REMOTE IMAGE ALIGNMENT USING THE DFT PHASE

Marco Dalai and Riccardo Leonardi

Department of Electronics for Automation, University of Brescia, Italy
email: {marco.dalai, riccardo.leonardi}@ing.unibs.it

ABSTRACT

In this paper we consider a problem of remote image alignment. Two users A and B have access respectively to two images X and Y, which differ only for a shift component. User B wants to know the shift difference between the two images. User A has thus to send useful information about image X to user B, so as to let him identify the value of the shift. The objective is to minimize the number of bits sent from A to B. In this paper we consider the above problem and we propose a framework based on the use of the phase of the DFT. Different possible decoding strategies are studied for user B, which depend on the trade off between complexity and performance.

Index Terms— Image processing, video signal processing.

1. INTRODUCTION

With the recent emerging interest for network and multiuser applications, signal processing technique specifically developed for network scenarios have been receiving increasing attention in the last years. Important examples are to be found for example in distributed sampling and signal processing, and distributed source coding (see for example [8], [1] and [5]).

The problem studied within the present paper can be considered either in the context of distributed signal processing, in the context of distributed source coding, or as a problem of distributed identification. Two images X and Y, which are obtained by cropping a common scene from two displaced positions, are made available to two distinct users, say A and B respectively. User B wants to know the shift between the two images and, having access only to the Y image, he needs information about the image X from A. The problem to be solved is the definition of a technique to extract meaningful information from X so as to allow B to correctly identify the shift between the images.

We remark that user A cannot himself evaluate the value of the shift, as he has no access to the Y image. Thus, the problem is a problem of distributed identification of the shift, in the sense that in order to evaluate the shift one needs information about both images X and Y while this information is not available in one single point. It is thus necessary to send some amount of information from one point to the other one, and the objective is to minimize the amount of data transmitted.

The motivations for the definition and the study of this problem, are to be found in different applications in multiuser scenarios. An example is the case of image and video coding, which is the application of distributed source coding principles to images and video coding problems. Distributed source coding is a branch of information theory which has been receiving increasing attention from the the publication of [9] by Slepian and Wolf. The main topic considered in this field is the possibility of separately encode correlated sources with the same encoding efficiency as if they were encoded jointly, provided one single decoder has access to the information communicated by every encoder. The application of this principle to the case of image and video coding leads to interesting problems that rely in the exploitation of the correlation between images that are available to remote users (see for example [6]). In this context, a natural problem of interest is to find a way to "align" two images in a remote fashion, i.e., without having both images available in one point, but allowing for the communication of a small amount of information. Thus, for example, it is of interest to study as a starting problem the scenario considered in this paper, where an image Y is completely available to user B (say the decoder), which wants to perform the registration, and an image X is available to user A (say the encoder) which has to send to B a concise description of X that brings enough information to perform the compensation.

In this paper we thus propose a first approach to the problem of distributed estimation of the shift between images. The described technique can be extended, by working on the Fourier-Mellin domain (see [3]) to the more general problem of estimation of rotation and scale between images. This paper is thus part of a more general work on the adaptation of image registration techniques (see [2, 4, 10]) to distributed scenarios.

In the whole paper '$\log(\cdot)$' indicates the base-2 logarithm. Given an integer m, '$\{\cdot\}_m$' indicates the modulo-m operation, while the symbol '$\overset{2\pi}{=}$' indicates a modulo-2π congruence. We consider phases to always take values on the interval $[-\pi, \pi)$ within the next sections. Finally, we use a binary sign function $\text{sign}(\cdot)$ defined as $\text{sign}(x) = 1$ if $x < 0$ and

$\text{sign}(x) = 0$ if $x \geq 0$.

