Cost Evaluation of Optical Packet Switches Equipped With Limited-Range and Full-Range Wavelength Converters for Contention Resolution[1]

V. Eramo, M. Listanti, A. Germoni and M.L. Foderà
INFOCOM Dept., University of Roma “La Sapienza”

Abstract- **An architecture is proposed for a Wavelength Division Multiplexed (WDM) optical packet switch equipped with both limited range wavelength converters (LRWCs) and shared full range wavelength converters (FRWCs). The FRWCs are used to overcome the performance degradation in terms of Packet Loss Probability due to the use of LRWCs only. A probabilistic model is proposed to dimension the number of shared FRWCs so that the same Packet Loss Probability of a switch equipped with only shared FRWCs is guaranteed. After introducing a cost model of the converters depending on the range conversion, we show that the architecture may allow a conversion cost saving in the order of 90%.**

I. Introduction

Wavelength Division Multiplexing (WDM) Optical Networks are believed to be backbone transport networks for the next generation Internet. WDM systems allow the enormous optical capacity of fiber to be utilized by transmitting multiple signals on a single fiber. Currently, commercially available optical fibers can support hundreds of channels, each operating at speed of several gigabits per second (e.g. OC-48, OC-192 and OC-768 in the near future) [1,2]. The concept of WDM has provided us an opportunity to multiply the network capacity. Current optical switching technologies allow us to rapidly deliver the enormous bandwidth carried out on any optical fiber. Among all of the switching schemes, photonic packet switching appears to be a strong candidate because of the high speed, data rate/format transparency and configurability it offers [3-7].

In general optical packet switched networks can be divided into two categories: slotted (synchronous) [8-12] and unslotted [13] (asynchronous) networks. In a slotted network the packets are aligned and have the same size. They are placed together with the header inside a fixed time slot, which has a longer duration than the packet header to provide guard time. In an unslotted network the packets may or may not have the same size. Packets arrive and enter the switch without being aligned. Therefore, the packet-by-packet switch action could take place at any instant. Obviously, in unslotted networks, the chance for packet contention is larger because the behavior of the packets is more unpredictable and less regulated. On the other hand, unslotted networks are easier and cheaper to be built because they do not need optical synchronizers.

In both type of network, wavelength conversion has been studied as an approach to resolve packet contentions [14-16]. Wavelength converters [17-20] are costly devices and therefore, a particular effort has been devoted to designing cost-effective switch architectures by minimizing the number of converters constrained to a prescribed Packet Loss Probability. A fairly well-popularized architecture that was first proposed in [21] relies on a bank of converters that is shared among all of the arriving packets. Probabilistic model proposed in [22-27] have shown that sharing can achieve a Packet Loss Probability that rivals earlier architectures in which a converter is dedicated to each wavelength channel of each input fiber, but with significantly fewer converters. The reason for this fact is simple: not all packets need conversion because not all packets encounter contention, so those packets not needing conversion should be switched directly to their desired output fiber rather than unnecessarily passed through a dedicated converter.

Experimental results showed that the performance of wavelength converters strongly depends on the combination of the input and output wavelengths. That is, for a given input wavelength, translations to some output wavelengths result in an output signal which is significantly degraded. A realistic all-optical wavelength translator may only allow for the translation of any given input wavelength to a limited range of output wavelengths. An ideal Full-Range Wavelength Converter (FRWC), therefore, has to be realized by cascading a given number of realistic Limited-Range Wavelength Converters (LRWCs). This makes the cost of a wavelength converter high. For this reason in [28-32] a WDM packet switch equipped with Limited-Range Wavelength Converters (LRWCs) is discussed. The optimum scheduling problem, allowing the packet loss probability to be minimized in slotted optical packet switch equipped with LRWCs, has been studied in [33,34] for bufferless optical packet switches equipped with FRWC. The authors in [33] show that the solution to the optimum scheduling problem is equivalent to finding a maximum matching in a bipartite graph called the conversion graph. An algorithm, called First Available Algorithm (FAA), with complexity *O(M)* is proposed in [34] for non-circular

[1] This work has been supported by the European Commission through the Network of Excellence e-Photon/ONE+.

symmetrical wavelength conversion, where M is the number of wavelengths per fiber. The performance of the scheduling algorithm proposed in [34] is evaluated in [35] by means of an analytical model validated by simulations.

The sharing problem of LRWCs has been coped with in both slotted and unslotted networks. In [36] a probabilistic model has been proposed to evaluate the performance of a synchronous switch equipped with shared LRWCs and working under a modified version of FAA to schedule the packets.

Studies in [37-38] have shown that the performance of LRWCs hardly reaches the same performance level offered by FRWCs, unless the conversion range of the LRWCs is high. For this reason in [37] a Two Layer Wavelength Conversion (TLWC) scheme is proposed. The first layer, equipped with LRWCs, performs the function of near wavelength conversion, thus cutting down the need for more expensive FRWCs performing such functions. The second layer, equipped with FRWCs, can perform far wavelength conversion and it is used when the limited range wavelength conversion of a packet is unsuccessful. The architecture proposed in [37] is asynchronous and the TLWC scheme considers the sharing of FRWCs only. Each input wavelength channel has one dedicated LRWC and FRWCs are shared per output fiber, that is all of the packets directed to the same output fiber share the same pool of FRWCs. A cost model of the converters is introduced in [37] and a cost evaluation of an optical switch adopting the TLWC scheme is made. The authors demonstrate that with TLWC, the conversion cost is reduced by as much as 40% under heavy-load condition with respect the case in which only shared FRWCs are used while achieving the same Packet Loss Probability.

Differently from [37] in which asynchronous switches have been taken into account, in this paper we study the performance of a synchronous optical packet switch adopting the TLWC scheme. A probabilistic approach is introduced to evaluate the performance of synchronous optical packet switches adopting the TLWC scheme. By means of the model we evaluate the conversion cost saving that the TLWC scheme allows us to obtain with respect to the case in which one conversion layer equipped with shared FRWCs is used.

The organization of the paper is as follows: in Section II we describe the switching architecture in which two layers of wavelength conversion are used; in Section III we discuss the control algorithm used in the architecture proposed; the probabilistic model, needed to evaluate the performance of the architecture, is introduced and validated by means of simulations in Section IV; Section V illustrates the obtained results: the conversion cost saving that the TLWC scheme allows us to obtain is evaluated; our main conclusions and further research topics are discussed in Section VI.

II. WDM PACKET OPTICAL SWITCHES ARCHITECTURES EQUIPPED WITH LRWCS AND SHARED FRWCS

We consider WDM optical switches with N input/output fibers. Each fiber supports M wavelengths channels. Let λ_i be the wavelength associated with channel i, i=0,…M-1, and assume the natural ordering $\lambda_0 < \lambda_1 < \ldots\ldots < \lambda_{M-1}$.

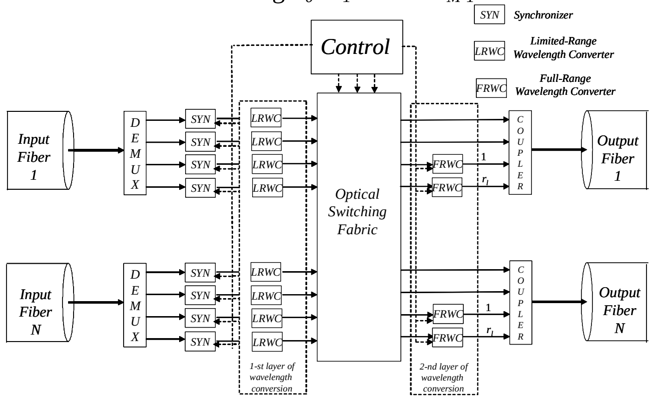

Figure 1 Two Layer Wavelength Conversion-Shared Per Output Fiber (TLWC-SPOF) Optical Packet Switch

The Optical Packet Switch (OPS) we want to analyze and compare is reported in Fig. 1. The operation mode of the proposed architecture is synchronous, meaning that all arriving packets have a fixed size and their arrival on each wavelength is synchronized on a time-slot basis [8], where a time slot is the time needed to transmit a single packet; the synchronization operation is realized by means of synchronizers [39] located at the ingress of the switch after the wavelength demultiplexers.

