Markov Chain-based Analysis of Multihop IEEE 802.15.4 Wireless Networks With Finite Node Buffers

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***Abstract*— In this paper, we propose a Markov chain-based analytical framework for modeling the behavior of the medium access control (MAC) protocol in IEEE 802.15.4 wireless networks. Two scenarios are of interest. First, we consider networks where the (sensor) nodes communicate directly to the network coordinator. Then, we consider scenarios where (sensor) nodes communicate to the coordinator through an intermediate *relay* node, which forwards the packets received from the sources (i.e., the sensors). In both scenarios, no acknowledgment messages are used to confirm successful data packet deliveries, and communications are *beaconed* (i.e., they rely on synchronization packets denoted as "beacons"). In all cases, our focus is on networks where the relay and the source nodes have finite queues (denoted as *buffers*) to store data packets. Network performance is characterized in terms of aggregate network throughput and packet delivery delay. Our results show a very good agreement between the proposed analytical model and realistic ns-2 simulation results. In particular, the impact of the buffer size is accurately taken into account in our model.**

I. Introduction

Wireless sensor networks (WSNs) have recently received a significant attention in the research community. In particular, the development of low-power and low-cost devices makes WSNs a promising technology for the future [1]. In this context, the IEEE 802.15.4 standard has been successfully proposed [2]. An IEEE 802.15.4-compliant network comprises the following two main types of nodes: (i) a central coordinator, which initializes the network and manages its parameters; and (ii) remote sensor nodes, which collect data on the status of the physical phenomena of interest and transmit them to the coordinator (acting, typically, as a sink) through (possible) *multihop* communications. A wireless sensor node is composed by two sub-units: (1) a sensing unit, which detects the status of the phenomenon of interest; and (2) a wireless transceiver unit, which transmits the collected data (after possible local processing). For the sake of conciseness, in the remainder of this paper the term "sensor" will be used interchangeably with "sensor node." Moreover, three kinds of topologies are envisioned: (i) star, (ii) cluster-tree, and (iii) mesh. The IEEE 802.15.4 standard refers to the first two layers of the ISO/OSI stack protocol, i.e., physical and medium access control (MAC) layers.

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Several approaches, based on simulations and experiments, have been proposed for performance evaluation of IEEE 802.15.4 networks (see, for example, [3] and references therein). However, it is of interest to determine analytical tools which can predict accurately the network performance without resorting to lengthy simulations or difficult experiments. Relevant network performance indicators, which will be considered in this paper, are *throughput* and *delay*. In [4], the author proposes an analytical framework, based on a Markov chain characterization of the MAC protocol, for IEEE 802.11 networks in saturation conditions [5]. Based on this pioneering work, several approaches have been proposed for the characterization of the MAC performance in IEEE 802.15.4 networks with a star topology. In [6], the authors consider a scenario with acknowledgement (ACK) messages and analyze network performance in both saturation and non-saturation regimes, trying to characterize the conditions under which the network enters the saturation region. In [7], a simple Markov chain theoretical model is proposed to characterize both the sensors and the channel statuses, showing a good agreement with ns-2-based simulations [8]. This model allows to investigate throughput and energy consumption metrics. In [9], we have extended the framework proposed in [7] to 2-hop network scenarios, i.e., networks where sensors communicate with the coordinator through an intermediate *relay* node, which forwards data packets from the sources (the sensors) towards the destination (the coordinator). As in [7], all the nodes of the network (both sensors and relay) are assumed to have no finite buffers. Finally, in [10], [11] the authors propose the use of a relay for interconnecting two different clusters in IEEE 802.15.4 networks and analyze the performance through a queueing theoretical analysis. However, the proposed scenario models the (simpler) cases where the relay does not contend the medium access to the sensors.

In this paper, we combine the theory of discrete-time Markov chains (DTMCs), considered in [9], and the theory of Geo/G/1/L queues [12], in order to take into account the presence of buffers (and their sizes) at source and relay nodes. Our framework can be generalized to evaluate the performance of multihop networks with arbitrarily complex topologies (e.g., mesh networks or networks with multiple sinks), where the relays may contend the access to the shared medium to the remote sensor nodes. No ACK messages are used, whereas *beacons* (i.e., synchronization packets in support of data transmission) are. Our Markov chain-based framework allows to evaluate the *aggregate network throughput* and the *packet delivery delay*.[1] The analytical results will be compared with extensive ns-2 simulation results, showing a very good agreement.

This paper is structured as follows. In Section II, we provide the reader with a quick overview of the IEEE 802.15.4 standard. Section III contains the derivation of the novel analytical models for the considered networking schemes: we start from single-hop schemes with bufferless nodes, to end with 2-hop schemes with buffers at the nodes. In Section IV, numerical results obtained with the proposed analytical models are shown, and their excellent agreement with realistic ns-2 simulation results is discussed. Finally, concluding remarks are given in Section V.

