

# Ultrawideband Communications in the Presence of Spatially Distributed Narrowband Interferers

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**Abstract**—This paper presents an analytical approach to derive the outage probability of an ultrawideband (UWB) communication in the presence of narrowband (NB) interference. In particular, we propose a closed-form expression for the bit error probability of UWB systems adopting a Rake receiver and affected by multiple NB interferers. The spatial distribution of the interferers is accounted for by resorting to a Poisson point process in the two-dimensional plane. The realistic scenario considered accounts for asynchronous interfering nodes in a wireless environment subject to path loss, log-normal shadowing, and fast fading: flat Rayleigh fading for the NB nodes, and frequency-selective Nakagami- $m$  fading for the UWB link. Through the derivation of the aggregate NB interference statistic, the proposed approach is able to predict the performance of the UWB link in terms of bit error outage. Therefore, the analysis can be useful to assess the possible coexistence between UWB and NB networks.

## I. INTRODUCTION

THE emerging interest in transmission systems with large bandwidth for both commercial and military applications has received an increasing attention. For example, ultrawideband (UWB) systems communicate with time-hopping (TH) or direct sequence (DS) spread-spectrum (SS) signals using a train of extremely short pulses, thereby spreading the energy of the signal very thinly over several GHz [1–4].

The use of large transmission bandwidths, on the other hand, introduces new challenges. In particular, the successful deployment of UWB systems requires that they coexist and contend with a variety of interfering signals over already populated frequency bands [5]. Intentional jammers are invariably present in many military scenarios and UWB systems must be robust against jamming. Therefore, it is apparent that a thorough performance analysis of such systems in the presence of narrowband (NB) interferers is essential for their efficient design and successful operation [6], [7].

In the existing literature, the performance analysis of transmission schemes in the presence of NB or tone interferers has been largely focused on additive white Gaussian noise (AWGN) channels [4], [8], [9]. Furthermore, the

literature does not take into account the spatial distribution of interferers.

The application of the Poisson field model to cellular networks was first investigated in [10], and later advanced in [11]. In [12–14], the authors characterize the impact of a Poisson field of NB interferers on a NB/UWB link, in terms of error probability, capacity, and power spectral density.

This paper analyzes the effect of NB interference on UWB communication systems, when the spatial distribution of the NB interferers is modeled by a Poisson field. In particular, we analyze the performance of a binary SS system in the presence of NB interferers. We consider independent, asynchronous NB interferers which are transmitting with the same frequency, and are approximated by sinusoidal tones as in [15], [16]. The NB interferers are scattered according to an infinite Poisson field, and are operating in a wireless environment subject to path loss, log-normal shadowing, and fast Rayleigh fading. Under this scenario, we derive closed-form expressions for the BEP of a general binary coherent system with Rake reception, subject to both noise and NB interference. To this goal, we extend the same framework of [15], by using perturbation theory to analyze UWB Rake reception subject to frequency-selective Nakagami- $m$  fading channel, in the presence of multiple NB interferers. Our approach is valid for binary systems with a broad range of signaling schemes, such as DS and TH, which have been under consideration for UWB systems.

## II. SYSTEM MODEL

### A. Spatial Distribution of Nodes

In the proposed model, we account for the spatial distribution of the NB interferers by assuming an infinite number of nodes distributed according to a homogeneous Poisson point process in the two-dimensional plane [13]. Typically, the interferer positions are unknown to the network designer a priori, so we may as well treat them as completely random and use a Poisson point process. Then, the probability  $\mathbb{P}\{k \text{ in } \mathcal{R}\}$  of  $k$  nodes being inside region  $\mathcal{R}$  depends only on the area  $A_{\mathcal{R}}$  of the region, and is given by

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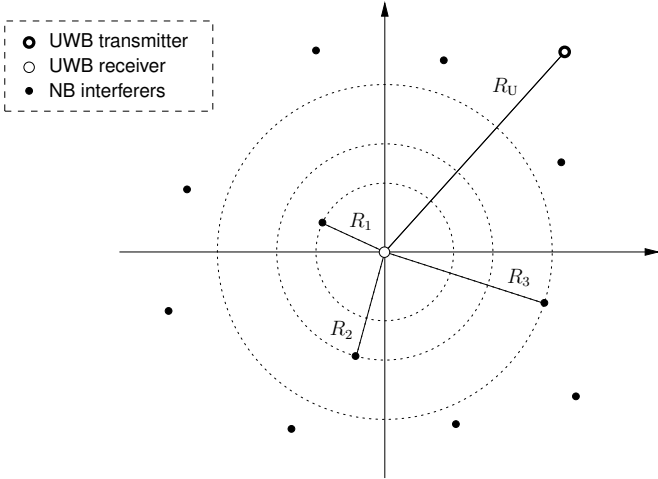


Fig. 1. Poisson field model for the spatial distribution of interferers.

