A DIFFERENT APPLICATION OF SAR IMAGES:

THE MEASUREMENT OF TITAN ROTATIONAL STATE

P. Persi del Marmo, L. Iess, G. Picardi, R. Seu

Dipartimento di Ingegneria Aerospaziale ed Astronautica, Università La Sapienza
Via Eudossiana 18, 00189, Rome, Italy
Phone:+390644585330, fax: +390644585952, email: paolo.persidelmarmo@uniroma1.it
web: www.diaa.uniroma1.it
Dipartimento di Ingegneria Aerospaziale ed Astronautica, Università La Sapienza
Via Eudossiana 18, 00189, Rome, Italy
Phone:+390644585336, fax: +390644585952, email: luciano.iess@uniroma1.it
web: www.diaa.uniroma1.it
Dipartimento di Scienza e Tecnica dell'Informazione e Comunicazione, Università La Sapienza
Via Eudossiana 18, 00189, Rome, Italy
Phone:+390644585455, email: giovanni.picardi@uniroma1.it
web: www.infocom.uniroma1.it
Dipartimento di Scienza e Tecnica dell'Informazione e Comunicazione, Università La Sapienza
Via Eudossiana 18, 00189, Rome, Italy
Phone:+390644585943, email: roberto.seu@uniroma1.it
web: www.infocom.uniroma1.it

ABSTRACT

The typical objective of the remote sensing and radar imaging analysis is the observation of the surface and subsurface of a body, to determine the roughness and the dielectric constant of its surface.

In this paper it is introduced a different use of the radar images, in particular it is described the methodology used to estimate the obliquity a celestial body (the angle between the rotational and orbital angular momentum) comparing the Synthetic Aperture Radar (SAR) images taken from different positions along the body's orbit with a pairs of surface landmarks. Following this methodology the vectorial angular velocity of the observed body and the corresponding body-fixed reference frame are obtained. SAR is the only instrument able to penetrate the atmosphere and observe the surface of bodies with a thick atmosphere,. The optical imaging is preferred for bodies without atmosphere, due to the high angular resolution of the planetary cameras. The rotational state of a planet or satellite provides crucial physical information on the body's dynamical evolution and deep internal structure. The determination of the moments of inertia value, combining the information on obliquity, physical librations and measurement of the second degree coefficients of the gravity field, could reveal the existence of a fluid core.

The topics presented in this paper refer to the analysis of Titan's surface and provide as main results, the non-synchronous rotation and pole position of Titan compatible with the occupancy of the Cassini state. Other results are the estimation of Titan's obliquity and the day length using a precise positioning of the Cassini spacecraft. Furthermore, the accuracy of the method in the estimation of the pole position and length of the day is suitable for geophysical interpretation.

1. INTRODUCTION

SAR images provide information about the morphology of the surface observed and chemical information tied to the dielectric constant of the material observed by the radar. In some cases, the radar observation allows even to get information about the subsurface and relevant material stratification.

This paper shows a new application for the SAR imagery. The aim is the determination of the rotational state of a body that provides important information on its internal structure.

It is possible to determine the spin vector of a body observing the movement of a point of the surface at different times and anomalies. It is well known that the obliquity of a solid planetary body is related to the polar moment of inertia *C* [1], a crucial data for constraining the geophysical models of the interior. This parameter can also be obtained trough the images provided by other instruments, but in presence of thick atmosphere the only possibility to observe the body surface is the use of a radar The results coming from the analysis of the SAR images of Titan, the largest moon of Saturn are shown in this paper. The Cassini Radar is a facility instrument on the Cassini Orbiter. It is capable of passive (radiometer) and active (scatterometer, altimeter, SAR imaging) operations. Interleaved passive measurements are obtained also during the active mode operation. Due to its thick hazy atmosphere, Titan's surface was not imaged successfully by the Pioneer and Voyager spacecrafts, though atmospheric "windows" in the near infrared have been exploited by the Hubble Space Telescope and earth-based telescopes to produce low-resolution albedo maps of a part of the surface. The Cassini radar instrument is obtaining backscatter and altimeter sounding measurements of Titan's surface. The high resolution synthetic aperture radar backscatter images of 15% of Titan's surface will be obtained.