2. EXTRACTING INFORMATION FOR SHIFT DETECTION: 1-D CASE

In order to introduce the method proposed for the distributed encoding of the shift information we first consider a simple 1-dimensional ideal problem. Suppose thus $X(\cdot)$ and $Y(\cdot)$ are two N-point signals, available to A and B respectively, which differ only by a circular shift s, with $0 \leq s < S$, $S < N$. We can write this in equations as

$$X(n) = Y(\{n - s\}_N), \quad n = 0, 1, \dots, N-1.$$

For the sake of simplicity, let us consider the case when both N and S are powers of 2. The problem to be considered by B is the identification of the value of s. Consider first the case when image Y is also available to user A. In this case user A can obviously detect the value of the shift between the two images and then communicate this values to B. If s is uniformly distributed between 0 and $S - 1$, this would require on average the transmission of $\log(S)$ bits from A to B. Now suppose instead that the image Y is not available to the decoder. Supported by distributed source coding theory, one may wonder whether it is still possible to transmit the value of s to B using only $\log(S)$ bits. In the remaining part of this session we show how this can be done, by properly working on the DFT of the signal X.

Let $\hat{X}(\cdot)$ and $\hat{Y}(\cdot)$ be the DFT of X and Y respectively, and, for every k, let $\Phi_{\hat{X}}(k)$ and $\Phi_{\hat{Y}}(k)$ be the phase of the coefficient $\hat{X}(k)$ and $\hat{Y}(k)$ respectively. From the shift hypothesis on X and Y, the phases of the DFT are related by the following equation

$$\Phi_{\hat{X}}(k) \overset{2\pi}{=} -\frac{2\pi s k}{N} + \Phi_{\hat{Y}}(k). \tag{1}$$

In the DFT domain it is evident that by knowing the phase $\Phi_{\hat{X}}(\cdot)$ and the phase $\Phi_{\hat{Y}}(\cdot)$ it is possible to recover the shift value s. What we want to find, however, is a technique to extract the minimal information from the phase $\Phi_{\hat{X}}(\cdot)$ that suffices, together with the knowledge of $\Phi_{\hat{Y}}(\cdot)$, to deduce s.

First note that, if we take $k = 1$ in (1), we have

$$\Phi_{\hat{X}}(1) \overset{2\pi}{=} -2\pi \frac{s}{N} + \Phi_{\hat{Y}}(1). \tag{2}$$

Now, given that $s < N$, for every value of s the value on the right hand side of the eq. (2) determines a different point in the range $[-\pi, \pi]$. The phases obtained for two different values of s differ by a multiple of $2\pi/N$. Hence, for every interval of width $2\pi/N$ there is only one possible value of s such that the right hand side of equation (2) lies in that interval. This implies that, when $\Phi_{\hat{Y}}(1)$ is known, a quantization $\tilde{\Phi}_{\hat{X}}(1)$ of $\Phi_{\hat{X}}(1)$ into N intervals of width $2\pi/N$ completely determines the value of s. Furthermore, note that it is not really necessary to distinguish all these N intervals. Indeed, given that $s < S$, intervals with distance multiple of $2S\pi/N$ can be grouped together. Only one of the s values indicated by these intervals, indeed, will satisfy the constraint that $0 \leq s < S$. This leads to the conclusion that in order to allow user B to understand the value of s, it is only necessary to communicate the value of $\{\tilde{\Phi}_{\hat{X}}(1)\}_S$. So, only $\log(S)$ bits are required in order to quantize $\Phi_X(1)$ so that user B can recover the value of s.

Even if the above exposed procedure theoretically allows to reach the objective of detecting the shift s by communicating only $\log S$ bits of information from A to B, it is important to remark that it is fragile in the case of non-ideal situation of noisy signals with non-circular shift. The fine quantization of the value of $\Phi_{\hat{X}}(1)\}_S$ implies in fact that even a small effect due to non-ideal situations can cause errors in the detection of the value of s. It is thus important to identify different quantization techniques that would lead to a more robust system. Here we propose a different method to extract the shift value, which is based on a coarse quantization of more coefficients, rather than on a fine quantization of only one coefficient.