[17]

$$\mathbb{P}\{k \text{ in } \mathcal{R}\} = \frac{(\lambda A_{\mathcal{R}})^k}{k!} e^{-\lambda A_{\mathcal{R}}}, \quad k \geq 0,$$

where  $\lambda$  is the (constant) spatial density of interfering nodes, in nodes per unit area. We define the *interfering nodes* to be all terminals which are transmitting within the frequency band of interest, during the time interval of interest (e.g., one symbol period), and hence are effectively contributing to the interference. Then, irrespective of the network topology (e.g., point-to-point or broadcast) or multiple-access technique (e.g., time or frequency hopping), the proposed model depends only on the density  $\lambda$  of interfering nodes.<sup>1</sup>

The proposed spatial model is depicted in Fig. 1. We assume there is a UWB link composed of two nodes: one receiver node, located at the origin, and one transmitter node, deterministically located at a distance  $R_U$  from the origin. All the other nodes ( $n = 1 \dots \infty$ ) are NB interfering nodes, whose random distances to the origin are denoted by  $\{R_n\}_{n=1}^{\infty}$ , where  $R_1 \leq R_2 \leq \dots$ . Our goal is then to determine the effect of the NB interfering nodes on the UWB link.

### B. Transmission Characteristics of Nodes

We consider the case where the NB interferers operate asynchronously, using the same frequency  $f_1$  and transmitted power  $I$ . This is a plausible constraint when power control is too complex to implement, and is applicable in decentralized ad-hoc networks, cellular systems, WLANs, WPANs, or electronic warfare scenarios where intentional jammers interfere with a UWB communications link.

The NB interfering signals are approximated by sinusoidal tones, which leads to a tractable problem. It is shown in [15] that such approximation is accurate if the bandwidth of the NB interferer is less than the bit rate of the UWB signal.

<sup>1</sup>Time and frequency hopping can be easily accommodated in this model, using the splitting property of Poisson processes to obtain the *effective* density of nodes that contribute to the interference.

The UWB link employs a binary signaling scheme, and the detection is performed by a coherent Rake receiver based on a matched-filter (MF). Typically, parameters such as the spatial density of interferers and the propagation characteristics of the medium (e.g., shadowing and path loss parameters) are unknown to the receiver. This lack of information about the interference, together with constraints on receiver complexity, justify the use of a MF detector, which is optimal in the presence of AWGN.

### C. Propagation Characteristics of the Medium

To account for the path loss affecting the NB interfering signals, we assume a  $1/R^\nu$  average signal strength decay with distance  $R$ . The parameter  $\nu$  is environment-dependent and can approximately range from 1 (e.g., hallways inside buildings) to 4 (e.g., urban environments).<sup>2</sup> With this model, the link between the  $n$ th NB interferer and the UWB receiver is characterized by a signal amplitude path loss  $\mathcal{L}_n = K/R_n^\nu$  with a given constant  $K$ . For generality, we do not make specific assumptions about the path loss model  $\mathcal{L}_U$  in the UWB link; as we shall see, this is achieved by considering the median received energy directly, instead of the transmitted energy.

To capture the shadowing effect affecting both UWB and NB propagation, we use a log-normal model where the probability density function (p.d.f.) of the received signal strength  $S$  is given by

$$f_S(s) = \frac{1}{s\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2} \ln^2\left(\frac{s}{\mu}\right)\right], \quad s \geq 0, \quad (1)$$

where  $\mu$  is the median of  $S$ . A useful fact is that a log-normal random variable (r.v.)  $S$  with parameters  $\mu$  and  $\sigma$  can be expressed as  $S = \mu e^{\sigma G}$ , where  $G \sim \mathcal{N}(0, 1)$ .<sup>3</sup> For generality, we assume that the UWB and NB signals are affected by the (possibly different) shadowing parameters  $\sigma_U$  and  $\sigma_I$ , respectively.

To account for the fading effect affecting the UWB link, we consider a fast, frequency-selective multipath channel with impulse response

$$h_U(t) = \sum_{k=1}^L h_k \delta(t - t_k), \quad (2)$$

where the  $h_k = \alpha_k \exp(j\theta_k)$ ,  $k = 1, 2, \dots, L$  are statistically independent r.v.'s describing the  $L$  path gains, and  $t_k$  is the delay of the  $k$ th path [15]. Specifically, the parameter  $\alpha_k$  represents the fading amplitude with arbitrary statistics, while  $\theta_k$  equals 0 or  $\pi$  with equal probability, to account for the random phase inversion due to reflections. In addition, we normalize the power dispersion profile (PDP) of the channel so that  $\sum_{k=1}^L \Omega_k = 1$ , where  $\Omega_k = \mathbb{E}\{h_k^2\}$ . Under

<sup>2</sup>We refer to  $\nu$  as the “amplitude loss exponent”, which corresponds to a decay in *signal amplitude*, not in *signal power*.

<sup>3</sup>We use  $\mathcal{N}(\mu, \sigma^2)$  to denote a real Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$ .

this model, the overall channel impulse response of the UWB link becomes

$$\tilde{h}_U(t) = \mathcal{L}_U \cdot e^{\sigma_U G_U} \cdot h_U(t), \quad (3)$$

where the median value of the shadowing is implicitly  $\mu = \mathcal{L}_U$ .