II. IEEE 802.15.4 Standard Overview

The IEEE 802.15.4 standard refers to the first two layers levels of the ISO/OSI stack (i.e., physical, PHY, and MAC) and guarantees (theoretically) a transmission data-rate equal to 250 kpbs in a wireless communication link. Three transmission bands are allowed by the Zigbee standard: (i) 2.4 GHz, (ii) 868 MHz, and (iii) 916 MHz. While the first transmission band is worldwide available, the second and third are available only in Europe and USA, respectively.

Since the main goal of a IEEE 802.15.4 network is data transmission under the constraint of maximum power saving, a *beacon* frame structure can be employed. The beacon frame is divided into two main periods, referred to as *active* and *inactive*, respectively. While in the latter period all nodes go into the sleeping state to preserve their battery energy, in the former period all nodes can transmit their data packets. In order to prevent collisions, two different access techniques can be employed. In the Contention Access Period (CAP) every node can transmit according to a *non-persistent* Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA MAC) protocol, with the use of a proper binary exponential *back-off* (BEB) algorithm. In the Contention Free Period (CFP), instead, only nodes with a reserved time slot, denoted as Guaranteed Time Slot (GTS), can try to transmit data packets, so that collisions can be avoided. In this portion of time, only these nodes can transmit, and they find, therefore, the channel free. The dimensions of the beacon frame (related to the parameter called Beacon Order, BO), the duration of the active phase (also called *Superframe Duration*, SD), and the GTS are defined by two parameters which are exchanged within the beacon signal. This signal is periodically sent by the coordinator in order to synchronize all remote nodes in the network and signal the beginning of the beacon frame. Finally, two kinds of CSMA/CA protocol are envisioned: (i) *slotted* and (ii) *unslotted*. In the rest of the paper, we will only consider scenarios with slotted CSMA/CA.

[1] In the remainder, the aggregate network throughput and the packet delivery delay will be simply referred to as throughput and delay, respectively.

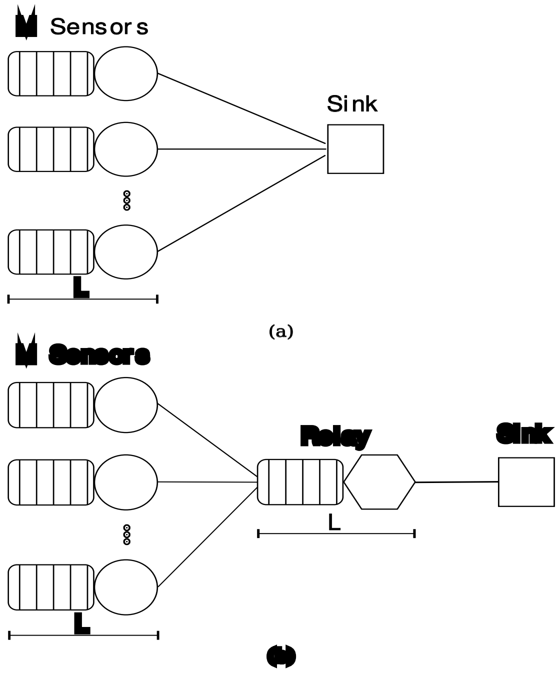


Fig. 1. Network scenarios of interest for (a) 1-hop and (b) 2-hop communications.

III. Markov Chain-based Models

The scenarios of interest for 1-hop and 2-hop networks are shown in Fig. 1 (a) and (b), respectively. All the nodes, except for the sink (coordinator), have a finite dimension buffer (at MAC level). Our analytical model allows to choose independently the buffer sizes of relay and source nodes. However, due to some limitations of the network simulator and for the sake of simplicity, all buffer sizes are fixed to the same value.

A. Single-hop Sensor Network Scenarios with Bufferless Nodes

The model for *bufferless* 1-hop networks with finite buffer is a direct extension of that proposed in [7]. We remark that by “bufferless” we mean that there is no queue for storing packets waiting to be scheduled for transmission, i.e., every time only the generated packet can be stored at the node. For the sake of clearness, we summarize the main assumptions behind this model. The simulation results will confirm the validity of these assumptions; in particular, the model is shown to be valid for values of BO larger than 5.