The OPS proposed uses a Two Layer Wavelength Conversion (TLWC). In the first layer, LRWCs are located to perform the near wavelength conversions, in the second layer, FRWCs are located to convert the packets requiring far wavelength conversions. All of the architectures are equipped with not circular LRWCs. The LRWCs allow the wavelength of a packet to be changed. An input wavelength may be converted to a set of adjacent outgoing wavelengths. We define the set of these outgoing wavelengths as the adjacency set of this input wavelength. We assume that the adjacency set of any wavelength λ_i for $i \in \{0,1,\ldots M\text{-}1\}$ is the interval $[u,v]$, where $u=\max\{0,i-d\}$ and $v=\min\{M-1,i+d\}$ and d is defined as conversion range. Notice that because a FRWC is able to convert a packet from any input wavelength to any output wavelength, it can be considered as the particular case of a LRWC with conversion range d=M-1.

In Figures 2a, 2b we show, in the case M=6 and when a LRWC with d=1 is used, the output wavelengths to which can be changed the input wavelengths λ_3 and λ_0 respectively. In this case the λ_3's and λ_0's adjacency sets are $\{\lambda_2, \lambda_3, \lambda_4\}$ and $\{\lambda_0, \lambda_1\}$respectively.

Fig.1 shows the architecture of a TLWC-SPOF node using one LRWC per each input wavelength channel and one pool of rl FRWCs in each OF. The incoming optical signal from a fiber is first wavelength-demultiplexed in separate wavelengths. The packets are then aligned by the optical synchronizers [39], wavelength converted by an LRWC if it needs near wavelength conversion and switched to the OF which they are directed to. If a packet needs a far wavelength conversion, one FRWC is used. Finally, various wavelengths are coupled on an output fiber link by an optical coupler. It should be noted that the

TLWC-SPOF switching fabric can be configured appropriately to direct a packet toward an OF, either with or without wavelength conversion performed by an FRWC. We assume a nonblocking switching fabric, this means that it is not possible for the switch to drop optical packets due to the lack of switch resources. If optical packets are dropped by the switch fabric, then it must be due to the lack of output wavelengths or failure in the two-layer wavelength conversion.

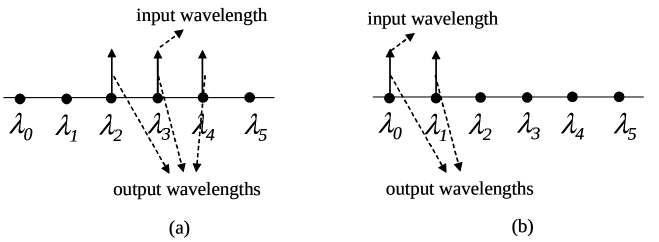

Figure 2. Adjacency sets of wavelength λ_3 (a) and λ_1 (b) when d=1 and M=6

In order to compare the implementation costs of the considered architecture, a simple linear cost structure is adopted such that the cost of a LRWC or FRWC is linearly proportional to its conversion range, i.e. the costs of an LRWC and an FRWC are assumed to be d and M-1 respectively. The overall-cost calculation of a TLWC-SPOF OPS is done by taking the sum of the costs of all of the converters used. Because the switch is equipped with $N{\cdot}M$ LRWCs and $N{\cdot}rl$ FRWCs the implementation cost $Cost_{TLWC\text{-}SPOF}$ of a TLWC-SPOF OPS can be expressed as follows:

$$Cost_{TLWC\text{-}SPOF} = d{\cdot}N{\cdot}M + (M-1){\cdot}N{\cdot}rl \qquad (1)$$

III. Scheduling Algorithm

The scheduling algorithm decides which packets arriving at a given time-slot can be transmitted, which packets have to be wavelength converted, whether to use a near or far wavelength conversion and which Output Wavelength Channels (OWCs) are assigned to the packets to transmit. The Scheduling Algorithm (SA) for TLWC-SPOF is composed by three phases: Initialization, LRWCs' Assignment, FRWCs' Assignment. To describe the Scheduling Algorithm (SA) we define the following sets and variables:

-$K_{i,j}$: the number of packets arriving on wavelength λ_j (j=0,…M-1) and directed to OF i-th (i=0,1,…,N-1)

- $\boldsymbol{a_{i,j,h}}$: the h-th packet (h=1,…,$K_{i,j}$) arriving on wavelength λ_j (j=0,…M-1) and directed to OF i-th (i=0,1,….N-1);

-$\Psi_i \equiv \{ \boldsymbol{a_{i,j,h}} \;/\; h=1,\ldots K_{i,j}\;;\; j=0,\ldots, N-1\}$: the set of packets directed to OF i-th (i=0,….,N-1);

-$\boldsymbol{o_{i,j}}$: the OWC of OF i-th (i=0,1,….N-1) on wavelength λ_j (j=0,…M-1);

-Λ_i : the set of OWCs belonging to OF i-th (i=0,….,N-1) and for which packets are not yet scheduled after the phase of assignment of LRWCs has been performed;

-rl_i: the available number of FRWCs in OF i-th (i=0,1,….N-1) during the execution of the SA for TLWC-SPOF OPS; it is initialized to rl, the available number of FRWC for each OF in TLWC-SPOF switch, and decremented by 1 each time one of the FRWCs is engaged.

The proposed SA is executed in every time slot and as shown in Fig. 3 it is composed by three phases referred to as: *Initialization*, *LRWCs' Assignment* (LCA) and *FRWCs' Assignment* (FCA).

In the *Initialization* phase the variables rl_i (i=0,1,….N-1) are initialized to rl, and the Ψ_i (i=0,,…N-1) sets are determined. The Λ_i, (0=1,…,N-1) sets are initialized to the empty set {∅ }.

After algorithm variables and sets have been initialized, the control unit performs the actions related to the LCA phase. In this phase the packets needing near wavelength conversions are scheduled by using the LRWCs. After the LCA phase is performed, the shared FRWCs are used in FCA phase to schedule the packets remaining on the OWCs not engaged in the LCA phase. The actions performed in the LCA and FCA phases are discussed in Sections A and B respectively.

A. LRWCs' Assignment(LCA) phase

The objective is to schedule the maximum number of packets using only LRWCs. In [34,35] it is shown that the problem can be resolved by evaluating the solution of a maximum matching problem in a bipartite graph. In the bipartite graph $G=[V,E]$ for OF i-th, the graph vertices in set V are partitioned in two subsets: subset V_I, whose vertices represent the set $\{\boldsymbol{a_{i,j,h}}\; h=1,\ldots K_{i,j}\; j=0,1,\ldots.,M-1\}$ of the packets directed to OF i-th and subset V_O, whose vertices represent the set $\{\boldsymbol{o_{i,j}}\; j=0,1,\ldots.,M-1\}$ of the OWCs in the OF i-th.

