) = \{f1, f1\}$ and $S_n(2) = \{f2, f2\}$ are equivalent, since they both lead to equivalent equilibria in the underlying INSG. Figure 6 reports the NSRAG for the network scenario in Figure 5 in its extended form, whereas Figure 7 gives the reduced game where equivalent strategy profiles of the access networks have been pruned.

The INSG equilibria can then be determined for the points in the reduced access networks strategy space (resource allocation) using the technique proposed in the previous section. Algorithm (1) formalizes the solution procedure in a pseudocode. Each iteration of the $for$ cycle in Algorithm (1) requires the solution of the INSG which leads to an assignment relationships among end users and access networks. The quality of such assignment is then evaluated with respect to the access networks utility, thus determining the NE of the overall NSRAG.

**Algorithm 1** Solve NSRAG

$\mathscr{S}_n^{reduced}$ = Reduce Strategy Space
**for** $S_n(i) \in \mathscr{S}_n^{reduced}$ **do**
  Solve INSG($S_n(i)$)
**end for**
Search for Network Equilibria

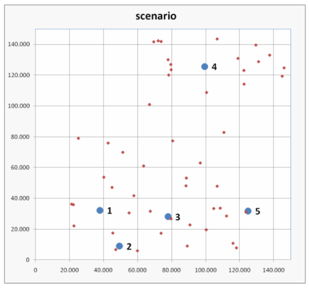


Fig. 8. *Reference Network Scenario for the NSRAG.* $n = 5$, $m = 50$, $L = 150m$, $d = 50m$.

From the access network point of view, a strategy profile is a Nash equilibrium if there is no player (i.e., access network) which has advantage in modifying its strategy unilaterally. Therefore, a strategy profile $S = (S_j, \mathbf{S_{-j}})$ is a Nash equilibrium for the access networks if and only if:

$$p_j((S_j, \mathbf{S_{-j}}), INSG) \geq p_j((S_j^*, \mathbf{S_{-j}}), INSG) \quad \forall S_j^* \in \mathscr{S}_n^j, \forall j \in \mathscr{N},$$

where $S_j$ is the strategy played by access network $j$, and $\mathbf{S_{-j}}$ indicates the set of strategies played by all the other access networks but $j$.

From the considerations above, the procedure to determine the Nash equilibria of the NSRAG can be formalized in the following Algorithm (2). We have applied the solution method

**Algorithm 2** Search for Network Equilibria

$S_{eq} = \{\emptyset\}$
**for** $S = (S_j, \mathbf{S_{-j}}) \in \mathscr{S}_n^{reduced}$ **do**
  $flag = 1$
  **for** $j \in \mathscr{N}$ **do**
    **for** $S_j^* \in \mathscr{S}_n^j$ **do**
      **if** $!(p_j((S_j, \mathbf{S_{-j}}), INSG) \geq p_j((S_j^*, \mathbf{S_{-j}}), INSG))$ **then**
        $flag = 0$
      **end if**
    **end for**
  **end for**
  **if** $flag = 1$ **then**
    $S_{eq} += S$
  **end if**
**end for**
**Return** $S_{eq}$

to the network scenario reported in Figure 8, featuring 5 access networks with coverage radius of 50 m providing access to 50 users with 3 available frequencies to be allocated. The strategy space of the access networks is therefore composed of $3^5 = 243$ strategy profiles. After reducing the strategy space by identifying equivalent strategies, the number of remaining strategy profiles to be analyzed scales down to 41, in this case. Thus, we can solve 41 instances of the INSG and identify through Algorithm (2) the equilibria of the NSRAG.