- According to the standard specifications, if the remaining time in the current superframe (a superframe is defined as the period between two successive beacons) is not sufficiently long for a data packet transmission to be completed, the node must defer it to the next superframe. When the duration of the superframe is sufficiently long and the traffic load is not too high, the above situation rarely appears and, therefore, can be neglected in the analytical model.
- The probability that the channel is sensed idle in a given backoff slot can be approximated by the steady state probability that the channel is idle. This assumption has

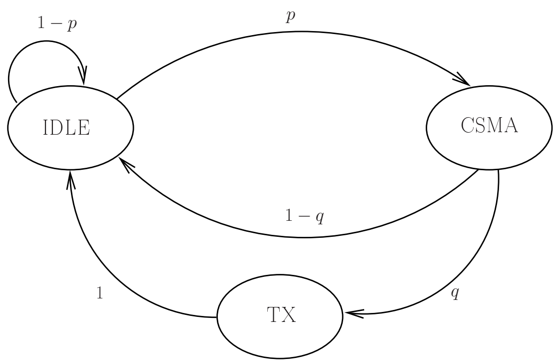


Fig. 2. DTMC model for a *bufferless sensor node* adhering to the IEEE 802.15.4 standard.

already been considered in the literature [13]. We remark that the channel idleness in one sensing backoff slot is not independent of that in the consecutive slot.

- The probability that a node begins its transmission in any generic backoff slot can be approximated by the steady state probability that a node transmits.

On the basis of these assumptions, it is possible to identify two semi-Markov processes (SMPs) and, consequently, two embedded DTMCs mutually coupled which model the system behavior with reasonable accuracy. Note that in [7] the network coordinator is not modeled, since it acts merely as a sink. Moreover, in [7] the authors assume that the distribution of the backoff duration is geometric, in order to take advantage on its memoryless property. However, since for SMPs only the average sojourn time is meaningful and not the entire distribution, it is possible to show that this assumption can be relaxed, leading to a model with lower complexity.

In Fig. 2, the DTMC model for a sensor node is shown. As one can see, the sensor can be in three states, denoted as "IDLE," "CSMA," and "TX," respectively. When the sensor is in the IDLE state, it can generate data packets according to a Poisson distribution and insert them in its buffer, which is supposed to contain only one packet ($d_{\rm buff}^{\rm node} = 1$). The parameter λ of the Poisson traffic distribution is equal to pN, where p is the probability that a new packet arrives during a backoff slot and N is the packet size (in backoff units). In this paper, the packet size is fixed and expressed in terms of the number N of backoff units: since at 2.4 GHz it is possible to send 10 bytes in each backoff unit, a packet size $N = 10$ means that the effective packet size is equal to 100 bytes.[2] When a packet is scheduled for transmission, the sensor moves into the CSMA state with probability p, and this probability depends on two parameters: (i) the probability that the channel is idle in a generic slot (referred to as $p_{\rm i}^{\rm c}$) and (ii) the probability that the channel is idle conditionally on the fact that it was also idle in the previous slot (referred to as $p_{\rm i|i}^{\rm c}$). The CSMA state embeds the details of the CSMA/CA MAC protocol and the BEB algorithm; these details are omitted here and can be found in [7]. Once the sensor has moved into the CSMA state, two transitions are possible: (i) the CSMA/CA MAC protocol succeeds in taking hold of the communication medium and the sensor moves to the TX state; (ii) if the CSMA/CA MAC protocol does not succeed (i.e., the backoff algorithms fails), then the packet is discared and the node comes back into the IDLE state. The first transition happens with probability q, and the second, obviously, with probability $1 - q$.

[2]In this work, no distinction is made between useful payload and packet headers.

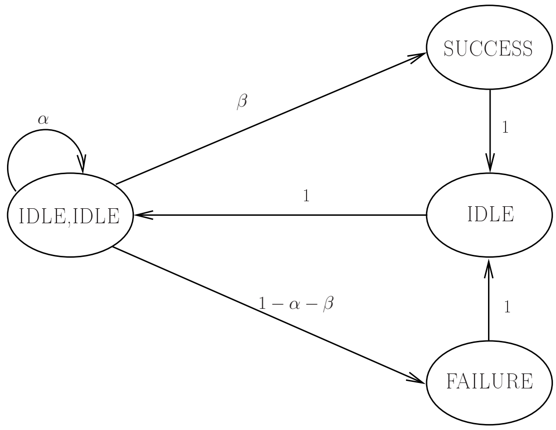


Fig. 3. DTMC model for the *physical communication channel* in IEEE 802.15.4 networks.

