Efficient Synchronization and Equalization for Short Bursts

Pietro Savazzi, Paolo Gamba, and Lorenzo Favalli

Dipartimento di Elettronica, Università degli Studi di Pavia
name.surname@unipv.it

1 Abstract

In this work a joint clock recovery (CR) and equalization scheme for short burst transmissions is presented. This approach may be useful in modern wireless packet data networks and the joint optimization performance may be pursued by means of both data aided and decision directed solutions. The novel algorithm is based on an iterative scheme, exploiting a timing error function sampled at symbol rate. The symbol timing adjustment is implemented by an interpolation filter, built according to the Farrow structure. Equalization is obtained by baud spaced zero forcing (ZF) and minimum mean square error (MMSE) linear filtering. Performance is evaluated by simulating a QPSK transceiver and simulations results are compared with the ideal solutions, for both symbol timing recovery and channel equalization, under frequency selective multipath fading channel conditions.

2 Introduction

In digital communication receivers, frequency selective fading channel impairments may be counteracted by employing a linear baud spaced equalizer, under the assumption of a perfect symbol timing recovering. In order to mitigate the strong dependance on symbol timing errors, a fractionally spaced equalizer working at least at two samples per symbol can be used [1][2]. Clock synchronization can be accomplished through an analog loop, controlled by a digital error, which tracks the received symbol timing and usually samples the incoming signal at twice the symbol rate [3]. An all digital implementation can be based on filter interpolation which is efficiently hardware implemented by the Farrow structure, in which the only variable parameter is the fractional phase delay [4]. Farrow interpolation filters may be realized as all-pass [5] or they can be included in the Nyquist receiver filter [6]. In [7] a polynomial-based maximum-likelihood technique is presented for open loop synchronization in an all digital receiver which uses a Farrow interpolator, while in [8] a similar receiver structure is exploited for designing a very fast digital symbol timing recovery (STR) by using a second order interpolator, obtaining, as a unique drawback, a relatively poor performance due to the low filter order. Following this idea, a data aided STR scheme for burst receiver was presented in [9], based on using a second order interpolator for fast estimating the correct receiver fractional delay while a higher 4th or 6th order filter is used for increasing the interpolation accuracy as well as the CR performance. A recent paper revealed that the joint design of both the interpolation filter and a decision feedback equalizer can improve the receiver performance [10]. According to these premises, the basic ideas shown in this paper are to derive a new and fast STR scheme useful for burst transmission, starting from [8][9], and including it in the equalization process, by an iterative algorithm, in order to better exploit the information contained in the received burst sequence. The new STR burst-by-burst algorithm is built using a timing error function found in [11], adapted to work in a burst receiver. In this sense the scope is to extend the results found in [10] to an adaptive situation where an iterative receiver exchange information between the STR interpolator and the equalizer, in a way similar to those shown in [12]. The algorithm proposed in this paper, instead of attempting to directly estimate the channel delay and equalizer coefficients, is based on the iterative re-filtering of the received burst data. At each iteration the symbol timing phase and equalizer tap values move towards increasing values of likelihood, according to the generalized expectation maximization (EM) framework [13].

3 The proposed scheme

The baseband received signal can be modeled as:

$$r(t) = \sum_{k=-\infty}^{\infty} d(k)h(t - kT - \tau T) + w(t) \tag{1}$$

where $d(k)$ are the transmitted symbols (assumed QPSK although the results can be easily extended to M-QAM modulations) with timing period T, $h(t)$ comprises the channel and all the transmitter and receiver filters and $w(t)$ represents the AWGN (Additive White Gaussian Noise) contribution. The imperfect sampling time is taken into account by the unknown fractional delay parameter τ and the cascade transceiver filters are modeled as a raised cosine frequency response filter. The channel fading is modeled as frequency selective while time selectivity can be negligible according to the short burst framework. After the analog to digital conversion, the signal is sampled at twice the symbol rate and can be represented by considering separately the two symbol halves:

$$r(2n) = \sum_{k=-\infty}^{\infty} d(k)h(k-2n-\tau/T) + w(2n)$$
$$r(2n+1) = \sum_{k=-\infty}^{\infty} d(k)h(k-2n-1-\tau/T) + w(2n+1) \tag{2}$$

where $r(n) \equiv r(nT)$.