Table IV reports the characteristics of the Nash equilibria, by reporting the frequency allocation pattern, the number of

TABLE IV

*Characteristics of the NSRAG Nash equilibria in the network scenario represented in Figure 8.*

| | Frequency Allocation | Number of Customers per AN | Average Interferers per user |
|---|---|---|---|
| **NE 1** | <f1,f2,f3,f1,f1> | <8,7,8,19,8> | 11.04 |
| **NE 2** | <f1,f2,f3,f1,f2> | <8,7,8,19,8> | 11.04 |
| **NE 3** | <f1,f2,f3,f2,f1> | <8,7,8,19,8> | 11.04 |
| **NE 4** | <f1,f2,f3,f2,f2> | <8,7,8,19,8> | 11.04 |
| **NE 5** | <f1,f2,f3,f3,f1> | <8,7,8,19,8> | 11.04 |
| **NE 6** | <f1,f2,f3,f3,f2> | <8,7,8,19,8> | 11.04 |

captured customers for each access network and the average number of interferers per user. As clear from the table, the reduced NSRAG has 6 equivalent equilibria where the access networks tend to choose frequencies as diverse as possible with respect to surrounding access networks. Namely, AN1, AN2, and AN3 always choose non-overlapping frequencies in every equilibrium, whereas AN 5 always picks a frequency which is non interfering with AN 3. On the other side, the choice of AN 4 is independent on the other AN's frequency pattern. Similar results, not reported here for the sake of brevity, have been obtained also in other network scenarios.

The general concept coming form the aforementioned results is that even if the game is non-cooperative and the players (ANs and users) potentially can act selfishly and competitively, the NSRAG features equilibria situation where both the end users and the access networks tend to play strategies which do not hinder the other players. In fact, the users tend to spread uniformly amongst the different access networks, and the access networks themselves tend to choose resource allocation strategies which lower the perceived/induced interference to other access networks.

## VI. Concluding Remarks

Motivated by the proliferation and widespread deployment of heterogeneous wireless access technologies, we have addressed in this paper the problems of network selection and radio resource allocation resorting to game theory. Different from other approaches, we have tackled the two problems jointly by casting a non-cooperative game where end users and access networks play selfishly strategy profiles to achieve maximum utility (quality of service, for the end users, and number of customers, for the access networks).

We have formalized the joint Network Selection and Resource Allocation Game (NSRAG) as a non-cooperative bi-level stage game, and we have characterized its Nash equilibria. Moreover, we have proposed a solution method to obtain the Nash equilibria based on mathematical programming. Finally we have commented on the quality of the Nash equilibria of the NSRAG in case of synthetic network instances representing realistic network scenarios. The numerical results have highlighted the fact that, even if the game is non-cooperative, the NSRAG features equilibria where both the access networks and the end users tend to be fair with respect to the other players. Namely, the end users spread uniformly among the different access networks, and the access networks themselves go for radio resource allocation strategies which minimize the inter-access networks interference.

## References


[1] E. Gustafsson and A. Jonsson, Always best connected, *IEEE Wireless Commun.*, Feb. 2003, Vol. 10, Page(s): 49-55.

[2] I.F. Akyildiz, Won Yeol Lee, M.C. Vuran, S. Mohanty, NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey, *Computer Networks*, Vol. 50, No. 13, Sept. 2006, Page(s): 2127-2159.

[3] S. Bosio, A. Capone, M. Cesana, Radio Planning of Wireless Local Area Networks, *ACM/IEEE Trans. on Networking*, Vol. 15, No. 6, Dec. 2007, Page(s): 1414-1427.

[4] A. Raniwala, K. Gopalan, T. Chiueh, Centralized channel assignment and routing algorithms for multi-channel wireless mesh networks, *ACM SIGMOBILE Mobile Computing and Communications Review*, April 2004, Vol. 8, No. 2, Page(s): 50-65.

[5] E. Amaldi, A. Capone, and F. Malucelli, Radio Planning and Coverage Optimization of 3G Cellular Networks, *Wireless Networks*, to appear.

[6] P. Vidales, J. Baliosian, J. Serrat, G. Mapp, F. Stajano, and A. Hopper, Autonomic system for mobility support in 4G networks, IEEE Jour. on Sel. Areas in Comm., Dec. 2005, Vol. 23, No. 12, Page(s): 2288-2304.