In Fig. 3, the DTMC model for the physical communication channel is shown. As one can see, in this case there are four states, denoted as "IDLE,IDLE," "SUCCESS," "IDLE," and "FAILURE." The first state of the channel DTMC is denoted as IDLE,IDLE, since before every transmission the channel has to remain idle for two consecutive slots, as stated by the IEEE 802.15.4 standard [2]. When a new packet is transmitted, the channel schedules a new transmission with probability $1 - \alpha$. With such probability, the channel can move into either one of two states: (i) if there is a single node transmission (i.e., no collision can happen) the channel moves to the SUCCESS state, and this happens with probability β; (ii) if, on the other hand, a collision happens, the channel moves to the FAILURE state with probability $1 - \alpha - \beta$. After a packet transmission has been carried out, either successfully or unsuccessfully, the channel moves to the IDLE state with probability 1 and then[3] to the IDLE,IDLE state with probability 1. The values of α and β are functions of the probability that a node transmits in a generic time slot, which can be obtained by solving the node DTMC. Therefore, the two DTMCs are mutually coupled.

B. Single-hop Sensor Network Scenarios with Buffered Nodes

In order to extend the original model to account for the presence of buffers and their sizes, making it more flexible, the following modifications, with respect to [9], are considered:

- we switch from continuous to discrete time regime;
- we insert finite size buffers at the MAC level of sources and relay.

The two modifications can be justified as follows. In the original model in [9], we implicitly use a continuous time

[3]One may think that the IDLE state could be merged with the IDLE,IDLE state in the DTMC shown in Fig. 3. However, the presence of the IDLE state is expedient for modelling the IEEE 802.15.4 standard, where a few free time slots are inserted before a new superframe is scheduled. The presence of the IDLE state, on the other hand, simplifies also the throughput calculation.

model with Poisson traffic sources. But, due to the slotted nature of the CSMA/CA mechanism and having fixed a packet size multiple of the slot period, it is also possible, by considering Bernoulli traffic sources, to study the system with a full discrete time model. With the original model it is not possible to handle the presence of a buffer with a finite size[4] L (dimension: [pck/s]).

A group of queues sharing the same channel cannot be analyzed with traditional tools of queueing theory, since all queues are reciprocally correlated. The key assumptions made in [9], namely the facts that (i) every source node has the same probability to access the channel in every time slot (denoted as $p_{\mathrm{t}}^{\mathrm{n}}$) and (ii) the probability that channel will be idle in a certain time slot (denoted as $p_{\mathrm{i}}^{\mathrm{c}}$) is always the same, lead to a decorrelation between the nodes' queues. Therefore, it is possible to use classical queueing theory tools to study a source node and determine the distribution of the number of costumers in its queue in correspondence to a slot boundary.

In the discrete time realm, a source node can be modeled as a Geo/G/1/L queue. The overall packet service time is composed of the time necessary to transmit a packet and the time spent during the channel access mechanism. Since the packet size is fixed (as stated in Section III-A), the distribution of the packet service time depends only on $p_{\mathrm{i}}^{\mathrm{c}}$ and on the probability (denoted as $p_{\mathrm{i|i}}^{\mathrm{c}}$) that the channel is idle, conditionally on the fact that it was idle in the previous slot. The maximum number of backoff attempts is $m = 4$ and the maximum duration of the waiting window for the i-th backoff attempt is $W_i = 8, 16, 32, 32, 32$ for $i = 0, 1, 2, 3, 4$, respectively.

The probability generating function (PGF) of the packet service time (denoted as $T_{\mathrm{t}}(z)$) can be expressed as a function of the PGFs of the i-th backoff period duration (denoted as $B_i(z)$) and the time necessary to transmit a packet (denoted as $T_{\mathrm{tx}}(z) = z^N$). Due to the uniform distribution of the waiting time of a generic backoff attempt, the following expression for $B_i(z)$ can be obtained:

$$B_i(z) = \frac{z^{W_i} - 1}{W_i(z-1)} \qquad i = 0, \ldots, m.$$

Finally, the PGF of the packet service time for the Geo/G/1/L queue can be expressed as

$$\begin{aligned} T_{\mathrm{t}}(z) &= \underbrace{\sum_{i=0}^{m} (1 - p_{\mathrm{i}}^{\mathrm{c}} p_{\mathrm{i|i}}^{\mathrm{c}})^i p_{\mathrm{i}}^{\mathrm{c}} p_{\mathrm{i|i}}^{\mathrm{c}} T_{\mathrm{tx}}(z) \prod_{j=1}^{i+1} B_{j-1}(z)}_{T_{\mathrm{succ(z)}}} \\ &\quad + \underbrace{(1 - p_{\mathrm{i}}^{\mathrm{c}} p_{\mathrm{i|i}}^{\mathrm{c}})^{m+1} \prod_{i=0}^{m} B_i(z)}_{T_{\mathrm{fail}}(z)} . \end{aligned} \tag{1}$$

Note that the first term at the right-hand side of (1) allows to account only for the packets effectively transmitted, regardless of their delivery status, while the second term accounts only for the packets discarded because of a CSMA/CA failure. The PGF of the packet service time has to depend on both these terms, because all the packets contribute to the server load. At the opposite, in the computation of the delay we will take into account only the packets which successfully access the channel after the CSMA/CA procedure. Note that the delay is considered to be the same for both successfully delivered and collided packets, since we are not considering the use of ACK messages.