To account for the fading affecting the NB interfering tones, we consider fast, frequency-flat Rayleigh channel. Specifically, the fading affects the signal received from the  $n$ th interferer by introducing a random phase  $\phi_n$  that is uniformly distributed over  $[0, 2\pi)$ , as well as a Rayleigh-distributed amplitude factor  $\alpha_n$  that is normalized to have unit power gain, i.e.,  $\mathbb{E}\{\alpha_n^2\} = 1$ . We assume the shadowing and fading are independent for the different NB nodes. Under this model, the overall channel impulse response of the link between the  $n$ th interferer and the UWB receiver can be written as

$$\tilde{h}_n(t) = \mathcal{L}_n \cdot e^{\sigma_1 G_n} \cdot \alpha_n e^{j\phi_n} \delta(t), \quad (4)$$

where  $\delta(\cdot)$  is the Dirac-delta function, and the median value of the shadowing is implicitly  $\mu = \mathcal{L}_n$ .

### III. SIGNAL REPRESENTATION AND DISTRIBUTION

Typically, the movement of the NB interferers during the interval of interest (e.g., a symbol or packet time) is negligible. This has two implications: 1) the distances  $\{R_n\}_{n=1}^{\infty}$  of the interferers to the origin vary slowly; and 2) the shadowing  $\{G_n\}_{n=1}^{\infty}$  affecting those nodes also varies slowly, since the shadowing is itself associated with the movement of the nodes near large blocking objects. In this *quasi-static* scenario, it is insightful to condition the interference analysis on a given realization  $\mathcal{P}$  of the distances  $\{R_n\}_{n=1}^{\infty}$  and shadowing  $\{G_n\}_{n=1}^{\infty}$  of the interferers. This will enable the derivation of the error outage probability of the UWB link – a more meaningful metric than the average error probability, in the case of slowly-varying  $\mathcal{P}$ . Because of its fast nature, the fading affecting both UWB and NB nodes will be averaged out in the analysis.

The signal transmitted by the UWB node can be written in general as

$$s_U(t) = \sqrt{E_b} \sum_i b(t - iT_b; d_i), \quad d_i \in \{0, 1\} \quad (5)$$

where  $b(t; d_i)$  is a unit-energy waveform used to transmit  $d_i$  (the  $i$ th information bit), and  $T_b$  is the bit duration. Specifically,  $b(t; 0)$  and  $b(t; 1)$  are the UWB waveforms used to transmit bits 0 and 1, respectively. We assume that data bits  $d_i$  are i.i.d. and equiprobable.

The NB signal transmitted by the  $n$ th interferer is modeled as a tone, and can be written as

$$s_n(t) = \sqrt{2I} \cos(2\pi f_1(t - \tau_n)), \quad (6)$$

where  $\tau_n, n = 1 \dots \infty$  are i.i.d. time delays.

The overall received signal  $r(t)$  consists of the desired UWB signal, the NB interferer tones, and the additive

noise. It results from the convolution of (5) and (6) with the corresponding channel impulse responses (3) and (4), respectively, and can be written as

$$r(t) = \sqrt{E_b} \mathcal{L}_U e^{\sigma_U G_U} \sum_i r_b(t - iT_b; d_i) + \sum_{n=1}^{\infty} \sqrt{2I} \mathcal{L}_n e^{\sigma_1 G_n} \alpha_n \cos(2\pi f_1(t - \tau_n) + \phi_n) + z(t), \quad (7)$$

where  $z(t)$  is the additive white Gaussian noise (AWGN) with two-sided power spectral density  $N_0/2$ ; and  $r_b(t; d_i)$  is the response of the UWB channel to the transmitted waveform  $b(t; d_i)$ , i.e.,

$$r_b(t; d_i) = h_U(t) * b(t; d_i), \quad (8)$$

with  $*$  denoting the continuous convolution.

Due to the heterogeneous nature of the considered scenario (consisting of both UWB and NB nodes), it is convenient to simplify the notation so that we can deal directly with received energy, instead of transmitted energy.<sup>4</sup> Specifically, without loss of generality and with slight abuse of notation, we let the term  $E_b$  absorb the expression  $E_b \cdot \mathcal{L}_U^2$ , and thus  $E_b$  becomes the median received energy per bit. Similarly, we let the term  $I$  absorb the expression  $I \cdot K^2$ , and thus  $I$  becomes the median received power from an interferer that is  $R_n = 1$  m away from the UWB receiver. Then, (7) can be rewritten as

$$r(t) = \sqrt{E_b} e^{\sigma_U G_U} \sum_i r_b(t - iT_b; d_i) + \sum_{n=1}^{\infty} \sqrt{2I} \frac{e^{\sigma_1 G_n}}{R_n^\nu} \alpha_n \cos(2\pi f_1(t - \tau_n) + \phi_n) + z(t). \quad (9)$$

Note that the signals in (9) are all real valued, since we are not using baseband equivalent notation. This approach allows us to study both carrier-based and carrierless systems.

The desired UWB signal in (9) is subject to both interference and AWGN. If only AWGN is present, the optimum receiver consists of a filter matched to  $v(t) = r_b(t; 0) - r_b(t; 1)$  or, equivalently, a correlator with template  $v(t)$ . In the presence of multipath, this adaptive MF is realized as the well-known Rake receiver.