A precise determination of the length of day and its variation could provide clues to the exchange of angular momentum between the atmosphere and the solid body of Titan. In the presence of the thick Titan's atmosphere the only onboard instrument which is able to penetrate the cloud layers and able to observe the surface with sufficient spatial resolution is the SAR.

Titan is thought to be totally locked to Saturn and occupies a Cassini state, in which the orbital angular momentum, the rotational angular momentum and the pole of the invariable plane are coplanar, and the rotational angular velocity is equal to the mean motion.

The oblate deformation results in an axisymmetric quadrupole field described by the coefficient

$$J_2 = \frac{C-A}{MR^2} \approx \frac{\omega^2 R_T^3}{GM_T} = \frac{M_S R_T^3}{M_T a^3} = \mu = 4\times 10^{-5} \tag{1}$$

Here C is the moment of inertia around the spin axis, A is the moment of inertia around the axis oriented toward the planet, M and R, R_T and M_T are the mass and the radius of Titan, G the gravitational constant, ω the rotational angular velocity, M_S the mass of Saturn and a the semimajor axis of the orbit. Neglecting the eccentricity and assuming synchronous rotation, Titan's rotation axis $\underline{e}=\underline{\omega}/\omega$ is subject to a precession at a rate

$$\sigma = -\frac{3}{2}\omega\frac{C-(A+B)/2}{C} = -\frac{3}{2}\omega\frac{J_2}{C/MR_T^2} \tag{2}$$

due to the torque exerted by Saturn on the oblate satellite, where B is the moment of inertia around the third axis. In case of a hydrostatic, homogeneous Titan, the corresponding precession period is about 300 years. The orbital plane of Titan, inclined at an angle I = 0.280 deg on the invariable plane of the Saturn system, precedes at a rate $d\Omega/dt \approx -0.5°/\mathrm{y}$ due to the perturbing effects from Saturn oblateness [2]. The theory establishes a simple relationship between the inclination I and the obliquity ε for a synchronous body:

$$\frac{C}{M_T R_T^2} = \frac{3}{2}\frac{\omega\varepsilon}{(I+\varepsilon)d\Omega/dt}(J_2+2C_{22}) \tag{3}$$

[4,5]. As I and $d\Omega/dt$ are known, the measurement of ε provides the moment of inertia factor if the quadrupole field is also known. The moment of inertia C follows from eq. 1, once J_2 is measured. It's possible to notice that the combination of the gravity field coefficients and obliquity provides the moments of inertia for bodies in a Cassini state. Fixing a theoretical value of 0.4 for $C/M_T R_T^2$ and 4*10^-5 for J_2, one expects an obliquity angle in the order of 7 arcmin., equivalent to a displacement of Titan's pole of about 5 km.

Titan should occupy a Cassini state of type 1, with the normal to the invariable plane, the normal to the orbital plane and the spin vector lying on the same plane in this order [4].

The results reported in this paper come from the analysis of Cassini SAR images for the determination of Titan's obliquity and spin period, equivalent results have been obtained by a JPL team of researchers adopting a different method of analysis [7]. The method is based upon precise inertial referencing of the surface landmarks observed at different anomalies. The analysis provides the direction and magnitude of the rotational angular velocity, allowing also the construction of a precise body-fixed reference frame. Another goal of the analysis is the verification of the occupancy of a Cassini state.

