Time-Frequency Packing for Linear Modulations: Spectral Efficiency and Practical Detection Schemes

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***Abstract*— We investigate the spectral efficiency, achievable by a low-complexity symbol-by-symbol receiver, when linear modulations based on the superposition of uniformly time- and frequency-shifted replicas of a base pulse are employed. Although orthogonal signaling with Gaussian inputs achieves capacity on the additive white Gaussian noise channel, we show that, when finite-order constellations are employed, by giving up the orthogonality condition (thus accepting interference among adjacent signals) we can considerably improve the performance, even when a symbol-by-symbol receiver is used. We also optimize the spacing between adjacent signals to maximize the achievable spectral efficiency. Moreover, we propose a more involved transmission scheme, composed by the superposition of two independent signals and a receiver based on successive interference cancellation, showing that it allows a further increase of the spectral efficiency. Finally, we show that a more involved equalization algorithm, based on soft interference cancellation, allows to achieve an excellent bit-error-rate performance, even when error-correcting codes designed for the Gaussian-noise-limited channel are employed, and thus does not require a complete redesign of the coding scheme.**

I. Introduction

We consider linear modulations over an additive white Gaussian noise (AWGN) channel. The transmitted signal is the superposition of time- and frequency-shifted replicas of a base pulse multiplied by channel symbols belonging to a complex constellation. As common in the literature (e.g., see [1]–[3]) a uniform rectangular tiling of the time-frequency domain is considered. Hence, the signal model is completely defined by the base pulse and the distance between adjacent symbols in the time and frequency domains, denoted by T and F, respectively. Note that, with respect to [1]–[3], we consider flat channels only. It is clear that T and F play an important role, since on one hand they establish the efficiency in the usage of the available time and frequency resources, and at the same time their values determine the amount of interference (if present) a signal brings to the adjacent ones.

A common choice in the literature is the use of orthogonal signaling, that ensures absence of interference. Rectangular or sinc pulses with orthogonal signaling are employed in orthogonal frequency division multiplexing (OFDM), which is known to be optimal from an information theoretic point of view when independent Gaussian distributed input symbols are used. Hence, constellations and practical coding schemes are often designed such as to ensure an approximately Gaussian input [4]. On the contrary, one of the aim of this paper is to show that, when finite-order constellations are considered (e.g., phase shift keying (PSK) or quadrature amplitude modulation (QAM)), it turns out that the efficiency of the communication system can be improved by giving up the orthogonality condition. To this end, we fix the base pulse[1] and we evaluate the information rate, achievable by a symbol-by-symbol receiver, as a function of the spacing values. Eventually, we optimize the spacings such as to maximize the achievable spectral efficiency of the communication system. We point out that improving the spectral efficiency without increasing the constellation order, e.g., by using a quaternary PSK (QPSK), can be considerably convenient, since the decoding complexity increases as the constellation size increases. Moreover, it is well known that low-order constellations are more robust to channel impairments such as time-varying fading or phase noise [5].

We remark that some of the above considerations have been already carried out in literature, but with substantial differences. For example, faster-than-Nyquist signaling [6] is a well known technique consisting of reducing the spacing in the time-domain well below the Nyquist rate, thus introducing controlled inter-symbol interference (ISI). [7] considers the joint optimization of both time and frequency spacing, by finding their smallest values that ensure no reduction of the minimum Euclidean distance, with respect to the Nyquist case. There are two fundamental differences between the proposed approach and those in the literature: first, we consider the spectral efficiency as our performance measure, rather than, for example, the minimum distance. Second, we consider a low-complexity symbol-by-symbol detection algorithm at the receiver, rather than more complex algorithms such as the Viterbi or linear equalizers.

Furthermore, we propose a more involved transmission scheme, composed by the superposition of two independent signals belonging to rectangular time-frequency lattices (being the two lattices properly staggered), with a suitable power allocation between the two signals, and a receiver based on successive interference cancellation. We show that the spectral efficiency achievable by the proposed scheme is considerably high, even when a low-order constellation, such as QPSK, is used. Finally, although all the information-theoretic results will be obtained assuming a symbol-by-symbol receiver, the bit-

[1]We consider pulses commonly employed in practical systems, namely rectangular (REC), Gaussian, and pulses whose spectrum is root raised cosine (RRC) shaped—for simplicity the latter will be denoted as RRC pulses.

error-rate (BER) performance of a suitably designed receiver based on soft interference cancellation (SIC) [8], [9] will be assessed as well. We will show that such a receiver exhibits an excellent BER performance despite the presence of strong interference, even when error-correcting codes designed for the AWGN channel are employed, thus without the need for a complete redesign of the coding scheme.

The remainder of this paper is organized as follows. In Section II we give the system model, while in Section III we address the related ultimate performance limits. In Section IV we describe an algorithm for optimizing the achievable spectral efficiency and in Section V a practical equalization algorithm for the considered problem, based on the SIC framework, is described. The effectiveness of the spectral efficiency optimization algorithm, as well as a BER performance of the SIC-based proposed equalization algorithm, are proved by the simulation results reported in Section VI. Finally, Section VII gives some concluding remarks.