We now consider the detection of information bit  $d_0$  and we assume that the pulses satisfy the Nyquist criterion (or introduce, in any case, negligible intersymbol interference). Considering perfect synchronization with the desired UWB signal, the matched filter output  $u(t)$  at the appropriate sampling instant  $t_0$  can be written as

$$u(t_0) = e^{\sigma_U G_U} s_0 + \zeta + n_0, \quad (10)$$

where the desired signal  $s_0$  is

$$s_0 = \sqrt{E_b} \int_{-\infty}^{t_0} r_b(t; d_0) v(t) dt; \quad (11)$$

<sup>4</sup>This also avoids making any specific assumption about the path loss  $\mathcal{L}_U$  of the UWB link.

$n_0$  is the Gaussian noise sample with zero-mean and variance  $\sigma_z^2 = (N_0/2) \int_{-\infty}^{\infty} v^2(t) dt$ ; and

$$\zeta = \sqrt{2I} \sum_{n=1}^{\infty} \frac{e^{\sigma_1 G_n}}{R_n^\nu} \alpha_n |H(f_1)| \cos \phi_n \quad (12)$$

is the interference term which depends on the transfer function of the MF,  $H(f)$ . In (12), and with a slight abuse of notation, the phase terms  $\arg\{H(f_1)\} + 2\pi f_1(t_0 - \tau_n)$  are absorbed by the random phases  $\phi_n$ .

The MF transfer function can be easily evaluated as  $H(f) = \mathcal{F}\{v(t_0 - t)\}$ , where  $\mathcal{F}\{\cdot\}$  is the Fourier transform operator. For the purpose of our analysis, this transfer function can be factorized as

$$|H(f)| = |H_0(f)| \cdot |\mathcal{F}\{h_U(t)\}|, \quad (13)$$

where  $H_0(f) = \mathcal{F}\{b(t; 0) - b(t; 1)\}$ . Note that (13) is composed of two factors: the first,  $|H_0(f)|$ , depends on the waveforms used, while the second depends on the channel impulse response of the desired signal.

The received and template waveforms can be written using (2) and (8) as  $r_b(t; d_0) = \sum_{k=1}^L h_k b(t - t_k; d_0)$  and  $v(t) = \sum_{k=1}^L h_k [b(t - t_k; 0) - b(t - t_k; 1)]$ , respectively. To evaluate the error performance, we assume without loss of generality that  $d_0 = 0$ .

Substituting the received signal  $r_b(t; d_0)$  and the template waveform  $v(t)$  into (11), the desired signal  $s_0$  can be written as

$$s_0 = \sqrt{E_b} (1 - \rho) \alpha_U^2, \quad (14)$$

where  $\alpha_U^2(\mathbf{h}) = \sum_{k=1}^L h_k^2$ , and  $\rho$  is the correlation coefficient between the two waveforms  $b(t; 0)$  and  $b(t; 1)$ , i.e.,  $\rho = \int_{-\infty}^{\infty} b(t; 0)b(t; 1) dt$ .<sup>5</sup> Note that  $\rho \in [-1, 1]$  and, for instance,  $\rho = -1$  corresponds to the antipodal modulation.

The noise power at the output of the Rake combiner can be evaluated as

$$\sigma_z^2 = N_0 (1 - \rho) \alpha_U^2. \quad (15)$$

As shown in (12), the interference term depends on the MF transfer function. To evaluate its impact in the performance, we use (13) to represent the Rake receiver as a filter with transfer function  $|H(f)| = |H_0(f)| \cdot |H_U(f, \mathbf{h}, \mathbf{t})|$ , where

$$H_U(f, \mathbf{h}, \mathbf{t}) = \mathcal{F}\{h_U(t)\} = \sum_{k=1}^L h_k e^{-j2\pi f t_k}, \quad (16)$$

with the random vectors (R.V.'s)  $\mathbf{h} = (h_1, h_2, \dots, h_L)$  and  $\mathbf{t} = (t_1, t_2, \dots, t_L)$  denoting the instantaneous path gains and delays, respectively. Note from (16) that  $|H(f_1)|$  is a r.v. which depends on the instantaneous channel impulse response  $h_U(t)$  through  $\mathbf{h}$  and  $\mathbf{t}$ .

As far as the contribution (12) from the interferers is concerned, when conditioned on the r.v.  $|H(f_1)|$ ,  $\mathcal{P}$  and

$G_U$ , the interferer term  $\zeta$  can be viewed as a weighted sum of i.i.d. Gaussian terms  $\alpha_n \cos \phi_n$ , since  $\alpha_n$  are i.i.d. Rayleigh distributed r.v.'s and  $\phi_n$  are i.i.d. random phases uniformly distributed over  $[0, 2\pi)$ , respectively. Therefore,  $\zeta$  is conditionally Gaussian with zero mean and variance  $\sigma_\zeta^2 = I |H(f_1)|^2 A$  where

$$A = \sum_{n=1}^{\infty} \frac{e^{2\sigma_1 G_n}}{R_n^{2\nu}}. \quad (17)$$

Since  $A$  depends on the position  $\mathcal{P}$  of interfering nodes (i.e.,  $\{R_n\}_{n=1}^{\infty}$  and  $\{G_n\}_{n=1}^{\infty}$ ), it can be seen as a r.v. whose value is different for each realization of  $\mathcal{P}$ . Furthermore, it can be shown [13] that  $A$  has a skewed stable distribution [18] given by<sup>6</sup>