[4] L is the capacity of the entire node, formed by a queue of size $L-1$ and a server able to process one packet at a time. Therefore, a node can store L packets.

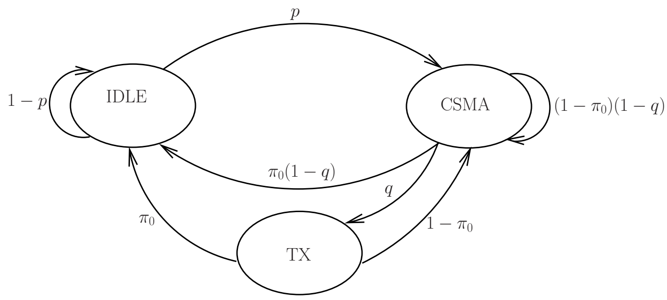


Fig. 4. Markov chain model for a *buffered* sensor node according to the IEEE 802.15.4 standard.

From (1), using the well-known procedure described in [12], one can derive the distribution of the number of customers that are queued at a generic instant (in correspondence to a slot boundary) and at the time of a packet departure (always in correspondence to a slot boundary). The latter can be represented through a vector of size L, denoted by $\boldsymbol{\pi} = [\pi_0 \pi_1 \ldots \pi_L]$, where π_i is the probability that the queue has i packets immediately after a packet departure.

At this point, the main challenge consists in inserting Geo/G/1/L queues in the original model presented in [9]. The presence of finite size buffers affects only the transitions exiting from the CSMA macrostate and the TX state of the node DTMC, while the channel DTMC is not affected at all. In particular, the presence of a queue modifies the behavior of a source node just after a packet departure, either a successfull departure or a packet drop (because of a CSMA/CA failure). In this situation, there are two possible exiting transitions from the CSMA macrostate:

- to the IDLE state, if the queue is empty at the time of the packet departure: this event happens with probability equal to π_0;
- to the CSMA macrostate, if the queue is not empty at the time of the packet departure: this event happens with probability equal to $1 - \pi_0$.

Due to the assumptions made previously, these variations do not affect the "nature" of the process governing the behavior of a source node, i.e., this process remains a SMP. For this reason, the embedded DTMC is still valid and its ergodicity property is maintained. The new node DTMC is shown in Fig. 4. When π_o is known, using the balance equations the stationary distribution of the DTMC can be computed in closed form. For lack of space, we do not report here the analytical details.

We now comment on the mathematical nature of the modelling problem at hand. In the original scenario formed by nodes without buffers, the two DTMCs (of source nodes and channel) are coupled [9]. In the current scenario with buffers, there is a three-element coupling, between the two DTMCs and the Geo/G/1/L queue system. This problem can be simplified by the following observations:

- the node DTMC depends on π_0 and p_i^c;
- the Geo/G/1/L queue depends on p_i^c;
- the channel DTMC depends on the distribution of the node DTMC through the parameter p_t^n.

Therefore, we have a non linear fixed-point equation, where the fixed point is p_i^c. In fact, by fixing this probability, it is possible to solve the Geo/G/1/L system and, consequently, the node DTMC. At this point, by solving the channel DMTC it is possible to compute the new value of p_i^c. By iterating this procedure until fixed and computed values of p_i^c are equal, the fixed-point equation can be solved. Although we cannot prove the existence and uniqueness of this solution, from the obtained results we conjecture that this problem admits, with high probability, a single solution.

Finally, we describe how to evaluate the throughput and the delay through the proposed model. The insertion of a finite buffer has no impact on the definition of the throughput, which can be computed as the average fraction of time (over a sufficiently long time horizon) spent in the SUCCESS state by the channel SMP, as stated in [7]. The computation of the delay is, instead, slightly different. Given the queue distribution at the time of a departure ($\boldsymbol{\pi}$), using Little's law the average waiting time can be computed as

$$\overline{W} = \frac{\sum_{\ell=1}^{L-1} \ell\pi_\ell + L(\pi_o + \rho - 1)}{p} - \overline{T}_\mathrm{t} \qquad (2)$$

where $\rho = p\overline{T}_\mathrm{t}$. The packets dropped due to CSMA/CA failures experience a different average *service* time ($\overline{T}_\mathrm{fail}$) with respect to the transmitted (successfully or not) packets ($\overline{T}_\mathrm{succ}$). However, the average *waiting* time is the same for all packets. Therefore, from (1) and (2) one can evaluate the average delay of a transmitted packet:[5]

$$\overline{D} = \overline{W} + \overline{T}_\mathrm{succ}. \qquad (3)$$

C. 2-hop Sensor Network Scenarios

In this section, we show how to modify the proposed DTMC-based models for 1-hop scenarios, in order to take into account the presence of an intermediate relay. In [9], one can find more details about a possible model for *bufferless* scenarios. However, this model is not of practical interest, since it has been created in an ad-hoc way and it can not be easily generalized. Therefore, in the following we will only present the model for *buffered* scenarios.