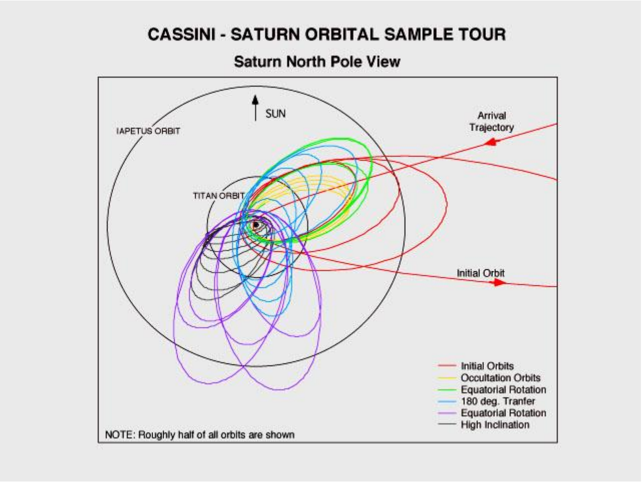

Figure 1- The Cassini Tour

2. METHOD

The nominal Cassini tour (see fig. 1) is used to determine the crossings of the SAR ground tracks in a putative body-fixed, Titan-centred reference frame (Titan has an orbital period of about 15.94 days).

The definition of the track crossings provides a first evaluation of the image resolution, which is affected by the altitude. On Cassini, the SAR high resolution mode required by our measurements (with pixel size of the order of 100-500 m) is only possible for altitudes below 1400 Km.

The BIDR (Basic Image Data Record) file, provided by the JPL and used in this method, are data files of single pass SAR images which are calibrated and gridded. These products are generated in more than one bit type at more than one resolution. Each BIDR product is produced in an oblique cylindrical coordinate system, defined to be 0 degrees latitude. Its extent is the minimum bounding rectangle of the area of coverage in this projection.

The method uses BIDR images normalized backscatter cross-section values corrected for the incidence angle effects (information from all five beams is recorded in the same file. Additionally, for the σ_0 images, a single pixel may contains information averaged from multiple beams). The method has to reference the images to a inertial reference frame and to do that it has before to project them to the body-fixed reference frame.

The coordinate systems and geodetic/cartographic parameters used in production of the Cassini Radar images are chosen to be consistent with the recommendations of the International Astronomical Union/International Association of Geodesy (IAU/IAG) Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites. The IAU/IAG regularly revises its recommended cartographic constants, based on the best available data. The cartographic constants for Titan, based on the astronomical observations and Voyager spacecraft data, are:

Right ascension α_0 and declination δ_0 of the axis of rotation at a time t.

$$\begin{aligned} \alpha_0 &= 36.41° - 0.036° t + 2.66° \sin(29.80° - 52.1° t) \\ \delta_0 &= 83.94° - 0.004° t - 0.30° \cos(29.80° - 52.1° t) \end{aligned} \qquad (4)$$

where t is the interval in Julian centuries from standard epoch (J2000) 2000 January. The rotation of Titan around this axis is given in terms of the angle W measured eastward from the intersection of the planetary equator with the standard Earth equator to the prime meridian

$$W = 189.64° - 22.5769768° d + 2.64° \sin(29.80° - 52.1° t) \qquad (5)$$

where d is the time in days from the standard epoch.

After the re-projection of the SAR images in a putative body-fixed frame, the method defines a crossing as two times at which the radar observes the same portion of the surface. These times has to be separated by a lapse enough to observe Titan at different anomalies. The images appear shifted by an amount which depends both on the rotational period and spin axis direction (see fig. 2). In other words, in the absence of measurement errors, the surface features appear displaced with respect to the expected position. The cause of this registration error, with a mismatch essentially in longitude, is the putative body-fixed reference frame not inertially fixed and that precedes at a period of 15.94 days (the Titan's rotation period) on a cone of semi-aperture ε.