$$A \sim \mathcal{S} \left( \alpha_A = \frac{1}{\nu}, \beta_A = 1, \gamma_A = \lambda \pi C_{1/\nu}^{-1} e^{2\sigma_1^2/\nu^2} \right), \quad (18)$$

for  $0 < \alpha_A < 1$  (or equivalently,  $\nu > 1$ ), and with  $C_x$  defined to be the function

$$C_x = \begin{cases} \frac{1-x}{\Gamma(2-x) \cos(\pi x/2)}, & x \neq 1, \\ \frac{2}{\pi}, & x = 1 \end{cases} \quad (19)$$

where  $\Gamma(\cdot)$  is the Gamma function.

#### IV. ERROR PROBABILITY

We now build on the results of the previous section and characterize the error performance of the UWB link, when subject to both NB interference and thermal noise. Again, in the *quasi-static* scenario of slowly-moving nodes it is insightful to analyze the error probability conditioned on a given realization  $\mathcal{P}$  of the distances  $\{R_n\}_{n=1}^{\infty}$  and shadowing  $\{G_n\}_{n=1}^{\infty}$  of the interferers, as well as on the shadowing  $G_U$  of the UWB link.

Since  $\zeta$  is conditionally Gaussian, the total disturbance due to interference plus noise is also conditionally Gaussian, with variance  $I |H_0(f_1)|^2 |H_U(f_1, \mathbf{h}, \mathbf{t})|^2 A + N_0 (1 - \rho) \alpha_U^2$ . Thus, when conditioned on the nodes position and shadowing ( $\mathcal{P}$  and  $G_U$ ) and on the instantaneous channel impulse response ( $\mathbf{h}$  and  $\mathbf{t}$ ), the BEP can then be written as  $P_{e|\mathbf{h}, \mathbf{t}, G_U, A} = \mathcal{Q} \left( \sqrt{2\tilde{\eta}_{G_U, A}(\mathbf{h}, \mathbf{t})} \right)$  where

$$\tilde{\eta}_{G_U, A}(\mathbf{h}, \mathbf{t}) = \frac{\alpha_U^2(\mathbf{h}) e^{\sigma_U G_U}}{\frac{N_0}{E_b} \frac{2}{1-\rho} + \frac{I}{C} \frac{2 |H_0(f_1)|^2 |H_U(f_1, \mathbf{h}, \mathbf{t})|^2}{T_b (1-\rho)^2} \frac{1}{\alpha_U^2(\mathbf{h})} A}, \quad (20)$$

and  $C = E_b/T_b$  denotes the median received power.

The next step is to perform expectations of  $P_{e|\mathbf{h}, \mathbf{t}, G_U, A}$  over R.V.'s  $\mathbf{h}$  and  $\mathbf{t}$ , to obtain the average performance over all the possible channel impulse responses  $h_U(t)$ . The contribution of the interference in (20) depends on the instantaneous channel impulse response of the desired signal,

<sup>5</sup>Distortions, e.g., caused by antennas in UWB systems can be taken into account by considering  $b(t; d_i)$  as the received waveform. This will be illustrated clearly in Section V.

<sup>6</sup>We use  $\mathcal{S}(\alpha, \beta, \gamma)$  to denote a real stable distribution with characteristic exponent  $\alpha$ , skewness  $\beta$ , dispersion  $\gamma$ , and location  $\mu = 0$ . When  $\alpha \neq 1$ , which is the case in (18), the corresponding characteristic function is  $\phi(\omega) = \exp[-\gamma|\omega|^\alpha (1 - j\beta \text{sign}(\omega) \tan \frac{\pi\alpha}{2})]$ .

making cumbersome and difficult to evaluate in closed-form the expectation  $\mathbb{E}_{\mathbf{h}, \mathbf{t}}\{P_{e|\mathbf{h}, \mathbf{t}, G_U, A}\}$ . Accordingly to [15], we can proceed with two steps: first we perform the average over the time delays  $\mathbf{t}$  for fixed path gains  $\mathbf{h}$ , and then we average over  $\mathbf{h}$ , i.e.,  $P_e = \mathbb{E}_{\mathbf{h}}\{\mathbb{E}_{\mathbf{t}}\{P_{e|\mathbf{h}, \mathbf{t}, G_U, A}\}\}$ . In this case, the inner expectation involves only the R.V.  $\mathbf{t}$  through the function  $|H_U(f_1, \mathbf{h}, \mathbf{t})|^2$ ; therefore, without loss of generality, in the following we define a r.v.  $\xi = |H_U(f_1, \mathbf{h}, \mathbf{t})|^2$  that depends on R.V.  $\mathbf{t}$  with fixed, but arbitrary  $\mathbf{h}$ . Hence,  $P_{e|\mathbf{h}, G_U, A} = \mathbb{E}_{\xi}\{\hat{P}(\xi)\}$  where  $\hat{P}(\xi) = Q\left(\sqrt{2\tilde{\eta}(\xi)}\right)$  with

$$\tilde{\eta}(\xi) = \frac{\alpha_U^2 e^{\sigma_U G_U}}{\frac{N_0}{E_b} \frac{2}{1-\rho} + \frac{I}{C} \frac{2|H_0(f_1)|^2}{T_b(1-\rho)^2} \frac{\xi}{\alpha_U^2} A}. \quad (21)$$