The first step of the algorithm is the choice of the half symbol sample closest to the right timing. This can be accomplished according to the maximum likelihood principle [1][3][7]:

$$L_0 = \Re\left[\sum_{n=0}^{N-1} r^*(2n)d(n)\right]$$
$$L_1 = \Re\left[\sum_{n=0}^{N-1} r^*(2n+1)d(n)\right] \tag{3}$$

L_0 and L_1 represent the likelihood functions computed for the even and odd samples while $d(n)$ are the preamble known symbol for the data aided (DA) case or the estimated ones at the receiver output for the decision directed (DD) mode. N is the number of symbols considered in the computation, and may coincide with the burst length for the DD operation mode, while "*" represents the complex conjugate.

3.1 Digital filtering by Interpolation

The Farrow structure of the interpolation filter consists of $L+1$ parallel FIR branch components with fixed coefficients having transfer functions $C_l(z)$, for $l = 0, 1, ..., L$, and only one variable parameter μ. The parameter L is the degree of the polynomial while μ represents the fractional delay timing error correction. The impulse response of the interpolator in each sampling time interval $T_s = \frac{T}{2}$, where T is the symbol time, is:

$$p(t) \equiv p(iT) = p\left[(k+\mu)T_s\right] = \sum_{l=0}^{L} c_l(k)\mu^l \tag{4}$$

The output of the Farrow scheme is:

$$y(n) = \sum_{k=-M/2}^{M/2-1} r(n-k)p\left[(k+\mu)T_s\right] = \sum_{l=0}^{L} f_l(n)\mu^l \tag{5}$$

where:

$$f_l(n) = \sum_{k=-M/2}^{M/2-1} r(n-k)c_l(k); \tag{6}$$

and M, the Farrow branch filter length, is equal to the filter order plus one. The basic idea of this structure is that the outputs $y(i)$ form a polynomial approximation for the continuous-time signal $r(t)$ at time instants

$iT = (n + \mu T_s)$. The obvious advantage, in terms of hardware implementation complexity, is that the filter coefficients are constant and the output time sampling is only controlled by the parameter μ. The design of Farrow interpolators can be done in several ways and traditionally it is based on Lagrange polynomials. In this work we consider the Farrow structure for the Lagrange interpolator polynomials, as reported in [5], which satisfies the following condition:

$$\mathbf{V}\mathbf{c} = \mathbf{z} \tag{7}$$

where:

$$\begin{aligned} \mathbf{c} &= [C_0(z)\ C_1(z)\ \cdots\ C_L(z)]^T \\ \mathbf{z} &= [1\ z^{-1}\ \cdots\ z^{-L}]^T \end{aligned} \tag{8}$$

and $\mathbf{V}$ is the Vandermonde matrix. The solution of (7) provides a filter structure in which the fractional delay $\mu \in [0, 1]$ and a constant phase delay with respect to the filter order.

A more efficient construction is suggested in [5], with a new parameter range equal to $[-0.5, 0.5]$. This can be pursued by employing the matrix transformation $\mathbf{T}$:

$$T_{n,m} = \begin{cases} \text{round}(\frac{L}{2})^{n-m} \binom{n}{m} & \text{for } n \geq m \\ 0 & \text{for } n < m \end{cases} \tag{9}$$

where $n,\ m = 0,\ 1,\ \cdots,\ L$ and the new filter is obtained by replacing the solution of (7) with:

$$\mathbf{c} = \mathbf{T}\mathbf{V}^{-1}\mathbf{z} \tag{10}$$

3.2 Generalized EM algorithm for joint STR and equalization

The EM algorithm is a very general framework that provides an iterative procedure for computing maximum likelihood estimation (MLE) in situations where some parameters are missing. Let $\mathbf{r}$ be the random vector corresponding to the received signal samples. This is the incomplete set, while $\mathbf{y} = (\mathbf{r}, \mu, \mathbf{b})$ represents the complete data set; the receiver must choose the estimated symbol vector which maximizes:

$$p(\mathbf{d}|\mathbf{r}) = p(\mathbf{d}|\mathbf{r}, \mu, \mathbf{b}) f(\mu, \mathbf{b}) \tag{11}$$

where the fractional timing delay μ and the equalizer coefficients $\mathbf{b}$ are the missing parameters and $f(\mu, \mathbf{b})$ is their joint probability density function.

The EM algorithm proceeds iteratively by replacing the complete data likelihood function by its conditional expectation given $\mathbf{r}$ using the current estimate of $(\mu, \mathbf{b})$, called $(\mu_k, \mathbf{b}_k)$. The maximization of (11) can be done by using the subsequent two-step iterative procedure:

1. E-Step.
   Calculate $Q(\mu, \mathbf{b}; \mu_k, \mathbf{b}_k) = E\{p(\mathbf{d}|\mathbf{r}, \mu, \mathbf{b})|\mathbf{r}_k, \mu_k, \mathbf{b}_k\}$

2. M-Step.
   Choose $(\mu_{k+1}, \mathbf{b}_{k+1})$ that maximize $Q(\mu, \mathbf{b}; \mu_k, \mathbf{b}_k)$, that is $Q(\mu_{k+1}, \mathbf{b}_{k+1}; \mu_k, \mathbf{b}_k) \geq Q(\mu, \mathbf{b}; \mu_k, \mathbf{b}_k)$.