II. System Model

We consider a linear modulation, where the base pulse $p(t)$ is regularly shifted in the time and frequency domains, of multiples of T seconds and F Hz respectively. We assume a perfect synchronization among the data streams (downlink assumption) and that the transmitted symbols $\{x_{n,k}\}$ (being n the time index and k the carrier index) belong to a given M-th order complex constellation and are independent and uniformly distributed (i.u.d.). In particular, it turns out that

$$\begin{aligned} E\{x_{n,k}\} &= 0 \\ E\{x_{n,k}x^*_{n',k'}\} &= \delta(n-n')\delta(k-k') \end{aligned} \tag{1}$$

where $\delta(\cdot)$ denotes the Kronecker delta. Under the above assumptions, the transmitted signal reads [1], [2]

$$x(t) = \sqrt{E_S TF} \sum_n \sum_k x_{n,k} p(t-nT) e^{j2\pi kFt} \tag{2}$$

where the factor $\sqrt{TF}$ has the important aim of normalizing the signal power, such as to ensure a constant average power spectral density (PSD) equal to E_S, irrespectively of T and F [1]. We point out that this normalization is arbitrary, and a different choice would have been acceptable as well. For example, when a frequency division multiplexed (FDM) multiuser scenario is considered, namely when the index k in (2) denotes signals coming from different users, a reasonable choice is to normalize the average energy of each user, rather than the average PSD. Hence, with this choice the normalization factor in front of (2) would become $\sqrt{E_S}$ instead of $\sqrt{E_S TF}$. This would lead to slightly different simulation results with respect to those obtained in this paper, although the general conclusions still remain unchanged. Note that the summations in (2) extend from $-\infty$ to $+\infty$, namely an infinite number of time epochs and of carriers are employed [1]. The base pulse $p(t)$ can be a RRC pulse with roll-off factor α, a REC pulse, or a Gaussian pulse. Gaussian pulses will be analyzed thanks to their time-frequency compactness property [10]. Without loss of generality, we will normalize the employed pulses in the time domain such that the RRC pulse has bandwidth $1+\alpha$, the REC pulse has a duration of 1, and the Gaussian pulse has the same standard deviation in both time and frequency domains. Moreover, the base pulse will be assumed of unit energy [1].

At the receiver, we assume that a filter matched to the time-frequency shifted replicas of the base pulse is employed, together with a symbol-by-symbol detection algorithm [2]. Note that this is the optimal choice in the maximum a posteriori (MAP) sense, only if the signals

$$\{p(t-nT)e^{j2\pi kFt}\}_{n,k} \tag{3}$$

are mutually orthogonal. On the contrary, when orthogonality is not ensured, interference among the signals arises and the MAP criterion leads to a more involved receiver, whose complexity depends exponentially on the cardinality of the interference set. The set of signals in (3) is denoted as a *Weil-Heisenberg* system of functions [1]. As it will be clear from the next sections, the product TF (i.e., the lattice size of the time-frequency grid) is of paramount importance in defining the performance of the considered communication system.

Since we assumed in (2) an infinite amount of signals, without loss of generality in the rest of this paper we will consider the problem of detecting the signal $x_{0,0}$ only. Since symbol-by-symbol detection is considered, the only observed sample employed in the detection reads

$$y_{0,0} = \int y(t)p^*(t)dt = \int [x(t)+w(t)]p^*(t)dt \tag{4}$$

where an AWGN channel is considered, $w(t)$ being a circularly symmetric zero-mean white Gaussian process with PSD N_0. By joining (2) and (4), we eventually obtain

$$y_{0,0} = \sqrt{E_S TF} \sum_n \sum_k x_{n,k} A_p(nT, kF) + z_{0,0} \tag{5}$$

where $A_p(\tau,\nu)$ is the *ambiguity function* [2], defined as

$$A_p(\tau,\nu) = \int p(t-\tau)p^*(t)e^{j2\pi\nu t}dt \tag{6}$$

that depends only on the base pulse $p(t)$ (e.g., if $p(t)$ is Gaussian, then $A_p(\tau,\nu)$ is a bi-variate Gaussian). The additive noise term $z_{0,0}$ is $z_{0,0} = \int w(t)p^*(t)dt$. We remark that $z_{n,k}$ is colored unless the signals (3) are orthogonal. However, the performance of a receiver which carries out symbol-by-symbol detection depends only the average power of $z_{0,0}$, given by N_0.

Note that (5) can be rewritten as

$$\begin{aligned} y_{0,0} = &\sqrt{E_S TF} x_{0,0} + \\ &\sqrt{E_S TF} \sum_{n,k\neq(0,0)} x_{n,k} A_p(nT,kF) + z_{0,0} \end{aligned} \tag{7}$$

where the two different impairments experienced by the receiver, namely the background noise and the interference due to adjacent signals, are pointed out [11]. The Gaussian pulse has the highest energy compactness jointly in time and frequency [10], and is therefore widely considered in the literature [1], [2].

Instead of simply neglecting the interference due to adjacent signals in (7), we pursue here a more general approach, which consists of modeling the interference as a zero-mean Gaussian process[2] with PSD N_I, of course independent of the additive thermal noise—we point out that this approximation is exploited only by the receiver, while in the actual channel the interference is clearly generated as in (7). Hence, the channel model assumed by the receiver is

$$y_{n,k} = \sqrt{E_S TF} x_{n,k} + v_{n,k} \tag{8}$$

where $v_{n,k}$ is a zero-mean circularly symmetric white Gaussian process with PSD $N_0 + N_I$. It turns out that

$$N_I = E_S TF \sum_{n,k \neq (0,0)} |A_p(nT, kF)|^2 \tag{9}$$

where the independence of the transmitted symbols has been used.