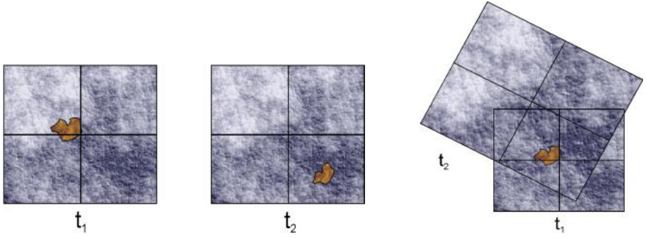

Figure 2 – Measurement concept: the two SAR images to the left (taken during different flybys) contain an identifiable landmark (brown spot). The centre of each image points to a known inertial direction and therefore has known coordinates in an inertial, Titan-centred reference frame. Due to the different altitude and ground track inclination of the two flybys, the landmark (brown feature) appears rotated, displaced and stretched. After applying appropriate transformations, the images are correlated to provide a vector representing the translation to be applied to the centre of the second image in order to target the same point on Titan's surface as the centre of the first image. The difference between the two resulting inertial vectors in the Titan-centric frame is orthogonal to the spin axis.

SAR imaging is based on the analysis of surface echoes. A major obstacle to the correlation of image pairs lies in the fact that the same scene seen in different view angles and altitudes may provide substantially difference returns and constraints on resolution. To obviate at these problems the method stretch and rotates pairs of image along the same point of view. It is necessary to fix the observation direction (normally choose this direction in one of the image) and re-project the image with respect to the fixed point of view, interpolating the image pixels along the new direction of observation and re-sampling them to the correct resolution.

The analysis of the ground tracks and simulation of the radar returns is important for the selection of the flyby pairs. The crucial parameters to be evaluated is the extent of the overlapping area and the angle between the ground tracks. In general, a small angle is associated to smaller effects due to the surface slope and shadowing in the image reconstruction. The other crucial problem is the correlation process, some easily identifiable features must be present in the crossing area. The best conditions are a uniform and flat surface. The optimum is to obtain images with flat surfaces and high radar contrast, such as shorelines of hydrocarbon lakes or oceans. These would be largely immune to shadowing effects and therefore provide excellent results.

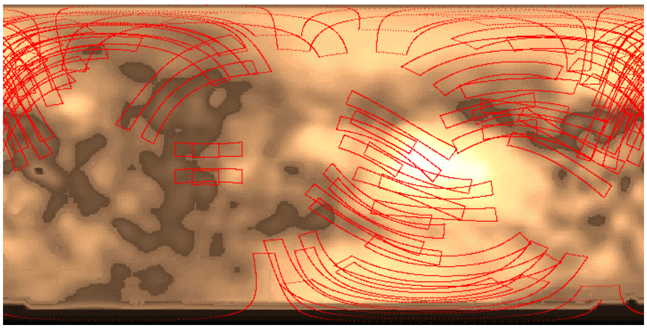

Figure 3 – Cassini SAR ground tracks on Titan.

The correlation is computed starting from the scattered field at the spacecraft antenna. Before the new approach is used on the real SAR images, the method has been tested with a simulated surface modelled by means of an analytic profile (representing the slow surface variations) and a superimposed set of random scatterers (distributed according to a Gaussian or fractal model). The bi-dimensional correlation function [2] depends on the electric field scattered by the surface.

The image correlation matrix provides information about the necessary shifting to enforce the images to obtain the maximum likelihood. This shifting vector **d** is calculated on the surface whose curvature is neglected. In other words, applying this displacement to the centre of the second image it is possible to target the same surface point as the centre of the other image.