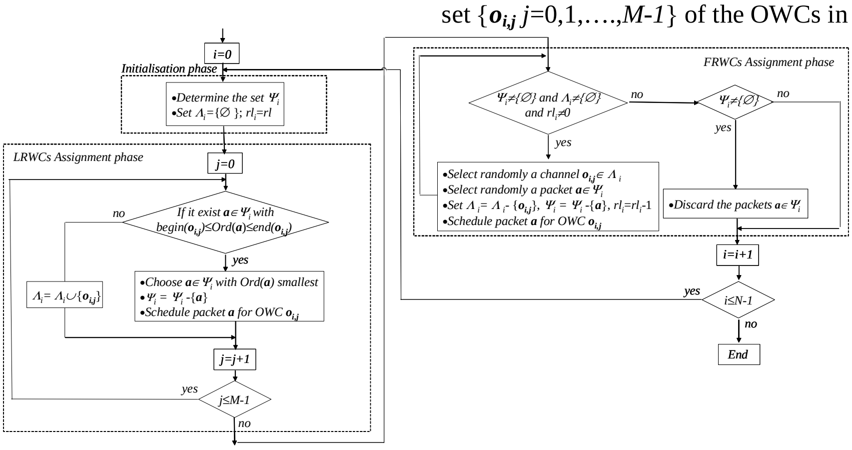

Figure 3. Scheduling Algorithm in TLWC-SPOF OPS

The edges in the set E represent the OWCs admissible assignments to the packets directed to OF *i-th* according to limited conversion range of the LRWCs. We include the edge $\{\boldsymbol{a_{i,j,h}}, \boldsymbol{o_{i,k}}\}$ in the graph if OWC $\boldsymbol{o_{i,k}}$ may be assigned to packet $\boldsymbol{a_{i,j,h}}$ according to the conversion constraint, that is if $k\in[u,v]$, where $u=\max\{0,j-d\}$ and $v=\min\{M-1,j+d\}$. Since under the unicast traffic assumption, the SA schedules the packets so that both one packet may be transferred on one OWC only and one OWC only may be assigned to one packet, the output of an admissible SA can be represented by a matching (*MA*) of the graph *G*. We remember that a *MA* of *G* is a graph G^* $[V,E^*]$ with $E^*\in E$ and the edges in E^* vertexes disjoint, that is no vertex of G^* is connected to two or more edges. Obviously an Optimum SA is represented by a maximum *MA* of *G*, that is an *MA* containing the maximum number of edges. The authors show in [34,35] that the graph *G* associated to scheduling problem previously described, has some nice properties. Let us order the left side and right vertices; in particular if we denote with $Ord(\boldsymbol{a_{i,j,h}})$ and $Ord(\boldsymbol{o_{i,k}})$ the order number of the vertexes associated to packet $\boldsymbol{a_{i,j,h}}$ and OWC $\boldsymbol{o_{i,k}}$, we have $Ord(\boldsymbol{a_{i,j,h}})=\sum_{p=0}^{j-1}K_{i,p}+h$ and $Ord(\boldsymbol{o_{i,k}})=k$ respectively. The graph *G* is a convex graph that is it has the following properties : i) the order numbers of the vertices in V_I adjacent to the vertex associated to OWC $\boldsymbol{o_{i,k}}$ constitute an interval whose begin and end are $begin(\boldsymbol{o_{i,k}})=\sum_{p=0}^{\max(0,k-(d+1))}K_{i,p}+1$ and $end(\boldsymbol{o_{i,k}})=\sum_{p=0}^{\min(M-1,k+d)}K_{i,p}$; ii) for two right side vertices associated to OWCs $\boldsymbol{o_{i,k}}$ and $\boldsymbol{o_{i,h}}$, with order numbers $k<h$, we have $begin(\boldsymbol{o_{i,k}})\leq begin(\boldsymbol{o_{i,h}})$ and $end(\boldsymbol{o_{i,k}})\leq end(\boldsymbol{o_{i,h}})$. For a convex bipartite graph, the First Available Algorithm (*FAA*) illustrated in [34,35] can be used to find a maximum matching. In this algorithm, we start matching the right side vertices associated to OWCs $\boldsymbol{o_{i,k}}$ in increasing order number (k=0,…M-1). In particular we match right side vertex associated to OWCs $\boldsymbol{o_{i,k}}$ with the left side vertex that is adjacent to it, not matched yet and with smallest order number; the right side vertex is left unmatched if such adjacent and unmatched left side vertex does not exist.

An example of scheduling of packets in an OF of a TLWC-SPOF OPS is shown in Fig. 4.a. The OF carries five wavelengths and six packets arrive in the considered time-slot. In particular, two packets are arriving on wavelength λ_1 and four packets are arriving on wavelength λ_2. According to the notations introduced, we have M=5, d=1, $K_{i,0}$=0, $K_{i,1}$=2, $K_{i,2}$=4, $K_{i,3}$=0 and $K_{i,4}$=0. In Fig. 5 the bipartite graph associated to the scheduling problem and the maximum matching are reported. Finally the packets scheduled at the end of the LCA phase are shown in Fig. 4.b.

The actions performed in the LCA phase are described in Fig. 3. For each OWC $\boldsymbol{o_{i,j}}$ of OF *i-th*, it is checked if there are packets not yet scheduled and which can be transferred on OWC $\boldsymbol{o_{i,j}}$ according to the conversion range of the LRWCs. If at least one of these packets exists, the one of smallest order number is scheduled for OWC $\boldsymbol{o_{i,j}}$, otherwise no packets will be transferred and the OWC $\boldsymbol{o_{i,j}}$ will be stored in set Λ_i containing the OWCs for which no packets have been scheduled in the LCA phase.

Notice that for each OF, the time complexity of the actions performed in the LCA phase, is O(M) and it is determined by the cycle for *j=0 to M-1* shown in the flowchart in Fig. 3.

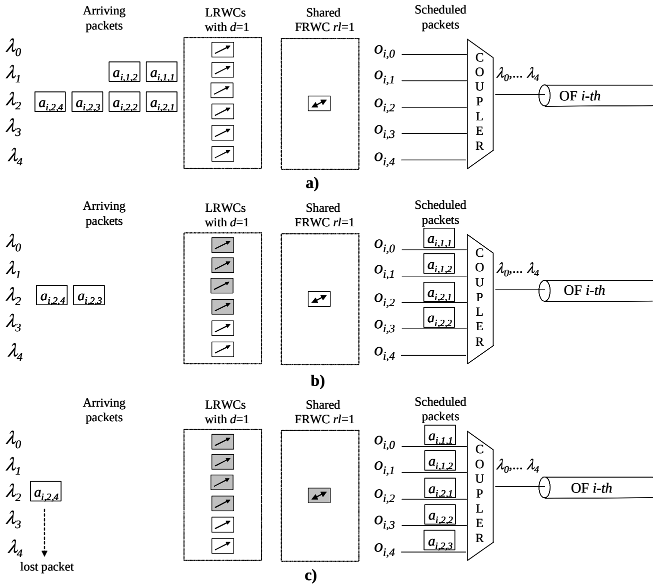

Figure 4. An example of packets scheduling for a generic OF carrying a number M=5 of wavelengths. In Fig. 4.a six arriving packets are shown. In Fig. 4.b the scheduled packets at the end of the LRWCs assignment phase are shown. In Fig. 4.c the scheduled packets at the end of the FRWCs assignment phase are shown.

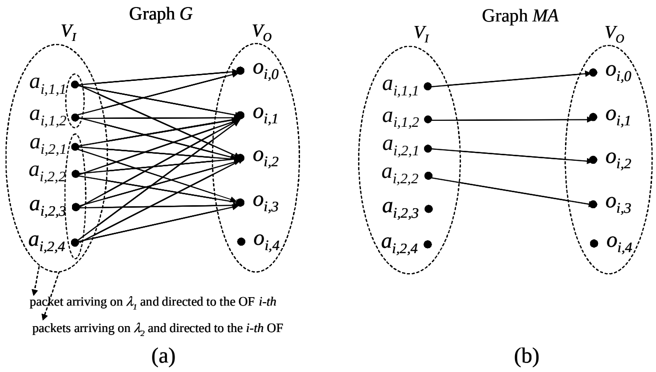

Figure 5. Bipartite graph and maximum matching associated to the scheduling problem of the LRWCs assignment phase. The OF considered is carrying a number M=5 of wavelengths. The conversion range d of the LRWCs used is 1.

B. FRWCs' Assignment (FCA) phase

In the FCA phase, illustrated in Fig. 3, the FRWCs are assigned to the packets not yet scheduled in the LCA phase. For OF *i-th* (i=0,…,N-1), the free OWCs and the packets not yet scheduled are stored in the sets Λ_i and Ψ_i respectively. One packet $\boldsymbol{a}\in\Psi_i$ is randomly selected and it is scheduled on one OWC $\boldsymbol{o_{i,j}}\in\Lambda_i$ by using one FRWC. When the action is performed, the variable rl_i is decreased by one. The FCA phase ends when either one the sets Λ_i, Ψ_i become empty or rl_i equals 0, i.e. all of the FRWCs have been used. When one of these events occurs, if any packet remains, it is discarded. Notice that in the FCA phase, the control unit discards the packets because of lack of FRWCs.

The scheduling of the packets in the FCA phase for the example reported in Fig. 4.a is illustrated in Fig. 4.c. At the end of the LCA phase, two packets remain to schedule and the OWC $\boldsymbol{o_{i,4}}$ is free. By using the only shared FRWC, one of the packets is forwarded. The remaining packet is discarded.

The computational complexity of the phase LCA is O(M), because in the worst case scenario both the number rl of FRWCs per each OF and the cardinality of the sets Λ_i and Ψ_i is lower than M. The computational complexity of the proposed SA for each OF is the sum of the complexities of phases LCA and FCA, i.e. O(M). Because all of the N OFs have to be scanned, the computational complexity of the SA in TLWC-SPOF architectures is $O(N{\cdot}M)$

IV. Probabilistic Model for the Evaluation of the Packet Loss Probability in TLWC-SPOF Switch

In this section we present the models we used to obtain the TLWC-SPOF architecture performance. Let M and N be the number of wavelengths and the number of OFs/IFs respectively. The Packet Loss Probability of the TLWC-SPOF switch is evaluated as a function of the number of FRWCs employed, under the following assumptions: i) the operation mode of the switching node is synchronous [3-23] on a time-slot basis; ii) packet arrivals on the $N{\cdot}M$ input wavelength channels at each time-slot are not dependent on each other; iii) packet arrivals occur with probability p on each input wavelength channel; iv) the destination of a packet is uniformly distributed over all N OFs, i.e., the probability that an arriving packet is directed to a given OF is equal to 1/N.