The main idea is to investigate the coefficients of the DFT that are more meaningful to the identification of s by considering the periodic variation of the phase of the coefficients at different frequencies when a shift component is introduced. Consider for example the coefficient of frequency $k = N/2$; for this coefficient equation (1) reduces to

$$\Phi_{\hat{X}}(N/2) \overset{2\pi}{=} -\pi s + \Phi_{\hat{Y}}(N/2) \tag{3}$$

i.e., $\Phi_{\hat{X}}(N/2)$ and $\Phi_{\hat{Y}}(N/2)$ differ by a multiple of π. This means that the sign of $\Phi_{\hat{X}}(N/2)$ is the same of the sign of $\Phi_{\hat{Y}}(N/2)$ if s is even, and it is the opposite if s is odd. Thus, by indicating to user B the sign of $\Phi_{\hat{X}}(N/2)$, he can decide if s is even or odd and, hence, he can determine the least significant bit s_0 of the binary representation of s. Note that this bit can be actually decoded as

$$s_0 = \text{sign}(\Phi_{\hat{Y}}(N/2)) \oplus \text{sign}(\Phi_{\hat{X}}(N/2)) \tag{4}$$

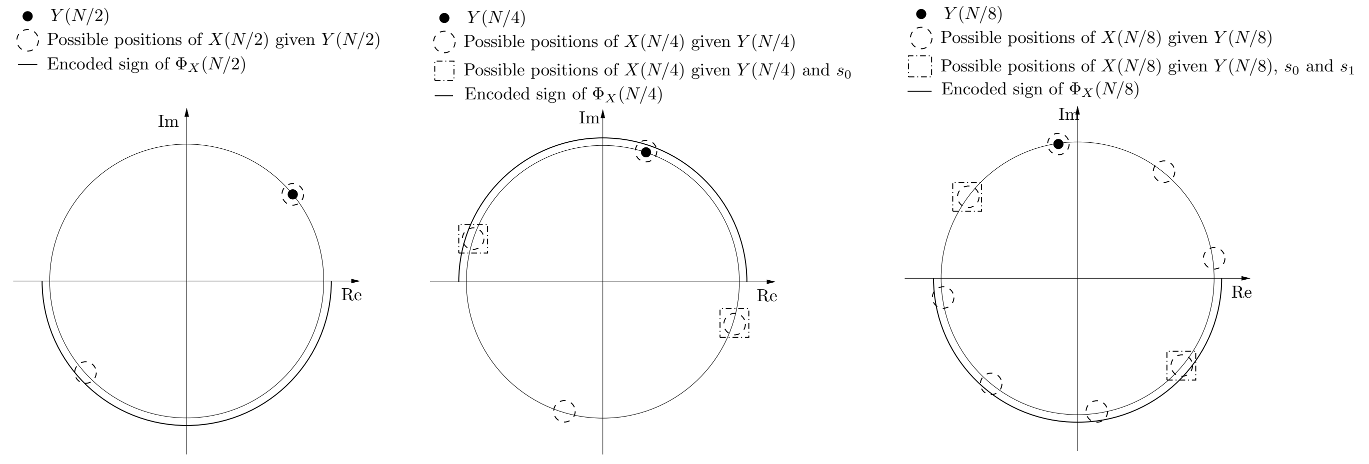
If we write $s = 2t + s_0$, it is then possible to use the sign of the phase $\Phi_{\hat{X}}(N/4)$ to detect if t is even or odd. Indeed, for $k = N/4$ equation (1) gives

$$\begin{aligned} \Phi_{\hat{X}}(N/4) &\overset{2\pi}{=} -\frac{\pi}{2} s + \Phi_{\hat{Y}}(N/4) \\ &\overset{2\pi}{=} -\pi t - \frac{\pi}{2} s_0 + \Phi_{\hat{Y}}(N/2). \end{aligned}$$

So again, the sign of $\Phi_{\hat{X}}(N/4)$ is sufficient to B to detect if t is even or odd, which corresponds to determine the second least significant bit s_1 of s. We explicitly have

$$s_1 = \text{sign}\left(\Phi_{\hat{Y}}(N/4) - \frac{\pi}{2} s_0\right) \oplus \text{sign}(\Phi_{\hat{X}}(N/4)) \tag{5}$$