III. Ultimate Performance Limits

We are interested in evaluating the ultimate performance limits achievable by a symbol-by-symbol receiver designed for the auxiliary channel (8) when the actual channel is that in (7), in terms of information rate and spectral efficiency. This issue is an instance of *mismatched* decoding [13] (see also [14]). The achievable information rate (AIR) for the mismatched receiver yields

$$I(x_{0,0}; y_{0,0}) = E_{x_{0,0}, y_{0,0}} \left\{ \log_2 \frac{M p(y_{0,0}|x_{0,0})}{\sum_x p(y_{0,0}|x)} \right\} \quad \left[\frac{\text{bit}}{\text{ch. use}} \right] \tag{10}$$

where $p(y_{0,0}|x_{0,0})$ is a Gaussian probability density function (pdf) of mean $\sqrt{E_S TF} x_{0,0}$ and variance $N_0 + N_I$ (in accordance with the auxiliary channel model (8)), while the outer statistical average, with respect to $x_{0,0}$ and $y_{0,0}$, is carried out according to the real channel model (7) [11], [14]. Eq. (10) can be evaluated efficiently by means of a Monte Carlo average. Let us recall that the mismatched receiver can assure error-free transmissions when the provided information rate does not exceed $I(x_{0,0}; y_{0,0})$ in (10).

From a system viewpoint, the spectral efficiency, that is the amount of information transmitted per unity of time and per unity of bandwidth, is a more significant quality figure than the information rate. Hence, the derivation of the spectral efficiency for the considered system is given in the following. First, we notice that the *overall* information rate (in bits per channel use) achievable by the symbol-by-symbol receiver when $2N+1$ time epochs and $2K+1$ carriers are employed, is given by

$$\sum_{n=-N}^{N} \sum_{k=-K}^{K} I(x_{n,k}; y_{n,k})$$

while the signal duration and bandwidth are proportional to $2N + 1$ and $2K + 1$, respectively, plus some additive terms independent of N and K and taking into account the pulse tails in time and frequency. Thus, for increasingly large values of N and K the boundary effects become negligible for the *overall* information rate, the *overall* duration, and the *overall* bandwidth. Hence, under the assumption of infinite transmission in both the time and the frequency domain, the achievable spectral efficiency (ASE) yields

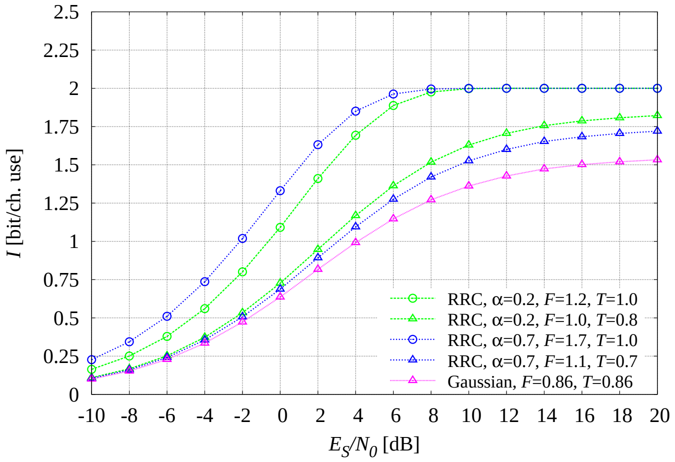
$$\eta = \frac{1}{FT} I(x_{0,0}; y_{0,0}) \quad \left[\frac{\text{bit}}{\text{s} \cdot \text{Hz}} \right]. \tag{11}$$

As a consequence, thanks to the assumption of a large number of signals in both time and frequency domains, we can keep focusing on the information stream $x_{0,0}$ even for evaluating the ASE.

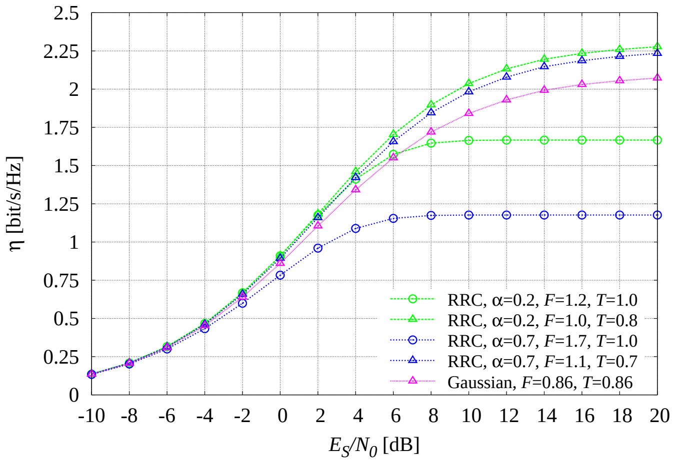
Some simulation results are reported in Fig. 1(a), which shows how the AIR varies with the value of E_S/N_0 when different values of the time and frequency spacings T and F are considered, for RRC and Gaussian pulses and QPSK symbols. Note that, for the RRC pulse with roll-off factor α, when $T = 1$ and $F \geq 1 + \alpha$ the resulting signals (3) are mutually orthogonal, thus $A_p(nT, kF) = 0$ for each $n, k \neq (0, 0)$ and the interference term disappears from (7). In this case, $N_I = 0$ and the auxiliary channel (8) becomes equivalent to the real channel (7). The two curves with $T = 1$ and $F = 1 + \alpha$ in Fig. 1(a) can therefore be interpreted as interference-free benchmark curves. We point out that the difference, in terms of AIR, between the two considered orthogonal signaling formats stems from the normalization factor $\sqrt{FT}$ introduced in (2). Fig. 1(a) gives a quantitative evidence of the fact that the lower the values of T or F, the larger the interference due to the adjacent signals, and therefore the smaller the resulting AIR. We remark again that the values of information rate shown in Fig. 1(a) are *achievable* by a MAP symbol-by-symbol receiver designed for the channel model (8).