No assumption is made for the mutual dependence of packet arrivals at different time-slots, since, due to the bufferless nature of the switch, the performance and the FRWCs dimensioning procedure depend on p only [21].

In order to carry out the analysis we introduce some notations. Let us introduce the random variables α and β denoting for a given OF, the number of packets to be still scheduled and the number of OWCs still available respectively, at the end of the LCA phase of the SA introduced in Section III. In particular they are the cardinality of the sets Ψ_k and Λ_k, introduced in the description of the flowcharts of Fig. 3 when the phase of scheduling of packets with LRWCs is ended. Let us denote with $p_{\alpha,\beta}(i,j)$ (i=0,…..,$N{\cdot}M$; j=0,….M) the joint probability mass function ($p.m.f$) of the random variables α and β. The probabilities $p_{\alpha,\beta}(i,j)$ are evaluated in Appendix and they will be used to calculate the Packet Loss Probability of the TLWC-SPOF switch.

Due to the symmetric traffic assumption we can express the packet loss probability $P_{loss}^{TLWC-SPOF}$ as follows:

$$P_{loss}^{TLWC-SPOF}=1-\frac{E\left[N_{a,LCA}^{TLWC-SPOF}\right]}{E\left[N_o\right]}-\frac{E\left[N_{a,FCA}^{TLWC-SPOF}\right]}{E\left[N_o\right]} \qquad (2)$$

where $E[x]$ denotes the expected value of the random variable x and

-$E[N_o]$ is the average number of packets offered to a particular OF; it is easy to show that:

$$E[N_o]=M{\cdot}p \qquad (3)$$

- $E\left[N_{a,LCA}^{TLWC-SPOF}\right]$ is the number of packets accepted in an OF in the LCA, that is the packets have been scheduled using only LRWCs. This term equals the average number of used wavelengths in an OF of a switch using LRWCs only and it has been evaluated in [35] by using a recursive method.

- $E\left[N_{a,FCA}^{TLWC-SPOF}\right]$ is the number of packets accepted in an OF in the FCA phase, that is the packets have been scheduled using only the shared FRWCs. If for a generic OF, $\alpha=i$ and $\beta=j$ are the number of remaining packets to be yet scheduled and the free number of OWCs respectively at the end of the LCA phase, then the number of packets accepted in the FCA phase is $min(i,j,rl)$, rl being the available number of FRWCs in an OF. Hence we can write:

$$E\left[N_{a,FCA}^{TLWC-SPOF}\right]=\sum_{i=0}^{N\cdot M}\sum_{j=0}^{M}\min(i,j,rl)\,p_{\alpha,\beta}(i,j) \qquad (4)$$

Finally evaluating the probabilities $p_{\alpha,\beta}(i,j)$ as described in Appendix, inserting (3) and (4) in (2) we are able to evaluate the Packet Loss Probability $P_{loss}^{TLWC-SPOF}$ of a TLWC-SPOF switch as a function of the available number rl of FRWCs used in each OF.

V. Numerical Results

In this section we verify the validity of the analytical model introduced in Section IV to evaluate the TLWC-SPOF switch performance. The TLWC-SPOF architecture is compared to the one in which only FRWCs shared per output fiber are used. Next this architecture, having one conversion layer only, will be referred to as OLWC-SPOF. The conversion cost $Cost_{OLWC-SPOF}$ *of* the OLWC-SPOF switch can be obtained by the expressions (1) when the conversion range d equals 0. The Packet Loss Probabilities $P_{loss}^{OLWC-SPOF}$ of the OLWC-SPOF switch can be evaluated by means of the analytical models proposed in [36] or by using the formulas introduced in Section IV with the conversion range d equal to 0.

In order to validate the analytical model proposed for the TLWC-SPOF, we report in Figs. 6-7 the analytical and simulation values of Packet Loss Probabilities as a function of the number of FRWCs used when the number N of input/output fibers equals 8, the number M of wavelengths equals 16 and 32, the conversion range d of the LRWCs equals 1 and 2, the offered traffic p to each input wavelength channel is varying from 0.2 to 0.8. We notice that the analytical results and the simulation results are very close. The analytical model can therefore be considered validated and accurate, and it is used in the following to carry out the comparative analysis of the TLWC-SPOF architecture to the OLWC-SPOF architecture in which only shared FRWCs per output fiber are used.

From Figs. 6-7 we notice that all of the sketched curves have the same trend, decreasing as the number of FRWCs used increases up to a threshold value for which the packet loss probability saturates: this saturation value for the probability denotes the packet loss probability of an Optical Packet switch using one FRWC for each input wavelength channel and referred to as Single Per Channel (SPC) architecture in [36]. In the SPC switch the packet loss is only due to lack of output wavelength channels. The threshold value for which the saturation packet loss probability is reached, denotes the lowest

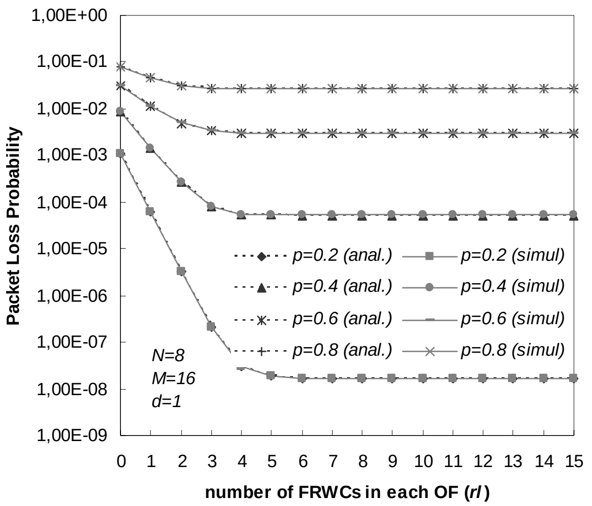

Figure 6. Comparison between analytical model and simulation results for the Packet Loss Probability in the TLWC-SPOF architecture, when: N=8, M=16, $d=1$ and p=0.2, 0.4, 0.6, 0.8.

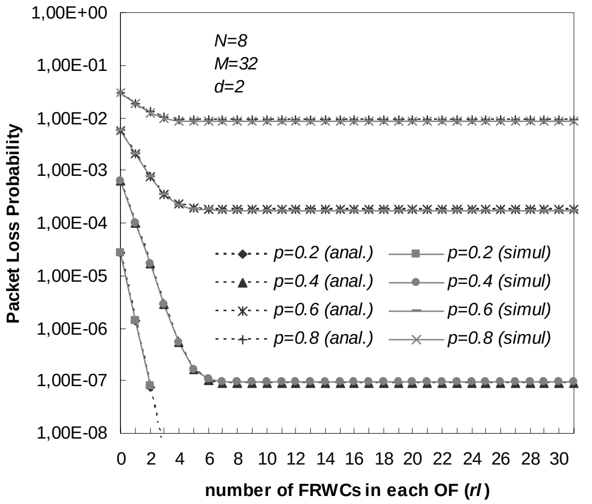

Figure 7. Comparison between analytical model and simulation results for the Packet Loss Probability in the TLWC-SPOF architecture, when: N=8, M=32, $d=2$ and p=0.2, 0.4, 0.6, 0.8.

number of FRWCs per output fiber needed in the TLWC-SPOF switch to reach the same packet loss probability of an SPC switch. Next this value, depending on the conversion range d of the LRWCs used, will be denoted with $r_{l,th}^{TLWC-SPOF,d}$. Obviously as the conversion range d of the LRWCs increases, lesser FRWCs are required to obtain the same performance of an SPC switch, that is the values $r_{l,th}^{TLWC-SPOF,d}$ decreases. This is shown in Fig. 8 where $P_{loss}^{TLWC-SPOF}$ is reported as a function of the number of FRWCs used for N=8, M=32, p=0.4 and d varying from 0 to 16. Notice as $r_{th}^{TLWC-SPN.d}$ decreases from 7 to 3 when d increases from 1 to 5. The result is obvious since increasing the conversion range of the LRWCs increases their capability to obtain suitable output wavelengths, resulting in fewer FRWCs needed. The curve for d=0 in Fig. 8 represents the packet loss probabilities of the OLWC-SPOF switch in which only FRWCs shared per output fiber are used. In this case we have the highest threshold values of FRWCs needed to reach the packet loss probability of an SPC switch. This value, next referred to as $r_{l,th}^{OLWC-SPOF}$, equals 14 for the OLWC-SPOF switch.