Modelling multihop networks with buffers at the nodes presents several difficulties: the numbers of customers in the network queues are strongly correlated; the service times of the relay queues are correlated with the arrival process distribution (the packet dimension is fixed and every node shares the same channel); the arrival process at the relay nodes cannot be characterized easily. Without proper approximations, almost every analytical model will become mathematically intractable. Roughly speaking, the modelling goal consists in reaching the best tradeoff between analytical complexity and accuracy of the results. The considered assumptions can be summarized as follows.

- Due to the Geo/G/1/L theory, it is possible to determine the output process of source nodes [12]. However, the arrival process at the relay is not a linear combination of the output process of each source node. Due to the collisions between transmitted packets, not all packet departures from a source node become new arrivals at the relay queue. Moreover, when the relay is involved on a Clear Channel Assessment (CCA) operation, it cannot receive any packet, regardless of its queue status [2]. Therefore, we arbitrarily assume that the arrival process at the relay has a Bernoulli distribution with a parameter, denoted as p^r, which is different with respect to the parameter p at the source nodes.
- Since the packet size is fixed and owing to the assumption that every node sees the same long-term channel condition, the packet service time is the same for relay and source nodes. Note however, that the corresponding queues have different behaviors since their arrival processes are different.
- We assume that the numbers of customers in different queues are independent and, therefore, we determine their stationary distributions in an independent way.
- The service time is independent from the arrivals. In fact, if a packet experiences a short channel access delay at the source nodes, no assumption about the channel access delay at the relay can be made.

Due to the above assumptions, we can study a two-hop scenario by simply adding a relay DTMC equal to the sources' DTMC, except for a different Bernoulli parameter $p^\mathrm{r} \neq p$. Moreover, at the relay we add a different Geo/G/1/K queue at the relay, equal to that of a source node except for the arrival process parameter. Therefore, we are in the presence of three DTMCs and two Geo/G/1/L queues mutually coupled. While in the one-hop scenario there is a single unknown parameter p_i^c, in this case there are two unknown parameters: p_i^c and p^r. To solve this problem, we extend the iterative algorithm introduced on Subsection III-A: we fix the initial values of p_i^c and p^r until we obtain the same values from the analytical system. In this case as well, we cannot prove the existence and uniqueness of the obtained solution, but we can heuristically claim its uniqueness in the interval of interest.

Finally, we can compute the throughput and delay for two-hop networks. As in [9], we define the throughput S as the long-term fraction of time spent for successfull transmissions, i.e., in the TX state, by the relay. The average end-to-end delay is simply obtained by adding the average delay experienced by a source node, denoted as $\overline{D}^\mathrm{s}$, and the average delay experienced by the relay node, denoted as $\overline{D}^\mathrm{r}$:

$$\overline{D} = \overline{D}^\mathrm{r} + \overline{D}^\mathrm{s}.$$

IV. Numerical Results

We now evaluate the performance of IEEE 802.15.4 wireless sensor networks with the proposed Markov chain-based ana-

[5] We assume that delay of a collided packet is the same of a packet successfully transmitted.

lytical models, verifying them through extensive ns-2 simulations. In particular, we use the ns-2 2.31 version and the built-in model, denoted as "wpan," available in this version [8]. In order to fulfill our requirements, we apply some modifications to the original source code of the simulator. In particular, we implement a Bernoulli traffic source, and install the No Ad-Hoc (NOAH) module to set up *static* routing in two-hop scenarios. Note that the CCA mechanism is slightly modified, since we think that its current ns-2 implementation is not compliant with the IEEE 802.15.4 standard specifications. The considered set-up is composed by $M = 12$ sensors, which communicate with the coordinator either directly (1-hop networks) or through a relay (2-hop networks). The traffic generation model is Bernoulli with parameter p and each node generates packets with constant size of 100 bytes ($N = 10$ backoff units). The other network parameters are set according to the IEEE 802.15.4 standard: the beacon order, as well as the superframe order, is set to 6, whereas the minimum and maximum backoff exponents are set to 3 and 5, respectively. The duration of each simulation run is set to 1000 seconds and the presented results are obtained by averaging over 10 simulation runs in order to eliminate statistical fluctuations. Note that this set-up is in agreement with that considered in [7].