The expectation of (21) over the r.v.  $\xi$  can be conveniently approximated by means of *perturbation theory* [19] without requiring integration. In fact, in [15] we prove that by expanding  $\hat{P}(\xi)$  in terms of central differences up to the third order we can obtain a closed-form approximation of  $P_{e|\mathbf{h}, G_U, A}$  that leads to very accurate results. Following this approach, the averaged BEP over the R.V.  $\mathbf{t}$  can be written in the form

$$P_{e|\mathbf{h}, G_U, A} \simeq \sum_{i=1}^N p_i \hat{P}(q_i \alpha_U^2), \quad (22)$$

where  $p_i$  and  $q_i$  are weights, and  $N$  is the number of terms in the expansion [15]. Considering the third-order expansion, we have  $N = 4$  terms with weights

$$\begin{aligned} \mathbf{p} &= (1/6 + b, 2/3, 1/6 - 2b, b) \\ \mathbf{q} &= (0, 1, 1 + \sqrt{3a}, 1 + 2\sqrt{3a}), \text{ where} \\ a &= 1 - \Upsilon_2, \text{ and} \\ b &= (1 - 3\Upsilon_2 + 2\Upsilon_3)/(18\sqrt{3}(1 - \Upsilon_2)^{3/2}) \end{aligned} \quad (23)$$

are functions of  $\Upsilon_2$  and  $\Upsilon_3$  that depend only on the normalized PDP of the channel, i.e.,  $\Upsilon_2 = \sum_{k=1}^L \Omega_k^2$  and  $\Upsilon_3 = \sum_{k=1}^L \Omega_k^3$ .

Note that the expression in (22) is a function of  $\mathbf{h}$  through the term  $\alpha_U^2$ , and hence the outer expectation,  $P_{e|G_U, A} = \mathbb{E}_{\mathbf{h}}\{P_{e|\mathbf{h}, G_U, A}\}$ , can be written as the expectation of (22) with respect to  $\alpha_U^2$ . The distribution of  $\alpha_U^2$  depends on the type of channel. For UWB channels, it has been shown recently that the amplitude distribution of the resolved multipaths can be modeled by the Nakagami- $m$  distribution [20]. Accordingly, we consider independent Nakagami distributed paths,  $\alpha_k$ , with average power  $\Omega_k = \mathbb{E}\{h_k^2\} = \mathbb{E}\{\alpha_k^2\}$  and Nakagami parameter  $m_k$ . Using the alternative expression for the Gaussian  $Q$ -function, one can derive the expectation of (22) over  $\mathbf{h}$  as

$$P_{e|G_U, A} \simeq \frac{1}{\pi} \int_0^{\pi/2} \sum_{i=1}^N p_i \prod_{k=1}^L \left( \frac{\eta_{G_U, A}(q_i)}{\sin^2 \theta} \cdot \frac{\Omega_k}{m_k} + 1 \right)^{-m_k} d\theta, \quad (24)$$

where

$$\eta_{G_U, A}(x) = \frac{e^{\sigma_U G_U}}{\frac{N_0}{E_b} \frac{2}{1-\rho} + \frac{I}{C} \frac{|H_0(f_1)|^2}{T_b} \frac{2x}{(1-\rho)^2} A} \quad (25)$$

is the signal-to-interference-plus-noise ratio (SINR) as a function of the signal-to-noise ratio  $\text{SNR} = E_b/N_0$ , and the signal-to-interference ratio  $\text{SIR} = C/I$ .

In our *quasi-static* model, the conditional error probability in (24)-(25) is seen to be a function of the slow-varying user positions and shadowing (i.e.,  $G_U$  and  $\mathcal{P}$ ). Since these quantities are random, the error probability itself is a r.v. Then, with some probability,  $G_U$  and  $\mathcal{P}$  are such that the error probability of the probe link is above some threshold probability  $p^*$ . The system is said to be *in outage*, and the error outage probability is

$$P_{\text{out}} = \mathbb{P}_{G_U, A} \{P_{e|G_U, A} > p^*\}. \quad (26)$$

In the case of slowly-varying user positions, the error outage probability is a more meaningful metric than the error probability averaged over  $G_U$  and  $\mathcal{P}$ .

## V. EXAMPLE: DS-BPAM SYSTEM

We now particularize the general analysis developed in the previous sections to a DS-BPAM system. Using the bit waveforms  $b(t; d_i) = d_i b(t)$  for BPAM, the transmitted signal can be written as

$$s(t) = \sqrt{E_b} \sum_i d_i b(t - iN_s T_f), \quad (27)$$

where the unit-energy waveform for each bit is given by

$$b(t) = \sum_{k=0}^{N_s-1} c_k w(t - kT_f). \quad (28)$$

Here,  $N_s$  is the number of pulses required to transmit a single information bit  $d_i \in \{-1, 1\}$ , and  $w(t)$  is the transmitted pulse shape with energy  $1/N_s$ . The pulse repetition time (frame length)  $T_f$  and the bit duration  $T_b$  are related by  $T_b = N_s T_f$ . Finally,  $\{c_k\}_{k=0}^{N_s-1}$  is the spreading sequence.