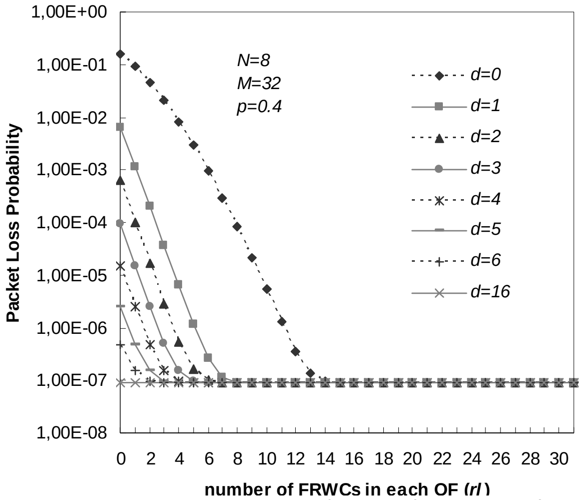

Figure 8. Packet Loss Probability as a function of the number of FRWCs used in each OF for a TLWC-SPOF. The switch and traffic parameters are N=8, M=32, p=0.4 and d varying from 0 to 16.

In order to evaluate the saving in terms of conversion cost of the TLWC-SPOF switch with respect to the OLWC-SPOF switch, we report in Figs. 9-10 the minimum conversion costs of the TLWC-SPOF switch as a function of the conversion range d of the LRWCs for N=8, p=0.6,0.8 and M varying from 16 to 512. The reported values have been obtained as follows: when the TLWC-SPOF switch is considered in Figs. 9-10, for a given conversion range d, the minimum number $r_{l,th}^{TLWC-SPOF,d}$ of FRWCs per output fiber needed to reach the packet loss probability of an SPC switch, is obtained. Hence the conversion cost of the TLWC-SPOF switch expressed by formula (1) is evaluated. Then the expression of the minimum conversion cost $Cost_{TLWC-SPOF,d}^{\min}$ for the TLWC-SPOF switch is the following:

$$Cost_{TLWC-SPOF,d}^{\min} = d \cdot N \cdot M + (M-1) \cdot N \cdot r_{l,th}^{TLWC-SPOF,d} \qquad (5)$$

The minimum conversion costs reported in Figs. 9-10 are U-shaped for the following reasons. When d is small, the conversion cost is dominated by the need for more FRWCs since the LRWCs range capability is too small to create a significant impact. When the conversion range of the LRWCs is increased, there is no need so many FRWCs as before and hence the conversion cost drops. However as d is further enlarged, the LRWCs becomes more like FRWCs and this will cause the overall cost to rise again.

From Figs. 9-10, we can see clearly that there is an optimum value d_{opt} of the conversion range that allows the lowest conversion cost to be obtained. Notice further that if LRWCs with conversion range d_{opt} are used in a TLWC-SPOF switch, the conversion cost is lower than in an OLWC-SPOF switch using only FRWCs shared per output fiber. In particular in Figs. 9-10 the minimum conversion costs of the OLWC-SPOF switches are obtained when d equals 0 and they have the following expression:

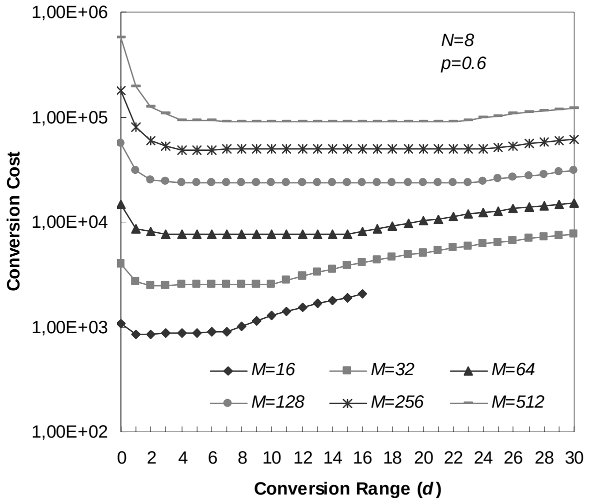

Figure 9. Conversion Cost of the TLWC-SPOF switch against the conversion range d for N=8, p=0.6 and M varying from 16 to 512.

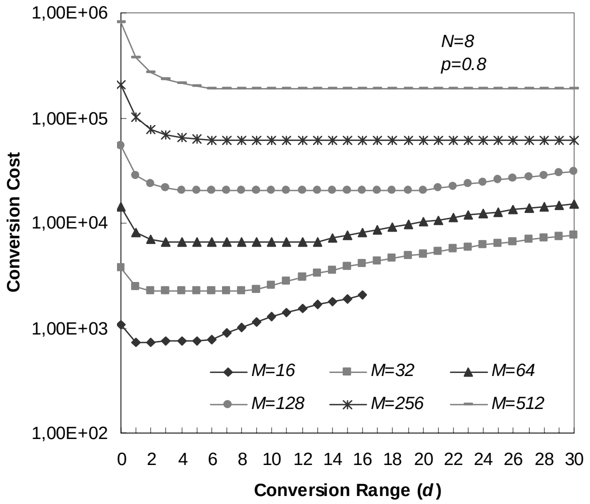

Figure 10. Conversion Cost of the TLWC-SPOF switch against the conversion range d for N=8, p=0.8 and M varying from 16 to 512.

$$Cost_{OLWC-SPOF}^{\min} = (M-1)\cdot N \cdot r_{l,th}^{OLWC-SPOF} \tag{6}$$

The final set of results focuses on the conversion savings made possible by the TLWC-SPOF switch compared to the OLWC-SPOF switch. We denote $\alpha_{TLWC\text{-}SPOF,d}$ the percentage of conversion saving of the TLWC-SPOF switch compared to the OLWC-SPOF switch. They are defined as follows:

$$\alpha_{TLWC-SPOF,d} = \left(1 - \frac{Cost_{TLWC-SPOF,d}^{\min}}{Cost_{OLWC-SPOF}^{\min}}\right) \times 100 \tag{7}$$

The conversion saving percentage $\alpha_{TLWC\text{-}SPOF,d}$ is shown in Figs. 11-12. The traffic and switch parameters are the same considered in Figs. 9-10. Notice as the higher the number M of wavelengths considered, the greater the conversion saving.

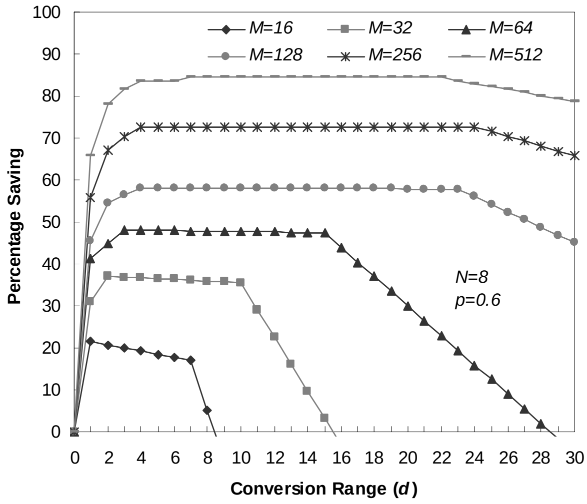

Figure 11. Percentage of the conversion saving of the TLWC-SPOF switch against the OLWC-SPOF for N=8, p=0.6 and M varying from 16 to 512.

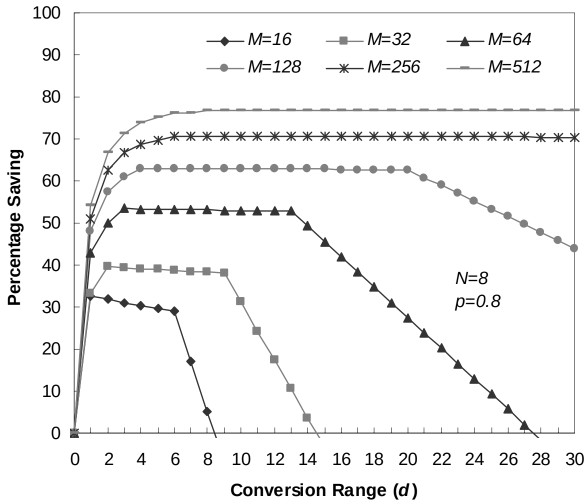

Figure 12. Percentage of the conversion saving of the TLWC-SPOF switch against the OLWC-SPOF for N=8, p=0.8 and M varying from 16 to 512.