On the other hand, very interesting insights are given by the results reported in Fig. 1(b), which shows the ASE corresponding to the same modulation formats as in Fig. 1(a). These results clarify that the values of T and F providing the best AIR are not those providing the best ASE, and thus that a careful design strategy, when the spectral efficiency is the key quality figure, should trade an intentional degradation in AIR for a larger ASE. For instance, in the case of the RRC pulse with roll-off factor $\alpha = 0.7$, the choice $T = 0.7$ and $F = 1.1$ provides a spectral efficiency significantly larger than the choice $T = 1.0$, $F = 1.7$, namely the minimum spacing that ensures orthogonality. Moreover, despite the ambiguity function $A_p(\tau, \nu)$ of the Gaussian pulse is strictly larger than 0 for each τ and ν (thus interference is always present, irrespectively of the values of T and F), the Gaussian pulse outperforms, in terms of achievable spectral efficiency, the orthogonal signal sets based on RRC pulses, at least for vanishing small noise power.

[2]Note that the interference is really Gaussian distributed only if the transmitted symbols $x_{n,k}$ are Gaussian distributed as well. Approximating the interference as Gaussian even when the constellation is finite, is common in the multi-user literature (see, e.g., [12]).



(a) AIR



(b) ASE

Fig. 1. AIR and ASE, for a QPSK modulation and several pulses and values of the spacing.

IV. Optimization of the Spectral Efficiency

Our aim is to find, for a given constellation and base pulse, the spacings T and F that provide the largest ASE. From previous considerations, we should expect that the optimal spacings depend on the signal-to-noise ratio (SNR). To this purpose, we plotted (not shown here for a lack of space) the ASE as a function of T and F and for different values of E_S/N_0. As the SNR increases, not only the ASE increases, but also the optimal values of the spacing change. For $E_S/N_0 \to \infty$, the maximal ASE for the RRC pulse with $\alpha = 0.2$ is achieved with $T = 0.8$ and $F = 1.0$, i.e., the values previously employed in Fig. 1. However, since we noticed that the dependence of the optimal spacing values on the SNR is usually very coarse, and in order to simplify computer simulations, we decided to carry out the optimization only asymptotically, i.e., for vanishing small noise power, and evaluate the ASE at any SNR using the spacings that are asymptotically optimal.

The properties of the function $\eta(T, F)$ cannot be easily studied in closed form,[3] but it reads clear, by physical arguments, that it is bounded, continuous in T and F, and tends to zero when $T, F \to 0$ or $T, F \to \infty$. Hence, the function $\eta(T, F)$ has a maximum value—according to our findings, we could also conjecture that there are no local maxima other than the global maximum. Formally, for a given modulation format and base pulse and a given value of E_S/N_0, the optimization problem consists of finding the maximal ASE

$$\eta_{\mathrm{M}}(E_S/N_0) = \max_{T>0,F>0} \eta(T, F, E_S/N_0) \tag{12}$$

which can be solved by evaluating $\eta(T, F, E_S/N_0)$ on a grid of values of T and F (coarse search), followed by an interpolation of the obtained values (fine search).

We now propose an effective way to improve the ASE, for a given modulation format and base pulse, with only a limited increase of the computation complexity at the receiver.

A. *Double Signaling with Interference Cancellation*

So far, a significant increase in the ASE was achieved by intentionally introducing controlled interference among adjacent signals, in order to make a better use of the available resources (time and frequency). Inspired by the same rationale, we propose a more involved way to improve the ASE, with only a minor increase of the computational complexity at the receiver. The idea is to combine two independent signals, each of them belonging to a rectangular time-frequency lattice (being the two lattices properly staggered), with a suitable power allocation between the two signals. The transmitted signal reads[4]

$$\begin{aligned} x(t) &= \sqrt{(1-\beta^2)E_S FT} \sum_{n,k} \dot{x}_{n,k} p(t - nT) e^{j2\pi kFt} \\ &+ \beta \sqrt{E_S FT} \sum_{n,k} \ddot{x}_{n,k} p(t - (n + 1/2)T) e^{j2\pi (k+1/2)Ft} \end{aligned} \tag{13}$$

where $\beta \in [0, 1]$ is the power allocation factor, $\{\dot{x}_{n,k}\}$ and $\{\ddot{x}_{n,k}\}$ both satisfy (1), and $E\{\dot{x}_{n,k}\ddot{x}^*_{n',k'}\} = 0$ for each n, n', k, k'. The constant multiplicative factors in (13) ensure a constant average PSD independently on T, F, and β. The sequences $\{\dot{x}_{n,k}\}$ and $\{\ddot{x}_{n,k}\}$ are the output of two separate encoders, with rate $\dot{R}$ and $\ddot{R}$ respectively. Note that, for $\beta = 0$ or $\beta = 1$, a signal equivalent to (2) results.