The transfer function  $H_0(f)$  for the DS system can be easily derived as

$$|H_0(f)| = 2|W(f)| \left| \sum_{k=0}^{N_s-1} c_k e^{j2\pi f k T_f} \right|, \quad (29)$$

where  $W(f)$  is the Fourier transform of the pulse  $w(t)$ .

Different pulse shapes are considered to satisfy FCC masks with the maximum transmitted power. Among these, in [21] the 5th derivative of a Gaussian monocycle is chosen, so the received pulse  $w(t)$  can be modeled as the 6th derivative. In this case, the Fourier transform of  $w(t)$  can be written as [21]

$$W(f) = \frac{8\pi^3}{3\sqrt{1155} N_s} \cdot \tau_w^{13/2} f^6 e^{-\frac{\pi}{2} f^2 \tau_w^2}, \quad (30)$$

where  $\tau_w$  is the pulse duration parameter. Note that the energy of  $w(t)$  is  $1/N_s$ , so that the received bit waveform  $b(t)$  has unit energy.

Both (29) and (30) can be used in (24)-(25) to evaluate the conditional BEP of the DS-BPAM system, as well as its error outage probability in (26).

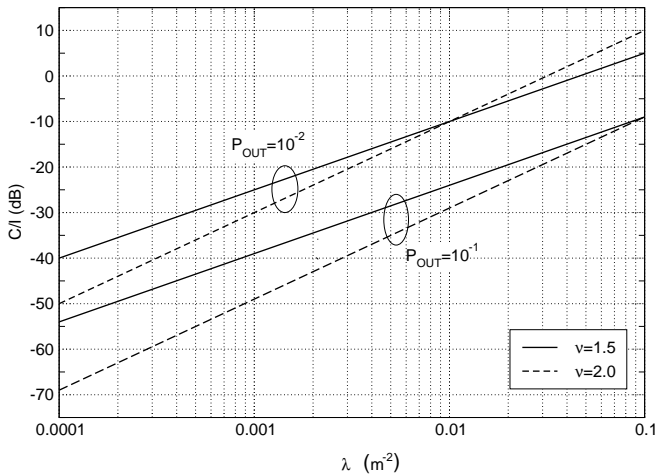


Fig. 2.  $C/I - \lambda$  curves of constant  $P_{\text{out}}$  with  $E_b/N_0 = 20$  dB.

## VI. NUMERICAL RESULTS

### A. The impact on a single UWB link

Figures 2 and 3 quantify the error performance of a DS-BPAM system subject to NB interference and noise, and illustrate its dependence on the various parameters involved, such as the SNR  $= E_b/N_0$ , SIR  $= C/I$ , signal loss exponent  $\nu$ , and interferer density  $\lambda$ . To evaluate the corresponding  $P_{\text{out}}$ , we resort to a hybrid approach where we employ the analytical results given in (24)-(25) and (26), but perform a Monte Carlo simulation of the stable r.v.  $A$  according to [22]. As an alternative, numerical integration of (26) is also possible, although computationally more involved. We emphasize that the error probability expressions given in Section IV completely replace the need for bit-level simulation of the system in order to compute  $P_{\text{out}}$ .

We consider a DS-BPAM system that adopts a 6th derivative Gaussian received pulse with duration  $\tau_w = 0.192$  ns, a frame length  $T_f = 50$  ns, and  $N_s = 16$  pulses/bit. Since the modulation is antipodal, the correlation parameter  $\rho$  is  $-1$ . The user has a DS sequence  $\{c_k\}_{k=0}^{N_s-1}$  with  $c_k = (-1)^k$ .

For the UWB link, we consider a frequency-selective multipath fading channel with  $L = 8$  independent Nakagami distributed paths having random delays and an exponential PDP [15], [20] given by

$$\Omega_k = \frac{e^{1/\epsilon} - 1}{1 - e^{-L/\epsilon}} e^{-k/\epsilon} \quad k = 1 \dots L, \quad (31)$$

where  $\epsilon = 3$  is a decay constant which controls the multipath dispersion. We assume also different Nakagami parameters for each path [15] according to

$$m_k = m_1 e^{-(k-1)/\gamma} \quad k = 1 \dots L, \quad (32)$$

where  $m_1 = 3$ , and  $\gamma = 4$  controls the decay of the  $m$ -parameters. The shadowing on this link is characterized by a standard deviation  $\sigma_U = \sigma_{\text{UdB}} \ln(10)/20$ , with  $\sigma_{\text{UdB}} = 3$ .

The NB interferers are characterized by a frequency  $f_I = 5010$  MHz and a standard deviation of the shadowing  $\sigma_{\text{IdB}} = 8$ . For all numerical results, we choose  $p^* = 10^{-2}$ .