The trick can be iterated again using the values of the phases $\Phi_{\hat{X}}(N/8), \Phi_{\hat{X}}(N/16), \cdots, \Phi_{\hat{X}}(N/S)$. We show indeed that



(a) Decoding of bit $s_0 = 1$ (b) Decoding of bit $s_1 = 1$ (c) Decoding of bit $s_2 = 0$

Fig. 1. Procedure for the decoding of the first three bits of s. In this case we had $s = 3$.

a 1-bit quantization of the above phases, for a total amount of $\log(S)$ bits, suffices to recover the value of s at the decoder.

Let us write the binary representation of s as $s_q s_{q-1} \cdots s_1 s_0$, $s_i \in \{0,1\}, i = 0, \ldots, q$, so that we have $\{s\}_{2^h} = s_{h-1}s_{h-2} \cdots s_0$. As explained above, knowing the sign of the phase of the $N/2$-th DFT coefficient of X, it is possible for B to recover the parity of s, and thus s_0. By using an iterated procedure, then, all the other bits $s_1, s_2, \ldots, s_q$ can be deduced by B when the signs of $\Phi_{\hat{X}}(N/4), \Phi_{\hat{X}}(N/8), \ldots, \Phi_{\hat{X}}(N/S)$ are known. We prove this by induction. Suppose in fact that the bits $s_0, s_1, \ldots, s_{h-1}$ has been determined using the signs of $\Phi_X(N/2), \Phi_X(N/2^2), \ldots, \Phi_X(N/2^h)$, and consider the coefficient $X(N/2^{h+1})$. We have that

$$\begin{aligned} \Phi_{\hat{X}}(N/2^{h+1}) &\overset{2\pi}{=} -\pi \frac{s}{2^h} + \Phi_{\hat{Y}}(N/2^{h+1}) \\ &\overset{2\pi}{=} -\pi s_h - \frac{\pi}{2^h}\{s\}_{2^h} + \Phi_{\hat{Y}}(N/2^{h+1}). \end{aligned}$$

Now, clearly $\{s\}_{2^h} = s_{h-1} \cdots s_1 s_0$ is known to the decoder, so that the only unknown term in the right hand side of the above equation is s_h. If the sign of the left hand side is known, s_h can be determined as

$$s_h = \mathrm{sign}\left(\Phi_{\hat{Y}}(N/2^{h+1}) - \frac{\pi}{2^h}\{s\}_{2^h}\right) \oplus \mathrm{sign}(\Phi_{\hat{X}}(N/2^{h+1})) \tag{6}$$

This proves that the $\log(S)$ bits that represent the signs of the phases $\Phi_{\hat{X}}(N/2^i), i = 1, \ldots, \log(S)$, allow the decoder to reconstruct the value of s. In Figure 1 an example of extraction of the first three bits of s is shown.

The reader may be interested in noticing that the problem we have considered in this section can also be reformulated as a problem of estimation of the frequency of a single tone signal in a distributed setting. Indeed, equation (1) can actually be rewritten as

$$\hat{X}(k) = \hat{Y}(k) \exp\left(-j\frac{2\pi s}{N}k\right) \tag{7}$$

When $\hat{Y}$ is known, the problem of identifying s is thus a problem of frequency estimation for user B. It is however interesting that the identification has to be performed with the smallest possible number of bits of information on $\hat{X}$. An in-depth study of the problem of frequency estimation from quantized uniformly spaced noisy samples can be found in [7]. In this section, however, the benefits of using logarithmically spaced samples have been shown. A detailed study of the best theoretical approach to frequency estimation using the least possible information is out of the scopes of the present paper.