When a closed form solution of the M-Step computation does not exist, it is possible to use the generalized version of the algorithm for which the maximization over all the possibile parameter values is replaced by [13, section 1.5.5, page 28]:

$$Q(\mu_{k+1}, \mathbf{b}_{k+1}; \mu_k, \mathbf{b}_k) \geq Q(\mu_k, \mathbf{b}_k; \mu_k, \mathbf{b}_k) \qquad (12)$$

That is choosing $(\mu_{k+1}, \mathbf{b}_{k+1})$ to increase the function over the parameter values $(\mu_k, \mathbf{b}_k)$ instead of maximizing it over all the possible outcomes.

In our scheme the E-Step corresponds to the interpolation of the matched filter output samples, using the Farrow structure described in (10), followed by the block-by-block equalizer. The M-Step is obtained in the generalized sense by using the type B timing error function, as called in [11], and the equalizer coefficients re-computation.

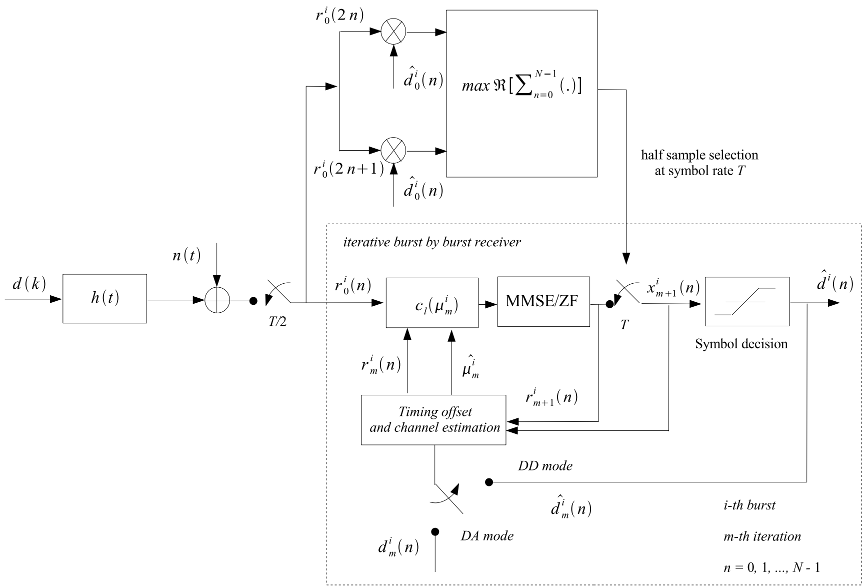

Fig. 1. The proposed scheme

3.3 The complete procedure

In fig. 1, the complete processing chain is shown while in the following each step of the iterative algorithm is explained in details. Please note that in the following equations superscript index represents the i-th received burst while subscript m is used for numbering the iterations; N is the burst length while N_p is the preamble length or the number of samples used for computing the timing delay and the equalizer filter coefficients in the DD operation mode. In the latter case N_p can be set $\leq N$.

Using the equations discussed in previous sections the detailed algorithm steps for the i-th burst are:

1. the half symbol closest to the right timing sampling is chosen according to (3) in the initialization step and it is kept fixed for all the i-th burst iterations;
2. According to the previous step the signal is down-sampled to T and the DD symbols are estimated if needed by the DD mode operation:

$$\begin{aligned} x^i_m(n) &= r^i_m(2n) \quad \text{if } L_0 > L_1 \\ x^i_m(n) &= r^i_m(2n+1) \text{ if } L_1 > L_0 \end{aligned} \tag{13}$$

$$\hat{d}^i_m(n) = g[x^i_m(n)] \tag{14}$$

   where the function $g(\cdot)$ is the output of the M-QAM symbol detector;
3. the channel impulse response is estimated using the symbol preambles or the DD ones from (14) [12], for
   $n = 0,\ 1,\ \cdots,\ N_p - 1$:

$$\hat{h}^i_m(k) = \frac{1}{N_p} \sum_{k=0}^{N_p-1} x^i_m(n) d^i_m(k-n); \tag{15}$$