Throughput and delay are evaluated as functions of the aggregate offered load and for different values of the buffer length. In particular, we define the average aggregate offered load (dimension: [bit/s]) as:

$$g = 80 \cdot M \cdot N \cdot p$$

where p, M, and N have been already introduced, while 80 is the number of bits transmitted in each slot. In order to derive a normalized traffic load, we divide g for the maximum transmission rate of IEEE 802.15.4 (i.e., 250000 b/s), defining the average normalized aggregate offered load as $G \triangleq \frac{g}{250000}$.

First, we present some results relative to the bufferless scenario. In the analytical case, the sensor DTMC model in Fig. 2 is used for the source nodes, whereas the model proposed in [9] is used for the relay. The channel model is that presented in Fig. 3. In Fig. 5, the throughput is shown, as a function of the per-node average offered load λ, in 1-hop and 2-hop communication scenarios. Both analytical and simulation results are presented. The analytical results are obtained with (node and relay) buffer length equal to 1, whereas it is 2 in the simulator.[6] The curves associated with the 1-hop scenario are in agreement with those presented in [7]: the throughput is increasing for small values of the offered load, whereas it is slightly decreasing for large values of λ. The throughput associated with 2-hop scenario is lower than that in 1-hop scenarios, because of the presence of the relay which increases the number of packets lost at the sensors–when the relay is transmitting data to the coordinator, it can not receive other incoming packets from the sensors. More precisely, there is a well pronounced maximum in a 2-hop scenario, beyond which the throughput rapidly reduces. One can also note a good agreement between simulation and analytical results in 1-hop scenario. However, the agreement in a 2-hop scenario is slightly worse. In particular, for low values of the offered load, the simulated throughput is higher than the analytical value since a larger number of packets can be correctly delivered to the destination. When the offered load is sufficiently high, instead, the throughput estimated by the ns-2 simulator is lower, since the relay is busy for a longer time and more packets, generated at the sensors, can be lost. Finally, similar considerations about the delay can be found in [9].

[6]In fact, the ns-2 module does not implement bufferless nodes, since there is a queue at the MAC level with at least size 1.

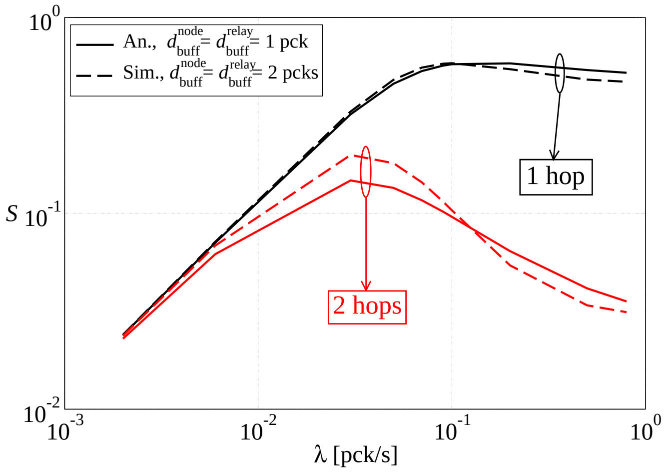
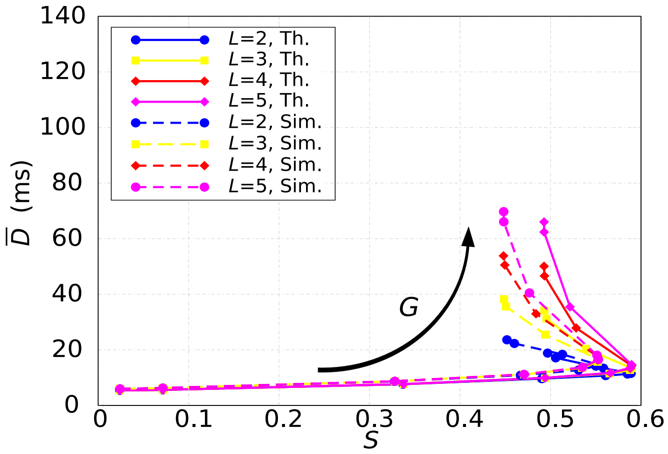


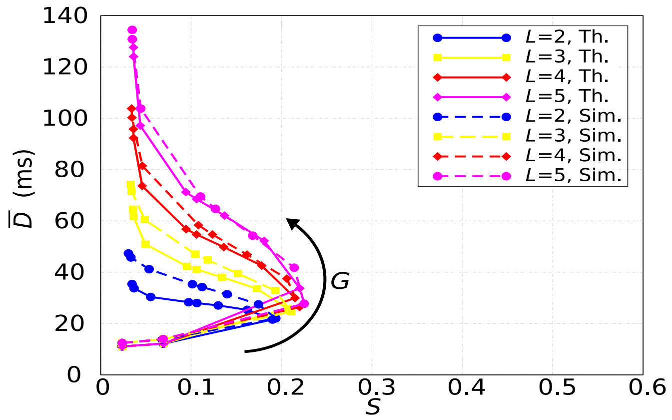
Fig. 5. Throughput, as a function of the per-node offered load, in a scenario with $M = 12$ sensors and 1-hop and 2-hop communications with the coordinator. Analytical (sollid lines) and simulation (dashed lines) results are shown, with buffer lengths equal to 1 pck and 2 pcks in the two cases, respectively.