VI. CONCLUSIONS

In this paper a WDM optical packet switching architecture denoted as TLWC-SPOF was discussed, equipped with Limited-Range Wavelength Converters and shared Full Range Wavelength Converters. A statistical approach was presented to provide theoretical drop performance of the TLWC-SPOF switch. The results obtained through analytical models were validated by those obtained through simulations. These results show that the TLWC-SPOF architecture, when compared to the OLWC-SPOF architecture using only FRWCs shared, allows for greater economies in terms of conversion cost.

REFERENCES

[1] R. Ramaswami and K.N. Sivarjan, Optical Networks (Morgan Kaufmann Publishers, San Francisco, 1998).

[2] B. Mukherjee, Optical Communication Networks, (Mc Graw-Hill, New York, 1997).
[3] S. Yao, B. Mukherjee and S. Dixit, "Advances in Photonic Packet Switching: an Overview", IEEE Communication Magazine, February 2000.
[4] M. Listanti, R. Sabella, V. Eramo, "Architectural and Technological Issues for Future Optical Internet Networks", IEEE Communications Magazine, pp.82-92, September 2000.
[5] B. Li, Y. Qin, X. Cao, K. Sivalingam, "Photonic Packet Switching: Architectures and Performance", Optical Network Magazine, January/February 2001, pp. 27-39.
[6] L. Xu, H.G. Perros and G. Rouskas, "Techniques for Optical Packet Switching and Optical Burst Switching", IEEE Communication Magazine, January 2001.
[7] M. Mahony, D. Simeoniddu, D. Hunter and A. Tzanamaki, "The Application of Optical packet switching in Future Communications Networks", IEEE Communication Magazine, March 2001.
[8] M. Renaud, C. Janz, P. Gambini and C. Guillemot, "Transparent optical packet switching: The European ACTS KEOPS project approach", in Proc. LEOS '99, Vol.2, pp. 401-402, 1999
[9] C. Guillemot et al., "Transparent Optical Packet Switching: The European ACTS KEOPS Project Approach", IEEE Journal of Lightwave Technology, Vol.16, No. 12, December 1998
[10] A.G. Rahbar and O. Yang, "Reducing Loss Rate in Slotted Optical Networks: A Lower Bound Analysis", IEEE ICC 2006, Istanbul, Turkey, June 2006.
[11] H. Mellah and F.M. Abbou, "Contention Resolution in Slotted WDM Optical Packet Switching Networks using cascaded auxiliary Switch", IEE Proceedings Communications, Vol. 152, No. 2, April 2006, pp. 205-206.
[12] F. A. Al-Zahrani, "Survivability Performance Evaluation of Slotted Multi-fiber Optical Packet Switching Networks with and without Wavelength Conversion", IEEE Information and Communication Technology, ICTTA 2006, April 2006.
[13] H. Harai, M. Suzuki and T. Ozeki "Buffer Management for Shared Feedback Buffer-Type Optical Packet Switches", IEEE ICC 2006, Istanbul, Turkey, June 2006.
[14] S. L. Danielsen, C. Joergensen, B. Mikkelsen and K. E. Stubkyaer, Optical Packet Switched Network Layer Without Optical Buffers", IEEE Photonic Technology Letters, Vol.10, No. 6, June 1998.
[15] S. L. Danielsen, C Jorgensen, B. Mikkelsen and K. E. Stubbkyaer, " Analysis of a WDM Packet Switch with Improved Performance Under Bursty Traffic Conditions Due to Tuneable Wavelength Converters", IEEE Journal of Lightwave Technology, Vol.16, No. 5, May 1998.
[16] A. Zalesky and R. S. Tucker, " Dynamically Concatenated Wavelength Converters", IEEE Photonics Technology Letters, Vol.18, No. 2, January 2006, pp.352-354.
[17] J. Zhou et al., "Four-Wave-Mixing Wavelength Conversion Efficiency in Semiconductor Traveling-Wave Amplifiers Measured to 65nm of Wavelength Shift", IEEE Photonic Technology Letters, Vol.6, No.8, pp. 984-987, August 1994.
[18] C. Joergensen et al., "4Gb/s Optical Wavelength Conversion Using Semiconductor Optical Amplifier", IEEE Photonic Technology Letters, Vol.5, No.6, pp. 657-670, June 1993.
[19] B. Mikkelsen et al., "Polarization Insensitive Wavelength Conversion of 10 Gb/s Signals with SOAs in a Michelson Interferometer", Electronic Letters, Vol. 30, No. 3, pp. 260-261.
[20] J.M.H. Elmirghani and H.T. Mouftah, "All-optical Wavelength Conversion Technologies and Applications in DWDM", IEEE Communication Magazine, Vol. 38, No. 3, March 2000, pp. 86-92.
[21] V. Eramo, M. Listanti, "Packet Loss in a Bufferless WDM Switch Employing Shared Tuneable Wavelength Converters", IEEE Journal Lightwave Technology, December 2000.
[22] V. Eramo, M. Listanti, P. Pacifici, "A Comparison Study on the Wavelength Converters Number Needed in Synchronous and Asynchronous All-Optical Switching Architectures", IEEE Journal of Lightwave Technology, February 2003.
[23] V. Eramo, M. Listanti ,"Comparison of Unicast/Multicast Optical Packet Switching Architectures using Wavelength Conversion," Optical Network Magazine, vol. 3, pp. 63-75, March-April 2002.
[24] C. Raffaelli, M. Savi, "Performance Modeling of Synchronous Buffer-less Optical Packet Switch with Partial Wavelength Conversion," IEEE ICC 2006, Istanbul, Turkey, June 2006.
[25] C. Raffaelli, M. Savi, A. Stavdas, "Performance of Scheduling Algorithms in Multi-Stage Optical Packet Switches with Sparse Wavelength Converters," IEEE Globecom 2006, San Francisco, USA, December 2006.
[26] C. Raffaelli, M. Savi, A. Stavdas, "Sharing Wavelength Converters in Multistage Optical Packet Switches," High Performance Switching and Routing, Poznan, Poland, June 2006.
[27] N. Akar, E. Karasan, K. Dogan, "Wavelength Converter Sharing in Asynchronous Optical Packet/Burst Switching: An Exact Blocking Analysis for Markovian Arrivals, IEEE Selected Areas in Communications, Vol. 24. No. 12, December 2006, pp. 69-80.
[28] G. Shen et al., "Performance Study on a WDM Packet Switch With Limited-Range Wavelength Converters", IEEE Communications Letters, Vol. 5, No. 10, October 2001.
[29] K. Dogan, N. Akar, "A Performance Study of Limited Range Partial Wavelength Conversion for Asynchronous Optical Packet/Burst Switching", IEEE ICC 2006, Istanbul, Turkey, June 2006.
[30] R.C. Almeida, J.F. Martins-Filho, H. Waldman "Limited-Range Conversion Modeling for Asynchronous Optical Packet-Switched Networks", SBMO/IEEE MTT-S International Conference on Microwave and Optoelectronics, July 2005.
[31] Z. Rosberg, A. Zalesky, H.L. Vu, M. Zukerman, "Analysis of OBS Networks with Limited Wavelength Conversion", IEEE/ACM Transactions on Networking, Vol. 14, No. 5, October 2006, pp. 1118-1127.
[32] Y. Mingwu, W. Aijun, L. Zengji, "Blocking Probability of Asynchronous Optical Burst/Packet Switches with Limited Range Wavelength Conversion", IEEE Photonics Technology Letters, Vol. 18, No. 12, June 2006, pp. 1302-1304.
[33] V. Eramo, M. Listanti, M. Di Donato, "Performance Evaluation of a Bufferless Optical Packet Switch with Limited-Range Wavelength Converters", IEEE Photonic Technology Letters, February 2004.
[34] Z. Zhang and Y. Yang, "Distributed Scheduling Algorithms for Wavelength Convertible WDM Optical Interconnects," IEEE IPDPS 2003, April 2003.
[35] Z. Zhang and Y. Yang, "Performance Modelling of Bufferless WDM Packet Switching Networks with Wavelength Conversion," IEEE Globecom 2003, December 2003.
[36] V. Eramo, M. Listanti, M. Spaziani, "Resources Sharing in Optical Packet Switches with Limited-Range Wavelength Converters", IEEE Journal of Lightwave Technology, Vol. 23, No. 2, February 2005, pp. 671-687.
[37] H. Li, I. Li-Jin, "Cost-Saving Two-Layer Wavelength Conversion in Optical Switching Network", IEEE Journal of Lightwave Technology, Vol. 24, No. 2, February 2006, pp. 705-712.
[38] Y.H. Choi, S.W. Seo, "A WDM Optical Packet Switch based on Wavelength Converter Blocks with Heterogeneous Conversion Capability", IEEE Globecom 2006, San Francisco, USA, December 2006.
[39] B. Bostica, M. Burzio, P. Gambini and L. Zucchelli, "Synchronization issues in optical packet switched networks", Photonic Networks, G. Prati, Sringer-Verlag, 1997
[40] A. E. Eckberg and T. C. Hou, "Effects of Output Buffer Sharing on Buffer Requirements in an ATDM Packet Switch", IEEE Infocom '88, pages 459-466
[41] M. Karol, M.G. Hluchyj, S. P. Morgan"Input Versus Output Queuing on a Space-Division Packet Switch", IEEE Transactions on Communications, Vol.Com-35, NO. 12, December 1987, pp. 1347-1357.
[42] J. A. Shormans, J. M. Pitts, "Overflow Probability in Shared Cell switched Buffers", IEEE Communications Letters, Vol. 4, NO. 5, May 2000, pp. 167-169.