At the receiver, whose frontend is again composed by a bank of matched filters, the following operations are carried out. First, the samples

$$\dot{y}_{n,k} = \int y(t) p^*(t - nT) e^{-j2\pi kFt} dt$$

[3] Although the functions AIR $I(\cdot)$ and ASE $\eta(\cdot)$ depend on various system parameters, in the following we will only explicitly indicate the parameters of interest for the relevant discussion.

[4] We point out that the transmitted signal in (13) falls under the general model of multi-pulse multi-carrier (MPMC) modulations of [3]. However, we remark that, as opposed to [3], we do not look for orthogonal pulses, we (partially) deal with interference at the receiver, and we employ finite-order constellations.

($y(t)$ being the continuous-time received signal) are employed to perform a symbol-by-symbol detection of the transmitted symbols $\dot{x}_{n,k}$. It is worth noting that, besides the self-interference as described in (7), samples $\dot{y}_{n,k}$ are affected by the interference due to the secondary symbols $\ddot{x}_{n,k}$ as well, thus the parameter N_I of the auxiliary channel model (8) must take into account the increased interference power. Therefore, we expect that the AIR $I(\dot{x}_{0,0};\dot{y}_{0,0})$, that of course depends on the SNR, the base pulse, and the spacings T and F, rapidly decreases for increasingly large values of β. However, assuming that $0 < \dot{R} < I(\dot{x}_{0,0};\dot{y}_{0,0})$, there exists a rate-$\dot{R}$ code that ensures an arbitrarily small probability of error for the hard decisions of the symbols $\dot{x}_{n,k}$. Consequently, after successful decoding of the symbols $\dot{x}_{n,k}$ has been carried out, the following interference-mitigation operation is performed

$$\ddot{y}(t) = y(t) - \sqrt{(1-\beta^2)E_S FT}\sum_{n,k}\dot{x}_{n,k}p(t-nT)e^{j2\pi kFt} \quad (14)$$

and the samples

$$\ddot{y}_{n,k} = \int y(t)p^*(t-(n+1/2)T)e^{-j2\pi(k+1/2)Ft}dt$$

are employed to perform symbol-by-symbol detection of the secondary sequence $\{\ddot{x}_{n,k}\}$. In order to ensure arbitrarily small probability of error for the detection of the secondary sequence as well, it must be $0 < \ddot{R} < I(\ddot{x}_{0,0};\ddot{y}_{0,0})$.

The overall AIR of the proposed method is the sum of the AIRs of the two sequences, and the ASE is the ratio between the overall AIR and FT. We remark that the power allocation parameter β plays a key role, and must be optimized in order to maximize the overall AIR. We denote by β_{OPT} the optimized parameter. For vanishing small noise power, the following proposition holds.

Proposition.: If $E_S/N_0 \to \infty$, $AIR(\beta)$ is continuous on $(0,1]$ (rather than on the closed interval $[0,1]$), it is strictly decreasing in $(0,1]$, and $AIR(\beta=0) = AIR(\beta=1)$. Its supremum

$$\sup_{\beta} AIR(\beta) = \lim_{\beta\to 0^+} AIR(\beta)$$

is doubled with respect to the case of a single signal (10), when the base pulse and the spacing values T and F are fixed. Moreover, the supremum is not attained by any value of $\beta \in [0,1]$.

Note that, when the SNR is bounded, the factor of increase of the AIR is strictly less than 2, and β_{OPT} must be optimized numerically.

In Fig. 2, the spectral efficiency achievable by the proposed receiver based on interference mitigation, when the transmitted signal is (13), is shown as a function of the power allocation factor β. A QPSK modulation and a RRC pulse with $\alpha = 0.2$, $T = 0.8$, $F = 1.0$, and several values of E_S/N_0 have been used. As it can be seen, for very small values of the SNR, the ASE is almost independent of β, and basically no improvements are obtained when a signal format as in (13) is

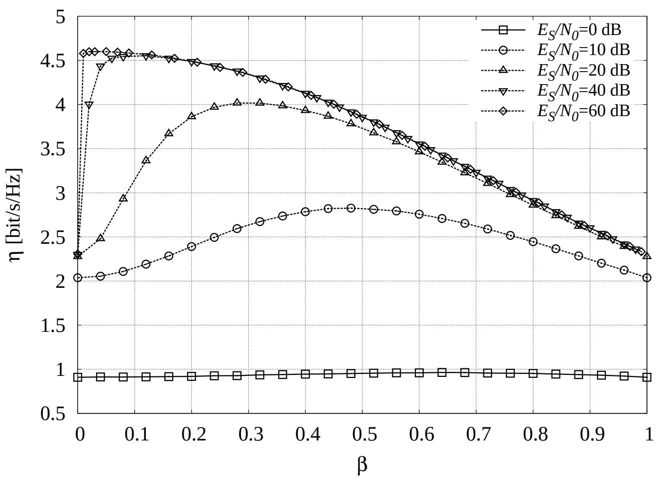


Fig. 2. ASE of the proposed interference-mitigation receiver and double signaling, as a function of the power allocation factor β, for a QPSK modulation and a RRC pulse with $\alpha = 0.2$, $T = 0.8$, and $F = 1.0$.

employed (i.e., when $\beta > 0$). On the contrary, for increasingly large values of the SNR, the gain provided by the proposed scheme becomes larger and larger, whereas β_{OPT} rapidly decreases to 0.