The position vector of a landmark observed in each image, as determined from the knowledge of the spacecraft trajectory, has coordinates $\mathbf{r}_1$ and $\mathbf{r}'_2$ in an inertial, Titan-centric reference frame (this is possible in absence of errors in the Titan-centric spacecraft trajectory and in the inertial radar orientation). The new vector provided by the correlation analysis is $\mathbf{r}_2=\mathbf{r}'_2+\mathbf{d}$, which represents the same surface point as $\mathbf{r}_1$. The identification of the landmark enables the link between the inertial and body-fixed frame. Defining the Titan motion as a simple rotation of its centre about a fixed axis, the difference $\mathbf{r}_1-\mathbf{r}_2$ is orthogonal to Titan's spin axis $\hat{\boldsymbol{\omega}}=\boldsymbol{\omega}/\omega$

$$(\mathbf{r}_1-\mathbf{r}_2)\cdot\hat{\boldsymbol{\omega}}=0 \tag{6}$$

This condition must be satisfied by all difference vectors obtained from each image pair. In practice the scalar product between the difference of the inertial vector position and the spin vector will be small, but not zero, due to the measurement errors. By means of a least square method applied to the set of all difference vectors, an estimation of the rotation axis is obtained: the spin vector is the one which minimizes the sum-square error. All observations are assumed to be affected by the same uncertainty. In a least square approach, it is required that the sum-square error

$$S^2=\sum_i\left\{\left[\mathbf{r}_i(t_{2i})-\mathbf{r}_i(t_{1i})\right]\cdot\hat{\boldsymbol{\omega}}\right\}^2=\sum_i e_i^2 \tag{7}$$

were a minimum. The observations $\mathbf{r}_i(t_{1i})$ and $\mathbf{r}_i(t_{1i})$ are the inertial positions of the i-th landmark at two different times. The set of equations in the components of $\hat{\boldsymbol{\omega}}$ are

$$\frac{\partial S^2}{\partial\omega_j}=\sum_i\left[\left(2\sum_k\Delta r_k^{(i)}\hat{\omega}_k\right)\Delta r_j^{(i)}\right]=2\sum_k\hat{\omega}_k\left(\sum_i\Delta r_k^{(i)}\Delta r_j^{(i)}\right)=0 \tag{8}$$

with

$$\begin{aligned}&\lfloor r_j(t_{2i})-r_j(t_{1i})\rfloor=\Delta r_j(t_i)\\&\left[r_k(t_{2i})-r_k(t_{1i})\right]=\Delta r_k(t_i)\end{aligned} \tag{9}$$

The goal is to reduce the solution to a homogeneous system $A_{jk}\hat{\omega}_k=0$, where the matrix A_{jk} represents all difference vector Δr_j and Δr_k.

The null vector will not belong to the range of **A** because of the matrix **A** will not be singular, due to measurement errors. The method needs to find the spin vector $\hat{\boldsymbol{\omega}}$ which is closer to the null vector. The solution is attainable by a singular value decomposition (SVD) of the system matrix. If the spin vector is represents by :

$$\hat{\boldsymbol{\omega}}=\left[\sin\varphi\sin\varepsilon;-\cos\varphi\sin\varepsilon;\cos\varepsilon\right] \tag{10}$$

where ε is the obliquity angle and φ the phase angle of the spin vector. It is hence determined by the set of equations:

$$\frac{\partial S^2}{\partial(\sin\varepsilon)}=0\qquad\frac{\partial S^2}{\partial(\cos\varepsilon)}=0 \tag{11}$$

where the solution of the (11) equations provide the angle ε (obliquity angle) and φ (phase angle) that minimize the derivatives.

Developing the (11) derivatives, giving the ***r*** vector in Cartesian coordinate and substituting the $\hat{\boldsymbol{\omega}}$ with the (10), the (11) reduce to

$$\begin{bmatrix}\sum_i(\sin\varphi x_i-\cos\varphi y_i)^2 & \sum_i z_i(\sin\varphi x_i-\cos\varphi y_i)\\ \sum_i z_i(\sin\varphi x_i-\cos\varphi y_i) & \sum_i z_i^{\,2}\end{bmatrix}\begin{bmatrix}\sin\varepsilon\\ \cos\varepsilon\end{bmatrix}=0 \tag{12}$$

where x_i, y_i, z_i are the components of the difference vectors.