[7] D. Niyato, E. Hossain, A Cooperative Game Framework for Bandwidth Allocation in 4G Heterogeneous Wireless Networks, *in proc. of IEEE ICC 2006*.

[8] D. Niyato, E. Hossain, A Noncooperative Game-Theoretic Framework for Radio Resource Management in 4G Heterogeneous Wireless Acces Networks, *IEEE Trans. on Mob. Computing*, March 2008, Vol. 7, No. 3, Page(s): 332-345.

[9] D. Niyato, E. Hossain, QoS-aware bandwidth allocation and admission control in IEEE 802.16 broadband wireless access networks: A non-cooperative game theoretic approach, *Computer Networks*, Aug. 2007, Vol. 51, No. 11, Page(s): 3305-3321.

[10] Q. Song, A. Jamalipour, Network Selection in an Integrated Wireless LAN and UMTS Environment using Mathematical Modelling and Computing Techniques, *IEEE Wireless Comm.*, June 2005, Page(s): 42-48.

[11] D. Charilasa, O. Markakia, D. Nikitopoulos, M. Theologoua, Packet-switched network selection with the highest QoS in 4G networks, *Computer Networks*, Jan. 2008, Vol. 52, No. 1, Page(s): 248-258.

[12] O. Ormond, J. Murphy, G. Muntean, Utility-based Intelligent Network Selection in Beyond 3G Systems, *in proc. of IEEE ICC 2006*.

[13] N. Blefari-Melazzia, D. Di Sorte, M. Femminella, and G. Reali, Autonomic control and personalization of a wireless access network, *Computer Networks*, July 2007, Vol. 51, No. 10, Page(s): 2645-2676.

[14] J. McNair, Fang Zhu, Vertical handoffs in fourth-generation multinetwork environments, *IEEE Wireless Comm.*, June 2004, Vol. 11, No. 3, Page(s): 8-15.

[15] M. Bernaschi, F. Cacace, G. Iannello, S. Za, A. Pescape, Seamless internetworking of WLANs and cellular networks: architecture and performance issues in a Mobile IPv6 scenario, *IEEE Wireless Comm.*, June 2005, Vol. 12, No. 3, Page(s): 73-80.

[16] Y.W. Lee, S. C. Miller, Network Selction and Discovery of Service Information in Public WLAN Hotspots, *in proc. of ACM WMASH 2004.*

[17] J. Antoniou, A. Pitsillides, 4G Converged Environment: Modeling Network Selection as a Game, *in proc. of IST Mobile and Wireless Communications Summit 2007.*

[18] L. Berlemann, G.R. Hiertz, B.H. Walke, S. Mangold, Radio Resource Sharing Games: Enabling QoS Support in Unlicensed Bands, *IEEE Network*, July/August 2005, Page(s): 59-65.

[19] M.J. Osborne, A. Rubinstein, *An Course in Game Theory*, the MIT Press, 1994.

[20] R. W. Rosenthal, A class of games possessing pure-strategy nash equilibria, *International Journal of Game Theory*, 1973, Vo. 2, Page(s): 65-67.

[21] B. Vöcking, Congestion games: Optimization in competition, *in proc. of the 2nd Algorithms and Complexity in Durham Workshop, 2006*

[22] I. Milchtaich, Congestion games with player-specific payoff functions, *Games and Economic Behavior*, 13:111-124, 1996.

[23] I. Milchtaich, Crowding games are sequentially solvable, *International Journal of Game Theory*, Dec. 1998, Vol. 27, No 4, Page(s): 501-509.

[24] R. Fourer, D. M. Gay, and B. W. Kernighan, *AMPL, A modeling language for mathematical programming*, 1993.

[25] *ILOG CPLEX 10.0 user's manual, http://www.ilog.com/products/cplex/.*