V. Low-Complexity Equalization

Although a symbol-by-symbol receiver has been assumed in this paper, more involved equalization algorithms could be employed to improve the receiver performance in practical coded schemes. In particular, as it will be clear from the next section, the interference among adjacent symbols, although small, may require a suitable and involved design of the coding scheme in order to obtain an acceptable performance in terms of convergence threshold and error floor.

In this Section we describe a more involved receiver, based on the well known soft interference cancellation principle, inspired by the wide literature on multiuser detection (see, e.g., [8], [9] and references therein). The resulting algorithm may be employed as a soft-input soft-output (SISO) block in iteratively decoded, concatenated "turbo" schemes, according to the turbo-equalization principle [15]. Although the algorithm is described for the single signaling case, its application to the double signaling scenario is straightforward and consists in two sequential executions of the algorithm, one for each signal component.

Assume that, at a given iteration, the equalization algorithm is activated with a set of a priori probabilities, coming from the SISO decoder, equal to $\{P(x_{n,k})\}$ (in such concatenated schemes, the code symbols are usually assumed independent). The algorithm proceeds as follows:

1) $iter = 0$ and the extrinsic probabilities $p^{(iter)}(\mathbf{y}|x_{n,k})$ are initialized to a constant value (we denote with $\mathbf{y}$ the set of all the observed samples belonging to the current codeword);
2) If the constellation is finite, as in the pratical schemes we have investigated so far, the a posteriori mean and

variance of every code symbol is evaluated according to

$$\mu_{n,k} = \sum_{\ell} x^{(\ell)} P(x_{n,k} = x^{(\ell)}) p^{(iter)}(\mathbf{y}|x_{n,k} = x^{(\ell)})$$

$$\sigma^2_{n,k} = \sum_{\ell} |x^{(\ell)}|^2 P(x_{n,k} = x^{(\ell)}) p^{(iter)}(\mathbf{y}|x_{n,k} = x^{(\ell)}) - |\mu_{n,k}|^2$$

where the complex values $x^{(\ell)}$, $\ell = 0, \dots, M-1$, denote all the constellation symbols of the M-ary alphabet;

3) the interference in removed from all the received samples according to

$$\tilde{y}_{n,k} = y_{n,k} - \sqrt{E_S TF} \sum_{(m,i)\in\mathcal{I}} A_p(mT, iF)\mu_{n+m,k+i}$$

where $\mathcal{I}$ denotes the interference set. Although in general $\mathcal{I} = \{(m,i) : m^2 + i^2 \neq 0\}$, in practice only the symbols adjacent to the considered one contribute to interference, since $A_p(mT, iF)$ rapidly decreases when $|m|$ or $|i|$ become large. Therefore, we will use $\mathcal{I} = \{(m,i) : |m| \leq L_T, |i| \leq L_F, m^2 + i^2 \neq 0\}$, being L_T and L_F design parameters;

4) a symbol-by-symbol evaluation of the extrinsic probabilities is carried out by assuming independent and Gaussian distributed $\{\tilde{y}_{n,k}\}$, namely

$$p^{(iter+1)}(\mathbf{y}|x_{n,k}) \propto \exp\left[-\frac{|\tilde{y}_{n,k} - \sqrt{E_S TF} x_{n,k}|^2}{\Sigma_{n,k}}\right]$$

where

$$\Sigma_{n,k} = N_0 + E_S TF \sum_{(m,i)\in\mathcal{I}} |A_p(mT, iF)|^2 \sigma^2_{n+m,k+i};$$

5) $iter = iter + 1$; if $iter < SI$ (the design parameter SI being the overall number of self-iterations) return to 2, otherwise continue;

6) the extrinsic probabilities fed to the SISO decoder are $\{p^{(SI)}(\mathbf{y}|x_{n,k})\}$.

We remark that the computational complexity of the proposed SIC algorithm is in general very limited, and only slightly larger than that of the plain symbol-by-symbol detector. In particular, the complexity depends linearly on the codeword size, the number of self-iterations, the constellation size, and the cardinality of the interference set $\mathcal{I}$. Moreover, we point out that the proposed algorithm does not involve any sequential operation, and can thus be implemented fully parallel, with the same latency of the symbol-by-symbol receiver. The performance of the proposed receiver in practical scenarios will be assessed in Section VI-A.

VI. Numerical Results

In this Section, the maximal ASE, obtained with a proper optimization of the spacing parameters T and F (and β when double signaling is considered), is evaluated for several modulation formats and base pulses. For comparison purposes, the capacity per unity of bandwidth $\eta_C = \log_2(1 + E_S/N_0)$ of the considered AWGN channel in terms of bit/s/Hz, is also shown. Note that the above spectral efficiency is achieved by Gaussian-distributed input symbols and orthogonal signaling.

TABLE I

Asymptotic ASE.