The solution of this set of equations, attainable by the SVD method, are two angle: ε the obliquity angle and φ the phase angle that by the (11) provides the spin vector.

Now, knowing the unit spin axis, it's possible to construct a cinematically defined, body-fixed frame. The rotation period is derived from the angle between the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ and the known time difference between the two observations:

$$\arccos\left(\frac{\mathbf{r}_1\cdot\mathbf{r}_2}{r_1\cdot r_2}\right)=\varphi\Rightarrow\omega=\frac{\varphi}{t_2-t_1}+2\pi n \tag{13}$$

where n is the number of the complete rotation between the two instant of observation.

The geodetic referencing of the SAR images is therefore complete.

3. SELECTION OF PAIR IMAGE

A number of geometrical constraints are required in order to carry out a successful SAR observation.

The core of the method is the correlation of the SAR images. It is important that the images are captured in similar geometry condition to obtain a correct estimation of the displacement between the pair of observation. First of all the altitude constraint: SAR can be operated in high resolution mode (with a resolution up to 170 m) only if the altitude is smaller than 1400 Km; the same observation altitude reduces the correlation problem due to different resolution for the two images. It may be difficult to determine a correlation matrix with an absolute noticeable maximum if the images are taken at widely differing altitudes. The effect of the shadow (that can be possible in spite of the image stretching correction) can mask the correct correlation peak.

Only the images taken at an altitudes below 1400 km and with a high resolution are used in the analysis.

The best crossings to be used in the analysis is an important aspect of the methodology because each crossing gives a different contribution to the evaluation of the obliquity, and in order to get the best information it is needed to focus on the crossings with the largest information content (crossings occurring at the same anomaly are less valuable).

(16)

The selection of the crossings is determined mainly by the geometry of the observation. The angle between the ground tracks at the intersection is the most important parameter to be evaluated because when this angle is small, the two observation geometries are similar and the radar imaging is less affected by shadowing and distortion effects.

Another interesting parameter to be taken into account is the minimum set of crossings needed to estimate the obliquity with the needed accuracy. Numerical simulations show that, an accuracy of 10% or smaller, can be obtained in the spin axis and moment of inertia factor estimation with four suitable crossings. The results showed hereinafter confirm the accuracy calculated in the simulation. However to have a large set of crossings at disposal could improve the determination of both the rotational state and the statistical characterization of the estimation errors; furthermore the observations carried out over a period of several years could allow a more accurate detection of the polar wander and the variations in the length of day. All these conditions and parameters are really crucial in the planning of the future radar observations in a mission extension

4. RESULTS AND CONCLUSION

The pairs of high resolution images provided by Cassini radar observations are 7, spanning a period of four years (2004-2007). Each image is referenced both in an inertial frame and in the IAU, Titan-centric, putative body-fixed frame. The position of Cassini relative to Titan is known with an accuracy of about 100 m in flybys (quite precise) when tracking is available from ground, and probably less than 300 m in the other flybys. It is more difficult to evaluate the effects of shadowing.

The result founded analysing four pairs of high resolution images are summarized in the table below and shown that the estimated pole position is different from the expected IAU reference pole (with an obliquity of 0.27 degrees)

Parameter Name and units	Estimate	1-sigma error	IAU_TITAN
Pole_RA α (deg)	39.3067	±0.028	37.56
Pole_DEC β (deg)	83.4689	±0.00023	83.67
Spin rate ω (deg/day)	22.5781	±0.00054	22.4795 NSR 0.36°/y
Obliquity ε (deg)	0.27	±0.02	0

Table 1 – Result of estimate. The 1-sigma error is derived from Gaussian interpolation of correlation matrix

In particular, the displacement in longitude is very important. The new Titan pole position follows the definition of a Cassini state, the main uncertainty is due to the unknown errors in the orientation of the Laplace plane. The results show that if Titan is in a Cassini state, surely it is not in state 1 but in state 4 where, the pole of the Laplace plane, the normal to the orbital plane and the spin vector appear in this order.