APPENDIX A: EVALUATION OF $p_{\alpha,\beta}(i,j)$ $(i=0,1,..,N{\cdot}M;\ j=0,\ldots,M)$

In the following we give a recursive method to calculate the probabilities $p_{\alpha,\beta}(i,j)$ (i=0,…$N{\cdot}M$; j=0,…M) analytically. Let us introduce the following random variables:

- R_k (k=0,…,M-1) denotes the number of packets arriving on wavelength λ_k and directed to a particular OF. R_k is a random variable whose probability mass function (*p.m.f.*) $p_r(i)$ (i=0,…,N) has binomial distribution with parameter (N, p/N); we can write:

$$p_r(i) = \binom{N}{i}\left(\frac{p}{N}\right)^i\left(1-\frac{p}{N}\right)^{N-i} \qquad i = 0,1,......N \qquad \text{(A.1)}$$

- $A^{(h)}$ (h=1,…,M) denotes the number of packets arriving on h wavelengths and directed to a particular OF. $A^{(h)}$ is a random variable whose probability mass function (*p.m.f.*) $p_{A^{(h)}}(i)$ (i=0,…$N\cdot h$) has binomial distribution with parameter ($N\cdot h$, p/N); we can write:

$$p_{A^{(h)}}(i) = \binom{N\cdot h}{i}\left(\frac{p}{N}\right)^i\left(1-\frac{p}{N\cdot h}\right)^{N-i} \quad i = 0,1,...N\cdot h \qquad \text{(A.2)}$$

- $\alpha_{p,q}$ and $\beta_{p,q}$ (1≤p≤M and 1≤q≤M) denote for a given OF, the number of packets arriving on wavelengths from λ_{M-p} to λ_{M-1} to be still scheduled and the number of OWCs from λ_{M-q} to λ_{M-1} still available respectively, at the end of the LCA phase of the SA under the conditions that: i) the output wavelengths λ_{M-q} to λ_{M-1} of the OF considered are only assigned to packets arriving on wavelengths λ_{M-p} to λ_{M-1}; ii) the packets arriving on wavelengths λ_{M-p} to λ_{M-1} are only assigned to output wavelengths λ_{M-q} to λ_{M-1}. $\alpha_{p,q}$ and $\beta_{p,q}$ are random variables taking value from 0 to $p\cdot N$ and from 0 to q respectively. We use $p_{\alpha,\beta}(p,q,i,j)$ to denote the p.m.f. of the random variables $\alpha_{p,q}$ and $\beta_{p,q}$. Figure A1 illustrates the definition of $\alpha_{p,q}$ and $\beta_{p,q}$. Clearly, when $p=q=M$, $\alpha_{M,M}$ and $\beta_{M,M}$ are simply α and β respectively. Hence the joint *p.m.f.* of $\alpha_{M,M}$ and $\beta_{M,M}$ is $p_{\alpha,\beta}(i,j)$.

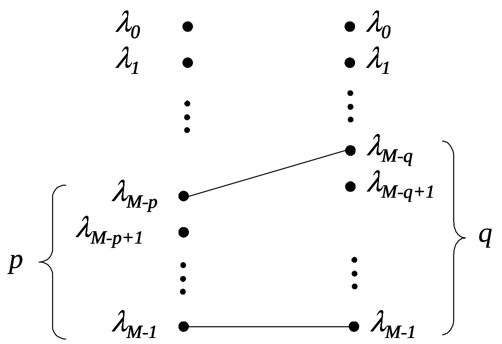

Figure A1 In evaluating the joint *p.m.f.* of $\alpha_{p,q}$ and $\beta_{p,q}$ output wavelengths λ_{M-q} to λ_{M-1} are only assigned to packets arriving on wavelengths λ_{M-p} to λ_{M-1}, and packets arriving on wavelengths λ_{M-p} to λ_{M-1} are only assigned to output wavelengths λ_{M-q} to λ_{M-1}

In the subsections A.1, A.2, and A.3 we evaluate $p_{\alpha,\beta}(p,q,i,j)$ in the three interest cases respectively: i) p=1 and 1≤q≤d+2; ii) 2≤p≤d+1 and 1≤q≤p+d+1; iii) p≥d+2 and p-d≤q≤p+d+1. Finally in subsection A.4 the iterative algorithm for the evaluation of the joint *p.m.f.* $p_{\alpha,\beta}(i,j)$ will be given.

A.1 Evaluation of the joint p.m.f. of $\alpha_{p,q}$ and $\beta_{p,q}$ for p=1 and 1≤q≤d+2

First we evaluate $p_{\alpha,\beta}(p,q,i,j)$ for p=1 and 1≤q≤d+1. This means that output wavelengths λ_{M-q} to λ_{M-1} are only assigned to packets arriving on wavelength λ_{M-1} and packets arriving on wavelength λ_{M-1} are only assigned to output wavelengths λ_{M-q} to λ_{M-1}. We have the following expression:

$$p_{\alpha,\beta}(p,q,i,j) = \begin{cases} p_r(q+i) & q+i \le N \quad j=0 \\ p_r(q-j) & i=0 \qquad j \le q \\ 0 & otherwise \end{cases} \qquad \text{(A.3)}$$

In fact when $q+i$ packets arrive on wavelength λ_{M-1}, according to the LCA phase of the SA, they are transferred on output wavelengths from λ_{M-q} to λ_{M-1}; in this case no output wavelengths will be available and i packets will be not yet scheduled at the end of the LCA phase. When q-j packets arrive on wavelength λ_{M-1}, according to the LCA phase of the SA, they are transferred on output wavelengths from λ_{M-q} to λ_{M-j+1} and the output wavelengths from λ_{M-j+1} to λ_{M-1} will be yet available at the end of the LCA phase. Further all of the arriving packets will be scheduled.

In order to evaluate the joint *p.m.f.* of the random variables $\alpha_{p,q}$ and $\beta_{p,q}$ for p=1 and q=d+2, we notice that, due to the limited range of the LRWCs, the arriving packets on wavelength λ_{M-1} may be forwarded on output wavelengths from λ_{M-d-1} to λ_{M-1} and wavelength λ_{M-d-2} always remains available at the end of the LCA phase. Hence we can write for p=1 and q=d+2:

$$p_{\alpha,\beta}(1,d+2,i,j) = \begin{cases} p_{\alpha,\beta}(1,d+1,i,j-1) & j \ne 0 \\ 0 & j = 0 \end{cases} \qquad \text{(A.4)}$$

A.2 Evaluation of the joint p.m.f. of $\alpha_{p,q}$ and $\beta_{p,q}$ for 2≤p≤d+1 and 1≤q≤p+d+1

First we evaluate $p_{\alpha,\beta}(p,q,i,j)$ for 2≤p≤d+1 and 1≤q≤p+d. By conditioning on r_{M-p} which is the number of packets arriving on wavelength λ_{M-p}, $p_{\alpha,\beta}(p,q,i,j)$ can be written as:

$$p_{\alpha,\beta}(p,q,i,j) = \sum_h \Pr\left(\alpha_{p,q}=i,\beta_{p,q}=j/r_{M-p}=h\right)\Pr\left(r_{M-p}=h\right) \text{(A.5)}$$

Notice that $Pr(r_{M-p}=h)$ is simply $p_r(h)$, the probability that there are h packets on wavelength λ_{M-p}. The term $\Pr ob\left(\alpha_{p,q}=i,\beta_{p,q}=j/r_{M-p}=h\right)$ is evaluated according to the following two remarks.