Constell.	Pulse	Sig. mod.	T_{opt}	F_{opt}	Orthog.	η
QPSK	REC	eq. (2)	1.00	1.00	Yes	2.00
QPSK	REC	eq. (2)	1.00	0.80	No	2.32
QPSK	RRC-0.2	eq. (2)	1.00	1.20	Yes	1.67
QPSK	RRC-0.2	eq. (2)	0.80	1.00	No	2.28
QPSK	RRC-0.7	eq. (2)	1.00	1.70	Yes	1.18
QPSK	RRC-0.7	eq. (2)	0.72	1.10	No	2.27
16-QAM	RRC-0.2	eq. (2)	1.00	1.20	Yes	3.33
16-QAM	RRC-0.2	eq. (2)	0.95	1.05	No	3.95
QPSK	Gauss.	eq. (2)	0.88	0.88	No	2.10
QPSK	Gauss.	eq. (13)	0.88	0.88	No	4.21
16-QAM	Gauss.	eq. (2)	1.15	1.15	No	5.43

Table I collects the performance, in terms of asymptotic (i.e., for vanishing small thermal noise power) maximal ASE, of a set of modulation formats and base pulses. For all the considered formats based on REC and RRC pulses, the performance for orthogonal signaling (i.e., when $T = 1$ and $F = 1 + \alpha$ for RRC and $T = F = 1$ for REC) is shown for comparison. This Table summarizes the significant gain in terms of ASE provided by the techniques proposed in this paper. Note that the asymptotic spectral efficiency, achievable by modulations based on REC pulse or RRC pulses with completely different values of the roll-off factor, is almost the same when optimization of the spacing values is carried out. This stems from the fact that pulses with a larger roll-off can be squeezed more in the time-domain, and viceversa. The product of the optimal T and F is almost the same in the considered scenarios, as well as the asymptotic AIR. Moreover, we point out that the larger the constellation size, the smaller the performance gain achieved with suitable optimization of the spacing values. An important fact, confirmed by these simulation results, is that the loss in terms of ASE of orthogonal signaling with respect to the optimized case rapidly decreases as the constellation size increases (we remark again that, when the input symbols are i.u.d. Gaussian, orthogonal signaling maximizes the ASE).

Similarly, Fig. 3 shows the ASE as a function of E_b/N_0, E_b being the received signal energy per information bit, for a set of selected cases. We remark again that the same values of T and F, optimized for vanishing small values of N_0, have been employed at each value of the SNR, while in the case of double signaling, the parameter β has been re-optimized for each value of the SNR. It is worth pointing out the large gain provided by the double signaling scheme proposed in Section IV-A, even at low and medium SNRs.

A. EXIT charts and BER performance

Having previously shown the potential gains in terms of ASE provided by the proposed technique, we now analyze the performance of a practical coded scheme in terms of

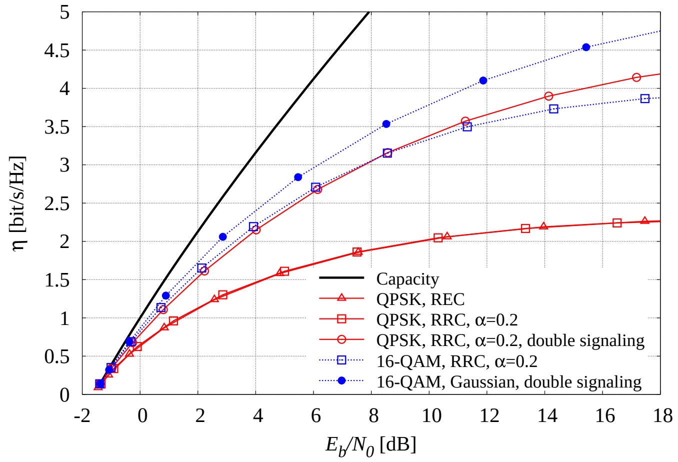


Fig. 3. ASE, as a function of E_b/N_0, for a set of selected cases.

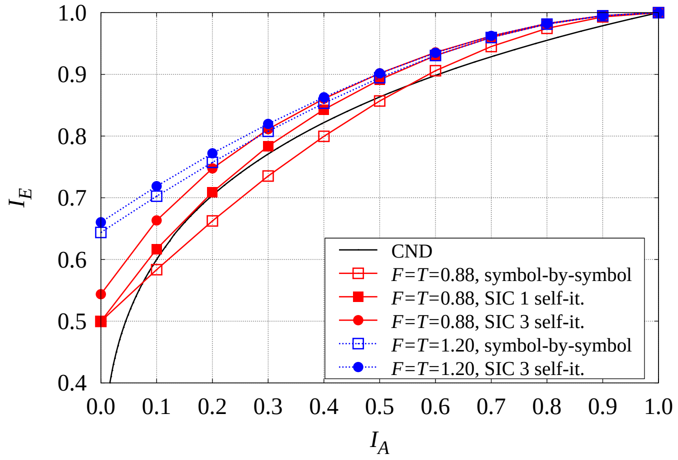


Fig. 4. EXIT chart corresponding to a VND and either the symbol-by-symbol detector or the proposed SIC equalization algorithm, when a Gaussian pulse and a QPSK modulation are employed. The curve corresponding to the CND is shown as well.