4. the fractional delay parameter μ^i_m is computed, following the timing error function reported in [11] [3], by means of

$$\mu = \tau/T \simeq \Re(\hat{h}(1)); \tag{16}$$

5. the estimated channel impulse response $\hat{h}^i_m$ is over-sampled, by zeros insertion, and filtered by the Farrow branch filters c_l, built according to (10):

$$y^i_m(j) = \sum_{l=0}^{L} \sum_{k=-M/2}^{M/2-1} \hat{h}^i_m(j-k) c_l(k) (\mu^i_m)^l, \tag{17}$$

   where $j = 2n$;
6. The equalizer impulse response is obtained from the previous equation by the application of the zero forcing criterion:

$$\mathbf{b}^i_m = (\mathbf{Y}^i_m (\mathbf{Y^H})^i_m)^{-1} \mathbf{y}^i_{\mathbf{D}m} \tag{18}$$

   or the MMSE one:

$$\mathbf{b}_m^i = (\mathbf{Y}_m^i(\mathbf{Y^H})_m^i + \sigma^2\mathbf{I})^{-1}\mathbf{y}_{\mathbf{D}m}^i \tag{19}$$

where $\mathbf{b}_m^i$ denotes the vector representation for the equalizer impulse response, $\mathbf{Y}_m^i$ is the convolution matrix of the channel impulse response provided by (17), down-sampled to symbol rate, while $\mathbf{y}_{\mathbf{D}m}^i$ denotes the dth column of $\mathbf{Y}_m^i$, representing the delay of the equalizer chosen in order to obtain a non-casual and symmetric impulse response. $\mathbf{I}$ is the identity matrix. In case of fractionally spaced equalization the rate of $y_m^i(j)$ is left unchanged;

7. The received signal $r_m^i(n)$ is filtered by $b_m^i(j)$, overs-ampled by two, where N_e denotes the equalizer filter length:

$$r_{m+1}^i(j) = \sum_{l=0}^{L_e} r_m^i(j-l)b_m^i(j) \tag{20}$$

All the steps from 2) are repeated until convergence.

The algorithm can be stopped when it reaches a maximum number of iterations or until

$$\sum_{j=0}^{N_e-1} |b_{m+1}^i(j) - b_m^i(j)| < \epsilon_{\mathsf{EQU}} \tag{21}$$

$$|\mu_m^i| < \epsilon_{\mathsf{STR}} \tag{22}$$

where $\epsilon_{\mathsf{EQU,STR}} \simeq 0$ represent the requested precisions, respectively for the estimated equalizer coefficients and the fractional timing error correction. Relation (22) is due to the fact that during iterations, the estimated fractional delay parameters go to zero as the received signal is re-filtered by the interpolator. This is similar to the behavior of a clock recovery loop using the Mueller and Müller TED [11].

Down-sampling by two prior to filtering, in steps 5) and 7) can be easily and jointly obtained by means of a polyphase implementation [14].

4 Performance Evaluation

Performance of the new scheme has been evaluated by simulating a QPSK transceiver with raised cosine pulse shaping, affected by frequency selective multipath fading and additive white Gaussian noise. Multipath fading is modeled as a two rays channel with impulse response equal to $\mathbf{h} = [1\ 0.5]$. The estimated channel impulse response has three tap coefficients while the equalizer length is set to nine. The order of the interpolation filter is four and it has been obtained by (10). The rolloff factor of the shaping filter is $\alpha = 0.35$

SER curves (see Fig. 2,3) have been obtained considering the worst case, corresponding to a channel delay $\tau = 0.25T$ and the channel estimator operating in the decision directed mode. The iterative algorithm stops when it reaches a maximum number of iterations equal to 10.

Simulation results of the new scheme are compared with those obtained assuming ideal clock recovery and known channel impulse response, considering the same equalizer filter length. In DD mode, all the transmitted burst symbols have been used for channel impulse response and delay computations ($N_p = N$).