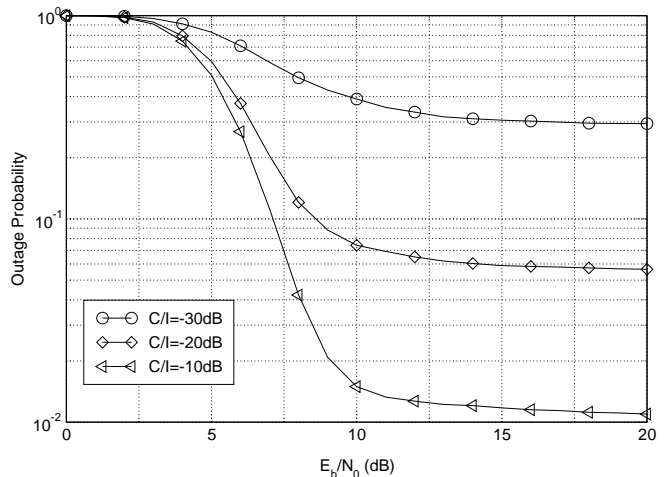


Fig. 3. Bit error outage probability of the UWB link, for various  $C/I$ ,  $\nu = 1.5$  and  $\lambda = 0.01 \text{ m}^{-2}$ .

The relation between SIR and  $\lambda$  is illustrated in Fig. 2, which plots this pair for a fixed  $P_{\text{out}}$ . The curve shows the tradeoff between the interferer density  $\lambda$  and the SIR: if  $\lambda$  increases,  $C/I$  must decrease accordingly, for a fixed error performance  $P_{\text{out}}$  [12].

From Figs. 2 and 3, we conclude that  $P_{\text{out}}$  deteriorates as  $\lambda$  or  $I$  increase. This is expected because as the interferers' density or transmitted power increase, the cumulative interference at the UWB receiver becomes stronger. Note in Fig. 3 that the UWB system is interference-limited for SNR greater than 10 dB, in this specific example.

### B. The impact on a UWB network

The analytical approach presented for a single UWB link can be useful also to assess the impact of the aggregate NB interference on a UWB network. To this aim, let us extend the analysis to a UWB network represented by  $M$  communications. Assuming that all the UWB links adopt power control and identical transmission parameters, irrespective of their locations, the  $M$  receivers experience the same SNR, SIR, and thus the same outage probability  $P_{\text{out}}$ .<sup>7</sup> Therefore, considering independent outage events among links, the cumulative distribution function (CDF) of the number of UWB communications in outage is

$$\mathbb{P}\{\#\text{ links in outage} \leq k\} = \sum_{i=0}^k \binom{M}{i} P_{\text{out}}^i (1 - P_{\text{out}})^{M-i}, \quad (33)$$

where  $P_{\text{out}}$  can be evaluated by (24)-(25) and (26). In Fig. 4, the CDF (33) for a network of  $M = 100$  UWB communications with the same system and channel parameters considered to plot Fig. 3 and with  $E_b/N_0 = 20$  dB, is considered. As the figure shows, the impact of the aggregate interference can compromise the UWB network operation

<sup>7</sup>Note that the impact of the interference is independent on the UWB receiver position, by virtue of the Poisson point process assumption for the interferers locations.

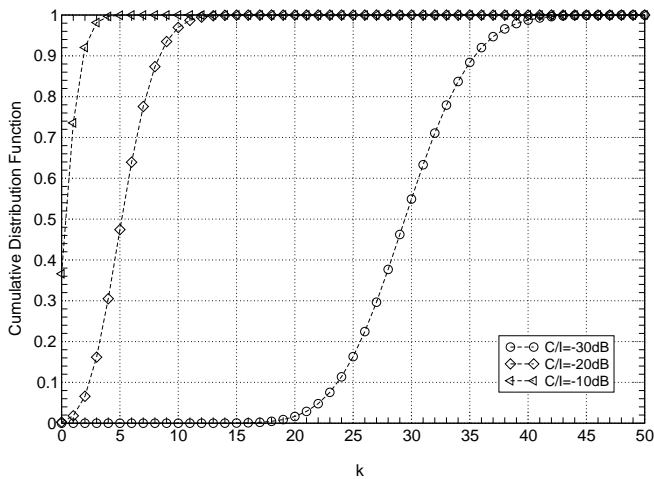


Fig. 4. CDF (33) of the number of UWB links in outage for various  $C/I$ ,  $E_b/N_0 = 20$  dB,  $\nu = 1.5$  and  $\lambda = 0.01$  m<sup>-2</sup>.

by increasing the number of communications affected by outage.

## VII. CONCLUSIONS

This paper investigates a mathematical model for UWB communication subject to spatially distributed NB interferers. We have considered a realistic scenario of asynchronous NB interferers in a wireless environment subject to path loss, log-normal shadowing and fast fading. Under this scenario, we derive the statistic of the aggregate interference and a semi-analytical expression for the corresponding outage probability. Through numerical examples, we quantify the error performance as a function of important parameters such as the SNR, SIR, path loss exponent, and the interferers' spatial density. Our analysis clearly shows how the UWB system performance depends on these parameters, thereby providing insights that may be of value to the network designer.

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