We now present results for finite size buffers at the sensors and the relay. In the analytical case, the sensor DTMC model in Fig. 4 is used for both source and relay nodes, except for different values of the DTMC parameter. The channel model is the same of Fig. 3. For the sake of conciseness, we do not show the throughput and delay as functions of the normalized traffic load G, but we combine them in a classical Throughput-Delay curve (S-D). Obviously, having fixed the number of nodes ($M = 12$) and the packet size (100 bytes, i.e., $N = 10$ backoff units), G is only influenced by p. In order to evaluate the effect of the buffer length on the network performance, we obtain a family of S-D curves by varying the buffer size L at the sources and the relay.

In Fig. 6 (a), we show the S-D curve for 1-hop scenarios. Simulation (dashed lines) and analytical (solid lines) results are obtained with (node and relay) buffer lengths in the set $\{2,3,4,5\}$. All curves show clearly that the considered networks have a bimodal behavior. For low values of G, the delay experienced by the packets is very short and the throughput is proportional to the aggregate traffic load. In this region, the buffer length has no impact on the network performance. When the aggregate traffic load is sufficiently high, the network enters into a saturation (or unstable) regime, where the delay quickly increases and the throughput is not proportional to G. In the saturation zone, the curves show that the dimension of the buffer has a slight impact on the



(a)



(b)

Fig. 6. Delay as a function of the throughput, in scenarios with (a) 1-hop and (b) 2-hop communications. In all cases, $M = 12$ and $N = 10$. The curves are parameterized respect to the average normalized aggregate offered load (G). G is increasing and assumes the following values, from the lower left corner to the upper side of the figure: $0.024, 0.072, 0.36, 0.6, 0.84, 1.08, 1.2, 2.4, 6, 9.6$. Analytical (solid lines) and simulation (dashed lines) results are shown. Different values for the buffer length are considered: 2, 3, 4, and 5.

throughput: in particular, its maximum and saturation values remain roughly the same—this behavior is due to the self-stability feature of the CSMA/CA MAC protocol adopted by the IEEE 802.15.4 standard. A buffer length increase, instead, leads to a rapid delay increase. Finally, we remark the excellent agreement between analytical and simulation results, considering that the curve trend is perfectly captured. The limited numerical discrepancy does not depend on L, and can be considered as a bias.

In Fig. 6 (b), we study the S-D behavior in 2-hop scenarios. Simulation (dashed lines) and analytical (solid lines) results are obtained with values of the (node and relay) buffer length in the set $\{2, 3, 4, 5\}$. As in the 1-hop scenarios, it is possible to identify two operative regions. In the former, the throughput increases proportionally to the load and the delay is bounded. In the latter (saturation region), the throughput quickly drops and reaches very low values for high traffic loads. The relay acts clearly as a bottleneck, limiting significantly the self stabilizing capacity of the CSMA/CA access mechanism. Note that the performance degradation, in terms of delay, is not as pronounced as in terms of the throughput. In fact, the delay is roughly twice that in the single hop scenarios. As for 1-hop scenarios, in the considered 2-hop scenarios one can observe the very good agreement between simulation and analytical results. Therefore, the approximations introduced in Subsection III-C are meaningful.

V. Conclusions

In this paper, we have presented a novel analytical framework that combines the theory of DTMCs and classical queueing theory for modelling the behavior of the MAC protocol in IEEE 802.15.4 multihop WSNs, with finite size buffers at the nodes. The throughput and delay have been evaluated and the performance predicted by our analytical model is in excellent agreement with realistic ns-2 simulations. Our results highlight that the presence of an intermediate relay drastically reduces the throughput with respect to 1-hop scenarios, i.e., the relay acts as a bottleneck. This suggests that the use of relays which share the wireless medium with source nodes is not effective in improving the network range. We point out that while we have limited our study to the topologies in Fig. 1, our framework is applicable to complex networks with multi-hop communications. In order to improve the performance in multihop scenarios, in the future we intend to pursue two different strategies: clusterization of source nodes and partition of the collision domain into smaller subsets (using the active and passive operative mode of IEEE 802.15.4).

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