The determination of the 1-sigma error come from the Gaussian interpolation of correlation matrix, it signals the error that come from the procedure of images correlation

Another remarkable news is the significant non-synchronicity of Titan's rotation. The result shows a super-rotation of the satellite of about 0.36 deg/year compared to the mean motion. This value is equivalent to a displacement on the surface of about 16 km per year, a value much larger than the measurement accuracy. The discovery of the non-synchronicity and the measurement of the obliquity have important physical implications for the interior structure of the satellite [9].

The moment of inertia factor estimation is a consequence of the result of the obliquity and shows a very large value (a body in hydrostatic equilibrium assume a moment of inertia factor near to 0.3). The result of the moment of inertia factor indicates that Titan is not in hydrostatic equilibrium. The first estimate of inertia factor comes from (2) and (3), combined with a measurement of J_2 and assuming that Titan's icy shell is rigidly coupled to the interior. The moment of inertia factor is

$$\frac{C}{M_T R_T^2} = 0.95\left(\frac{J_2}{4.910^{-5}}\right) \qquad (14)$$

Assuming a hydrostatic ratio J_2/C_{22}=10/3.

Replacing the new estimates of J_2, determined by the Cassini Radio Science Team in the (14), the unphysical result $C/M_T R_T^2 > 0.4$ is obtained. This value is unphysical because it would indicates a solid core comparable to a material more thick of the iron. This inconsistency may be attributed or to a more complex internal structure or to some spin-orbit resonance.

The failure of a standard approach in the determination of the moment inertia factor is also suggested by the difference between the spin period and the mean motion. The non-synchronicity is a violation of the first Cassini law and cannot be explained in any simple way in the framework of celestial mechanics.

A new explanation is necessary if Titan's icy crust is decoupled from the solid interior, for example by means of a global subsurface ocean.

The current work shows the value and power of georeferencing methods for the determination of the rotational state of Titan and the construction of a body-fixed reference frame using SAR images. The application, to this new research, of SAR data, with precise orbit and gravity field determination, are crucial tools to provide information to build geophysical models of Titan interior. Additional SAR observations in a extended mission may reveal important effects such as polar wander and variations of the spin period. It is also clear that the actual estimates of both the obliquity and spin period are quite reliable.

REFERENCES

[1] G. Tobie, J.I. Lunine, C. Sotin, “Episodic outgassing as the origin of atmospheric methane on Saturn’s moon Titan”, *Nature* 440, 61–64

[2] http://ssd.jpl.nasa.gov/?sat_elem#saturn

[3] G. Colombo, “Cassini’s Second and Third Laws”, *Astronomical Journal* 71, 891 (1966).

[4] Bills and Nimmo 2007, “Forced obliquity and moments of inertia of Titan”, *American Geophysical Union, Fall Meeting 2006*, abstract #P12A-08

[5] G. Picardi, INFOCOM unpublished report (send requests to the author, giovanni.picardi@uniroma1.it)

[6] W.R. Ward and D.P. Hamilton, “Tilting Saturn, I. Analytic model”, *Astronomical Journal*, 128, 2501, 2004.

[7] B.W. Stiles et al., “Determining Titan's Spin State from Cassini RADAR Images”, *Astronomical Journal*, in press

[8] N. Rappaport, B. Bertotti, G. Giampieri, J.D. Anderson, “Doppler Measurements of the Quadrupole Moments of Titan”, *Icarus* V. 126, I. 2, April 1997, pg. 313-323

[9] Ralph D. Lorenz, Bryan Stiles, Randolph L. Kirk, Michael Allison, Paolo Persi del Marmo, Jonathan I. Lunine, Steven J. Ostro, Scott Hensley, “Titan’s Rotation Reveals an Internal Ocean and Changing Zonal Winds”, *Science* 319, 1649 (2008)