3. EXTENSION TO 2-D SIGNALS: REMOTE IMAGE ALIGNMENT

3.1. Noiseless signals with circular shift

The theoretical development presented in the previous section is applied in this section for the 2-dimensional case in order to deal with the practical problem of encoding the relative shift between images. We first consider the ideal case where two to N by N images X and Y differ by a 2-dimensional circular shift. Let us call $\mathbf{v} = (r, c)$ the shift vector, where $0 \leq r < R$ and $0 \leq c < C$ - with $R < N$ and $C < N$ - are the value of the shift parameters along the rows and the columns respectively. In other words, the mathematical relation between the images is

$$X(n,m) = Y(\{n-r\}_N, \{m-c\}_N), \quad n,m = 0,1,\ldots,N-1,$$

and the corresponding relation between the 2-dimensional DFT's is thus

$$\Phi_{\hat{X}}(k,l) = -j\frac{2\pi kr}{N} - j\frac{2\pi lc}{N} + \Phi_{\hat{Y}}(k,l). \tag{8}$$

It is easily seen that the problem of determining r and c can be separated into two disjoint 1-dimensional problems. In fact, by setting $l = 0$ in (8), the term including c is canceled, and eq. (8) reduces to an equivalent of eq. (1), where r plays the role of s. In the same way, by setting $k = 0$ the term containing r is canceled and again an equivalent of eq. (1) is obtained, where now c plays the role of s. So, by taking respectively $l = 0$ and $k = 0$, we can solve the problem of extracting meaningful information from X so as to allow user B to detect the value of r and c independently. In this case, the bits extracted are the signs of the coefficients

$$\begin{gathered}\Phi_{\hat{X}}(N/2,0), \Phi_{\hat{X}}(N/4,0), \ldots, \Phi_{\hat{X}}(N/R,0), \\ \Phi_{\hat{X}}(0,N/2), \Phi_{\hat{X}}(0,N/4), \ldots, \Phi_{\hat{X}}(0,N/C).\end{gathered}$$

which amount to a total of $\log(R) + \log(C)$. So, the 2-dimensional problem in the ideal situation of noiseless circular shifts is optimally solved exactly in the same way as in the 1-D case.

3.2. Redundant information for concrete scenarios

The insight presented in the preceding section is now applied to a more realistic situation where the two images X and Y are not related by an ideal noiseless circular shift, but are instead obtained as noisy observations of a common scene from two shifted positions. We model this fact by saying that there exists a scene $Z(n,m)$ and independent noises n_x, n_y such that

$$\begin{aligned} Y(n,m) &= Z(n,m) + n_Y(n,m), & (9) \\ X(n,m) &= Z(n-r,m-c) + n_X(n,m). & (10) \end{aligned}$$

where again r and c nonnegative and bounded by R and C respectively. In this case, the values of R and C are assumed to be much smaller than N, because when R and C get comparable with N the overlap between the x and y image gets smaller and smaller.

Under these different assumptions, we are not interested anymore in the problem of detecting the shift using only exactly $\log(R) + \log(C)$ extracted from X. Due to the noise and to the boundary effects, it is reasonable to allow the use of more bits in order to detect the shift. In general, of course, it is reasonable to assume that the number of required bits to correctly detect the shift may depend on the strength of the additive noise on the X and Y images and on the amplitude of the shift components. So, for this practical situation, we relax the problem to more informal constraints and we aim at finding a robust strategy to detect the shift using only a small number of bits extracted from X.

The main idea that we adopt in this practical situation, then, is to use the insight given by the theoretical development proposed for the ideal case and "extend" the technique by increasing the robustness front to "errors" in the phase signs due to noise and to the boundary effects. In order to do this, it is necessary to add redundancy to the bits extracted from X. In the ideal case, when we considered the phase relation expressed by eq. (8), we clarified that it is possible to solve the problem separately for r and c by setting $l = 0$ and $k = 0$ respectively, so as to use a minimum number of bits. In a concrete non-ideal scenario, instead, in order to add robustness, it is useful to look at the situation from an opposite point of view. In particular, we note that when l and k are both different form zero the value of the resulting phase is affected by both r and c at the same time. So, rather than using only the coefficients of the form $\Phi_{\hat{X}}(N/2^i, 0)$ or $\Phi_{\hat{X}}(0, N/2^j)$, which represent the phases of perfect vertical and horizontal frequency harmonics, we can also consider the phases of "diagonal" frequency coefficient, i.e. the values of $\Phi_{\hat{X}}(N/2^i, N/2^j)$. By doing this, we actually add some sort of "parity-check" bits that can be used by user B to find a more reliable estimation of the shift.