Remark 1- When h≤q-1 packets arrive on wavelength λ_{M-p}, they are transferred on wavelength from λ_{M-q} to $\lambda_{M-q+h-1}$. Consequently $\Pr ob\left(\alpha_{p,q}=i,\beta_{p,q}=j/r_{M-p}=h\right)$ equals $p_{\alpha,\beta}(p-1,q-h,i,j)$, that is the probability that i packets are to be still scheduled and j OWCs are still available at the end of the LCA phase respectively under the condition that output wavelengths from λ_{M-q+h} to λ_{M-1} are only assigned to packets arriving on wavelengths from λ_{M-p+1} to λ_{M-1}, and packets arriving on wavelengths from λ_{M-p+1} to λ_{M-1} are only assigned to output wavelengths from λ_{M-q+h} to λ_{M-1}.

Remark 2- When h≥q packets arrive on wavelength λ_{M-p}, all of the OWCs from λ_{M-q} to λ_{M-1} will be engaged and no OWCs will be available at the end of the LCA phase. The number of packet not scheduled equals the number of packets not scheduled on wavelengths λ_{M-p}, that is h-q, plus the number of packets arriving on the (p-1) wavelengths from λ_{M-p+1} to λ_{M-1}.

According to the remarks 1, 2 it is possible to prove that for $p_{\alpha,\beta}(p,q,i,j)$ the following expression holds when 2≤p≤d+1 and 1≤q≤p+d:

$$p_{\alpha,\beta}(p,q,i,j)=\begin{cases}\sum_{h=0}^{q-1}p_r(h)p_{\alpha,\beta}(p-1,q-h,i,j)+ & \\ +\sum_{h=q}^{\min(N,i+q)}p_r(h)p_{A(p-1)}(i-(h-q)) & j=0 \\ \sum_{h=0}^{q-1}p_r(h)p_W(p-1,q-h,i,0) & j\neq 0\end{cases} \quad (A.6)$$

In order to evaluate the joint *p.m.f.* of the random variables $\alpha_{p,q}$ and $\beta_{p,q}$ for $2\leq p\leq d+1$ and $q=p+d+1$, we notice that, due to the limited range of the LRWCs, the arriving packets on wavelength λ_{M-p} may be forwarded on output wavelengths from λ_{M-p+d} to λ_{M-1} and wavelength $\lambda_{M-p+d-2}$ always remains available at the end of the LCA phase. Hence we can write for $2\leq p\leq d+1$ and $q=p+d+1$:

$$p_{\alpha,\beta}(p,p+d+1,i,j)=\begin{cases}p_{\alpha,\beta}(p-1,p+d,i,j-1) & j\neq 0\\ 0 & j=0\end{cases} \quad (A.7)$$

A.3Evaluation of the joint p.m.f. of $\alpha_{p,q}$ and $\beta_{p,q}$ for $p\geq d+2$ and $p-d\leq q\leq p+d+1$

The evaluation of $p_{\alpha,\beta}(p,q,i,j)$ for $p\geq d+2$ and $p-d\leq q\leq p+d$ can be carried out according to a procedure like the one followed in subsection A.2. The only difference is that in calculating the term $\Pr ob(\alpha_{p,q}=i,\beta_{p,q}=j/r_{M-p}=h)$ occurs to take into account that, due to the limited conversion range of the LRWCs, not all the *q* output wavelengths can be assigned to the *j* packets arriving on λ_{M-p}. We use $T=q-p+d+1$ to denote the number of output wavelengths on which the j packets arriving on λ_{M-p} can be transferred according to the limited conversion range of the LRWCs. The term $\Pr ob(\alpha_{p,q}=i,\beta_{p,q}=j/r_{M-p}=h)$ is evaluated according to the following two remarks.

Remark 1- If $h\leq T$, the first h output wavelengths, λ_{M-q} to $\lambda_{M-q+h-1}$, will be assigned to packets arriving on λ_{M-p}. Consequently $\Pr ob(\alpha_{p,q}=i,\beta_{p,q}=j/r_{M-p}=h)$ equals $p_{\alpha,\beta}(p-1,q-h,i,j)$, that is the probability that i packets are to be still scheduled and *j* OWCs are still available at the end of the LCA phase respectively under the condition that output wavelengths from λ_{M-q+h} to λ_{M-1} are only assigned to packets arriving on wavelengths from λ_{M-p+1} to λ_{M-1}, and packets arriving on wavelengths from λ_{M-p+1} to λ_{M-1} are only assigned to output wavelengths from λ_{M-q+h} to λ_{M-1}.

Remark 2- If $h>T$, the first *T* output wavelengths, λ_{M-q} to $\lambda_{M-q+T-1}$, will be assigned to packets arriving on λ_{M-p}. This is because by the definitions, none of the packets arriving on λ_{M-p} is assigned to output wavelengths with indexes smaller than *M-q* and output wavelengths λ_{M-q}, λ_{M-q+1}, λ_{M-q+2},…..are not assigned to packets on wavelengths with index smaller than *M-p*. Due to the limited range of the LRWCs, only the *T* output wavelengths from λ_{M-q} to $\lambda_{M-q+T-1}$ will be assigned to the packets arriving at wavelength λ_{M-p} and the remaining *h-T* packets will not scheduled at the end of the LCA phase. Consequently when $h>T$, $\Pr ob(\alpha_{p,q}=i,\beta_{p,q}=j/r_{M-p}=h)$ equals $p_{\alpha,\beta}(p-1,q-T,i-(h-T),j)$, that is the probability that $i-(h-T)$ packets are to be still scheduled and *j* OWCs are still available at the end of the LCA phase respectively under the condition that output wavelengths from λ_{M-q+T} to λ_{M-1} are only assigned to packets arriving on wavelengths from λ_{M-p+1} to λ_{M-1}, and packets arriving on wavelengths from λ_{M-p+1} to λ_{M-1} are only assigned to output wavelengths from λ_{M-q+T} to λ_{M-1}.

According to the remarks 1, 2 it is possible to prove that for $p_{\alpha,\beta}(p,q,i,j)$ the following expression holds when $p\geq d+2$ and $p-d\leq q\leq p+d$:

$$\begin{aligned}p_{\alpha,\beta}(p,q,i,j)=&\sum_{h=0}^{T}p_r(h)p_{\alpha,\beta}(p-1,q-h,i,j)+\\ &+\sum_{h=T+1}^{\min(N,i+T)}p_r(h)p_{\alpha,\beta}(p-1,q-T,i-(h-T),j)\end{aligned} \quad (A.8)$$

In order to evaluate the joint *p.m.f.* of the random variables $\alpha_{p,q}$ and $\beta_{p,q}$ for $p\geq d+2$ and $q=p+d+1$, we notice that, due to the limited range of the LRWCs, the arriving packets on wavelength λ_{M-p} may be forwarded on output wavelengths from λ_{M-p+d} to λ_{M-1} and wavelength $\lambda_{M-p+d-2}$ always remains available at the end of the LCA phase. Hence we can write $p\geq d+2$ and $q=p+d+1$:

$$p_{\alpha,\beta}(p,p+d+1,i,j)=\begin{cases}p_{\alpha,\beta}(p-1,p+d,i,j-1) & j\neq 0\\ 0 & j=0\end{cases} \quad (A.9)$$

A.4Iterative algorithm for the evaluation of the joint p.m.f. of α and β

To find the joint *p.m.f.* of α and β, which is the same of the joint *p.m.f.* of $\alpha_{M,M}$ and $\beta_{M,M}$, start with random variables with first index p=1: $\alpha_{1,1}$ and $\beta_{1,1}$, $\alpha_{1,2}$ and $\beta_{1,2}$, …, $\alpha_{1,d+2}$ and $\beta_{1,d+2}$, whose joint p.m.f. was evaluated in subsection A.1. Then use Equations (A.6), (A.7) in subsection A.2 and the joint *p.m.f.* of $\alpha_{1,1}$ and $\beta_{1,1}$, $\alpha_{1,2}$ and $\beta_{1,2}$, …, $\alpha_{1,d+2}$ and $\beta_{1,d+2}$ to find the joint *p.m.f.* for the random variables when p=2: α_{21} and $\beta_{2,1}$, $\alpha_{2,2}$ and $\beta_{2,2}$, …, $\alpha_{2,d+3}$ and $\beta_{2,d+3}$. Repeatedly we apply Equations (A.6), (A.7) and the joint *p.m.f.* found in the previous step to obtain the joint *p.m.f.* for random variables when p=3, p=4,….until $p=d+2$. Then use of Equations (A.7), (A.8) in subsections A.3 to obtain the joint *p.m.f.* for random variables for larger *p* until the joint *p.m.f.* of $\alpha_{M,M}$ and $\beta_{M,M}$ is found.