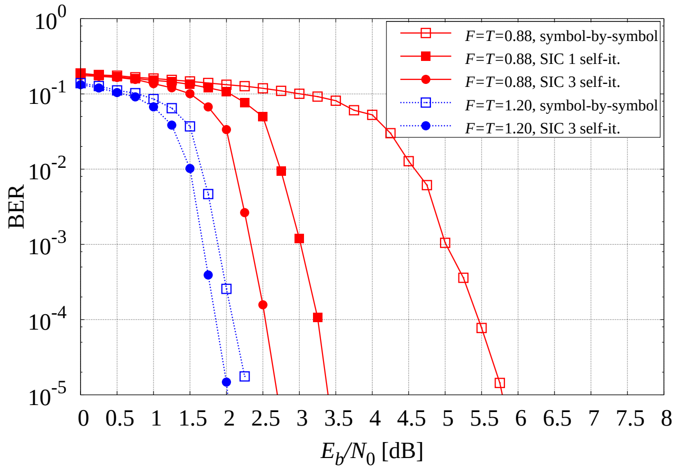


Fig. 5. BER performance of the considered LDPC-coded scheme, when a Gaussian pulse and a QPSK modulation are employed.

convergence threshold, by means of the powerful EXtrinsic Information Transfer (EXIT) charts analysis [16], as well as in terms of BER. In both cases we employed a rate $1/2$ $(3,6)-$regular Low-Density Parity Check (LDPC) code [17], with codeword length 4000 bits, and a QPSK modulation. The single-signaling scenario described by (2) have been used, although we point out that all the results discussed below can be generalized to the double-signaling scheme (eq. (13)) as well.

The EXIT charts depicted in Fig. 4 have been obtained as follows. The receiver can be seen as the serial concatenation of two component blocks, connected through an interleaver [18]. The outer block is denoted as Check Node Decoder (CND), and the relevant curve is depicted in black in Fig. 4. On the other hand, the inner block is the composition of the LDPC Variable Node Decoder (VND) and the employed equalization algorithm, namely, either the symbol-by-symbol or the proposed SIC algorithm. Two scenarios have been considered, both employing a symmetric Gaussian pulse. The first is characterized by $F = T = 0.88$ and a quite strong interference, is practically optimal in the sense described by (12), and the corresponding spectral efficiency (equal to 1.29 bit/s/Hz) is achieved at 2.2 dB (see Fig. 3). The latter, with $F = T = 1.20$ and a weak interference, is highly sub-optimal since at corresponding spectral efficiency (i.e., 0.69 bit/s/Hz, achieved at 0.5 dB) the optimal spacing evaluated according to (12) would result much smaller than 1.20. The relevant curves, corresponding to the two scenarios, have been obtained by means of numerical simulations working at $E_b/N_0 = 2.30$ dB. This value has been chosen since it is the convergence threshold, with the considered LDPC code, when the proposed SIC equalization algorithm, here implemented with $L_T = L_F = 2$ and only one self-iteration, is employed. From Fig. 4 some observations can be drawn: first, in the strong-interference scenario (i.e., FT small), the proposed SIC equalization algorithm strongly outperforms, in terms of convergence threshold, the symbol-by-symbol receiver. Hence, the latter requires a complete redesign of the coding scheme in order to ensure a convergence threshold in line with the information theoretic results obtained so far. Moreover, by increasing the number of self-iterations from 1 to 3, a further improvement is possible. No improvements have been observed when performing further self-iterations. On the other hand, in the weak-interference scenario (i.e., FT large), the improvements stemming from the use of the SIC receiver are much more limited, as expected.

In order to confirm the outcomes of the EXIT chart analysis discussed above, the BER performance is evaluated in the same scenario. At the receiver, the proposed SIC equalization algorithm is iteratively activated along with the LDPC decoder, for an overall amount of 25 iterations. The process also stops if, by checking the code syndrome, a valid codeword is found before the 25^{th} iteration. Fig. 5 shows the BER as a function of E_b/N_0. Note that the convergence thresholds are in line with the results highlighted by the EXIT chart analysis. For example, the BER for the strong-interference scenario and the single-self-iteration SIC receiver starts decreasing at about 2.5 dB. Note that, in the strong-interference scenario, the performance of the symbol-by-symbol receiver is very poor, and a suitable design of the coding scheme is advocated in this case to obtain a convergence threshold in line with the ASE results that pointed out a convergence threshold of 2.2 dB. However, the proposed SIC receiver is a low-complexity alternative, allowing the use of existing coding scheme with excellent performance.

VII. Conclusions

We have proposed a way to improve the spectral efficiency of linear modulations with finite order constellations on an AWGN channel, achievable by a symbol-by-symbol receiver. This is achieved by reducing the spacing, in both time and frequency domains, between adjacent signals, hence introducing controlled interference, but at the same time making a better use of the available time and frequency resources. With respect to [7], that pursued a similar approach, a symbol-by-symbol detection algorithm is employed at the receiver, and the achievable information rate is used as a performance measure.

Moreover, we have shown that when two independent signals, that suitably span the time-frequency domain with a proper power allocation, are employed, along with a successive interference cancellation receiver, the performance gain is remarkable. The simulation results clearly pointed out that, when finite-order constellations (e.g., QPSK) are employed, orthogonal signaling can be largely suboptimal from the spectral efficiency point of view. On the contrary, when higher-order constellations are considered, the gain of the proposed schemes decreases. In summary, the proposed techniques can be seen as a low-complexity alternative to Gaussian shaping [4].

Finally, a low-complexity equalization scheme, based on the soft-interference-cancellation principle, has been proposed, and its performance in an LDPC-coded iteratively-decoded scheme evaluated in terms of EXIT chart and BER. The proposed receiver has a computational complexity only slightly larger than that of the symbol-by-symbol receiver, but allows excellent performance, even in strong-interference scenarios, when error-correcting codes designed for the AWGN channel are employed, without the need for a complete redesign of the coding scheme.

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