In this case, it is interesting to study the problem of finding the best technique to be used by user B in order to find the most probable value of the shift given the values of the bits received by user A. In fact, in this case, contrarily to the ideal noiseless situation, there is no exact expression for the value of the shift components in terms of the phases of $\hat{Y}$ and on the signs of the phases of $\hat{X}$. It is instead necessary to investigate techniques for the identification of the shift that give a reliable estimate with high probability. In this context it is interesting to find that the ability of user B to detect the value of the shift depends on his computational power. Indeed, the more complicate operations on the available information are allowed, the more sophisticated the estimation procedure can be.

Here we outline a couple of different possible strategies for the identification of the shift. Both techniques are based on a full-search test for the possible value of r and c but a different approach is used on the operations performed in order to test every possible choice. In particular, the two methods differ for their computational complexity and, as a counterpart, for their performance. Before discussing these two methods, it is necessary to clarify an important point. What user B wants to do, in the end, is to estimate the shift that, applied to Y, gives an image which is similar to X. So, we can interpret the problem in the following way. User B tests the possible values of r and c by considering the result obtained when shifting Y by r and c and checking if the signs of the phases resemble the ones communicated by A. In the the real scenario, image X and the shift-compensated Y will always coincide only in the central part, as user B cannot recreate the portion of the X image located on the disappeared region. So, in order to reduce the boundary effects we can smooth the X

(a) Image X (b) Image Y

Fig. 2. Example of 256x256 x and y images cropped from the 512x512 *"goldhill"* image. Here we have $r = 21$, $c = 36$ and n_X and n_Y are independent white gaussian noises with $\sigma_{n_X} = \sigma_{n_Y} = 2$. In this case 69 bits suffice to correctly detect the shift with the computationally light decoder (in less than 1 second), while 39 bits suffice in the case of the complex decoder (in more than 300 seconds).

image so as to remove the information located on the borders of the area. Of course user B must take into account that this windowing operation is performed on the X image, for example by performing the same operation on the Y image. We now briefly describe the two techniques.

Complex method
User B consider all possible pairs of (r, c) values; for every one of them a circular shift by a (r, c) vector is applied to the image Y. The resulting image is multiplied by the window so at to remove the border effects and it is transformed with a DFT operation. A measure of distance between the obtained phases of the coefficients and the quantized ones (actually the signs) extracted from X is then computed, and the values of r and c that minimize this distance are kept as best estimate of the true shift components. Note that with this technique, when the correct value of r and c are checked, the shifted and windowed image Y differs from the windowed X mainly only for the noise, the border effects being smoothed by the window. This gives to the technique a great robustness. On the other hand, the main disadvantage is that for every (r, c) pair a DFT must be computed for the Y image. This lead to a high computational complexity of the method.

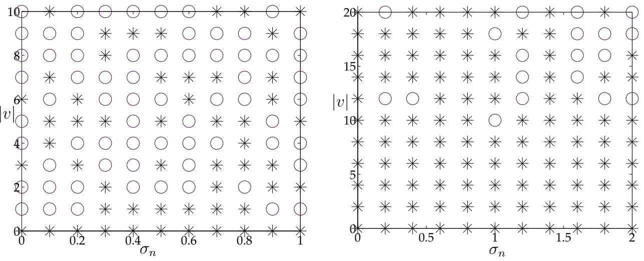
Fast method
An alternative choice, which cannot reach the same performance of the previous one, can be made in order to obtain a technique with a much lower computational complexity. In this case, the Y image is multiplied by the window only once at the beginning of the process and it is transformed. From the transformed image, the subset of meaningful frequency coefficients is extracted. Then, for every (r, c) pair, a circular shift on Y by a (r, c) vector is "simulated" on this subset of frequencies, by multiplying every coefficient by an appropriate exponential factor. This corresponds to perform a circular shift on the windowed version of image Y, and thus leads to a different result, because also the window is in this case shifted. A measure of distance between the so obtained phases of the meaningful coefficients and the signs received from user A and, again, the (r, c) pair that gives the minimum distance is kept as estimate of the shift vector. Note that in this case only one DFT is computed, and the operations required for every (r, c) pair have much lower computational complexity than in the previous method.