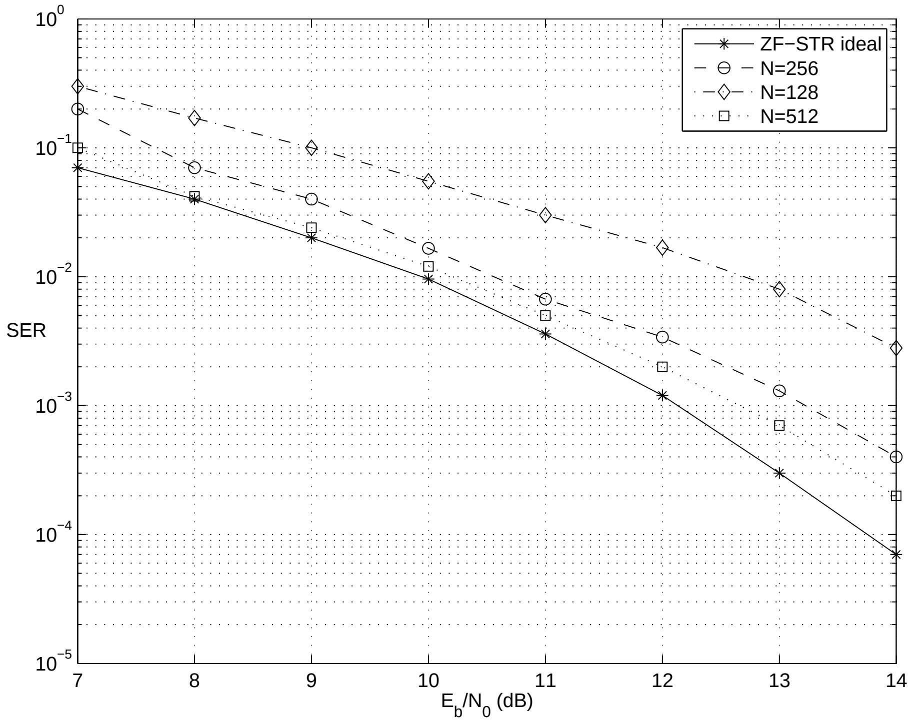

Fig. 2. SER versus E_b/N_0, DD case, $N_p = N = 128, 256, 512$.

5 Conclusion

This work introduces a novel burst-by-burst joint symbol timing recovery and linear equalization scheme, which makes use of an adaptive computation of

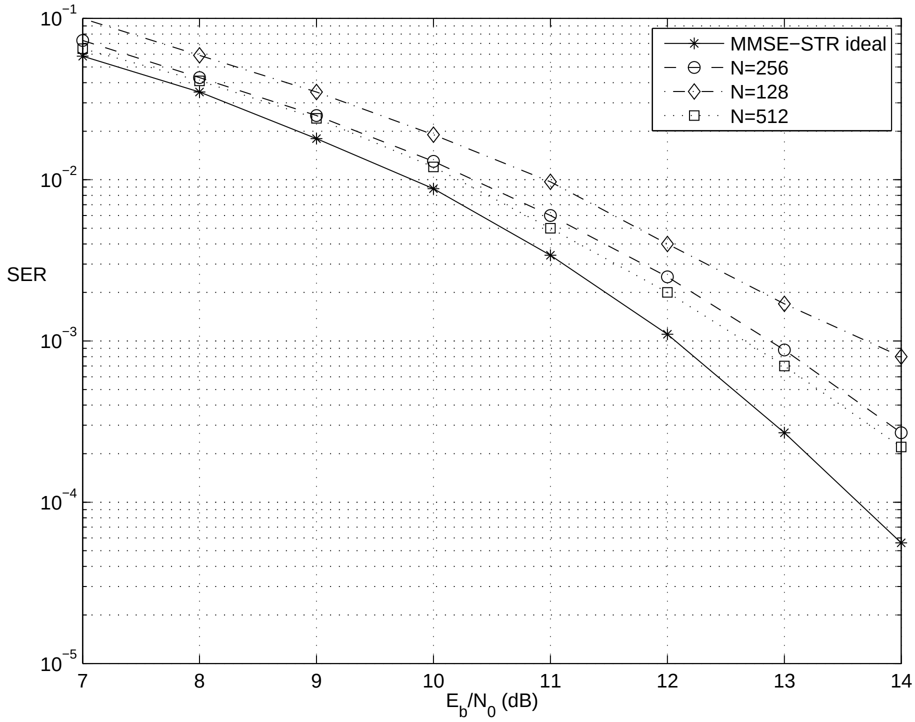

Fig. 3. SER versus E_b/N_0, DD case, $N_p = N = 128, 256, 512$.

the timing error function, by exploiting the impulse response channel estimate in an iterative manner. The timing error function output is used directly as fractional delay parameter of an all-pass Farrow polynomial interpolator while the linear equalizer coefficients are computed using the same estimated channel impulse response by the application of the ZF and MMSE criteria. The scheme convergence is assured by the generalized framework of the expectation maximization algorithm [13].

Presented results look promising, especially from the point of view of using this solution jointly with a more sophisticated symbol estimation technique, like decision feedback equalization (DFE) [10], decoding scheme or a combination of both [15].

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