Remark
In both the above proposed methods, we have assumed that a measure of distance between the phases of the compensated Y image and the 1-bit quantized phases of the X image is defined. It is worth noticing that the definition of such a distance can be made in many different ways. In particular, a distance from a sequence of real values and a sequence of binary values must be defined. It is not simple to establish the optimal distance measure to be used for this problem and in our experiment we have used different types of distances. In particular, we have tried both "hard-decision" distances and "soft-decision" distances. The "hard-decision" distance is actually the Hamming distance between the sequence of 1-bit quantized phases of the compensated Y image and the 1-bit quantized phases of X. Soft distances, instead, are real valued distances that try to fully exploit the actual values of the phases of Y. Even if "soft-decision" distances usually perform better than "hard-decision" distances, in our preliminary experiment we have not verified much gain of the first ones over the latter ones. This topic is however still under investigation since many different factors, such as the particular type of additive noise and the power spectrum of the used image, impact the results.

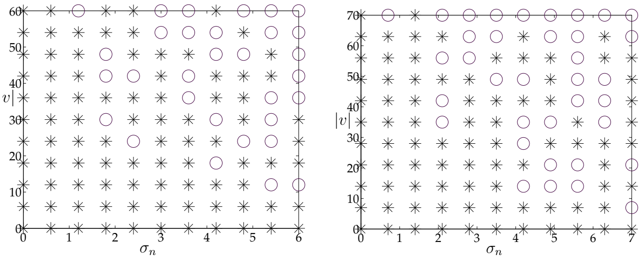
4. EXPERIMENTAL RESULTS

In order to show the effectiveness of the proposed method and to evaluate the performance in a practical situation, we have run some experiments on test images, and we report here one of these tests. We have only performed extensive simulations using the computationally-light proposed scheme, as the computationally complex scheme requires too many operation to extensively study the performance for different noise strength and shift amplitudes (see Fig. 2 for an example of difference of performance of the two methods).

So, we have taken the 512x512 *"goldhill"* image, and we have constructed the 256x256 x and y images by cropping portions of *"goldhill"* and by adding independent white gaussian noise to them. We have then applied the proposed computationally simple method and we have checked whether it gave the right result or not. The experiment was performed by testing, for different number of bits used for the code, various shift vector lengths and increasing noise amplitudes. The results are shown in Fig. (3), where we can see that by increasing the number of bits of the code progressively from 25 to 81 we are able to correctly detect shift vectors with increasing amplitudes and for increasing strength of the noise.



(a) 25 bits (b) 39 bits



(c) 61 bits (d) 81 bits

Fig. 3. Success/failures in the computationally light decoding of v, depending on the amplitude of the shift and on the noise strength, for different number of used bits. Images x and y were obtained here by cropping the image *"goldhill"* in random positions. An asterisk indicates a success while a circle indicates a failure. Note that the axis scale is different for different number of bits used.

5. CONCLUSION AND FUTURE WORK

In this paper we have studied the problem of distributed detection of the shift between two signals, and we have applied the theoretical discussion in order to construct a robust method for the distributed estimation of the shift between images. The method is based on the encoding of the sign of the phase of certain meaningful coefficients of the DFT. Further work will be devote to the study of more general phase sampling techniques and shift estimation methods. Extensions of the method to the case of rotations and scaling between images are under development and the application of the underlying ideas to the problem of remote motion compensation and remote disparity compensation between images is being considered.

6. ACKNOWLEDGMENTS

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