

Performance of Noncooperative Power Control Techniques for UWB Wireless Networks

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Abstract—This paper studies the performance of noncooperative game-theoretic power control techniques for wireless data networks in frequency-selective multipath. The uplink of an infrastructured network using ultrawideband signals as multiple access technique is considered. The effects of self- and multiple-access interference on the performance of Rake receivers are investigated for synchronous systems. Focusing on energy efficiency, a noncooperative game is proposed in which users in the network are allowed to choose their transmit powers to maximize their own utilities. A large system analysis is performed to derive the properties of the Nash equilibrium for the proposed game. Performance of this technique is also compared to that of the Pareto-optimal (cooperative) solution.

I. INTRODUCTION

Ultrawideband (UWB) technology is considered to be a potential candidate for next-generation multiuser data networks, due to its large spreading factor (which implies large multiuser capacity) and its lower spectral density (which allows coexistence with incumbent systems). The requirements for designing high-speed data mobile terminals include efficient resource allocation and interference reduction. These issues aim to allow each user to achieve the required quality of service at the uplink receiver without causing unnecessary interference to other users in the system, and minimizing power consumption. Scalable energy-efficient power control techniques can be derived using game theory [1]–[4].

Commonly, impulse-radio (IR) systems are employed to implement UWB systems [5]. To provide robustness against multiple access interference (MAI), every user in the network is assigned a pseudo-random time-hopping (TH) sequence. Recently, additional robustness against MAI is achieved by pulse-based polarity randomization [6], which also helps to optimize the spectral shape according to US Federal Communications Commission (FCC) specifications [7]. Due to the large bandwidth, UWB signals have a much higher temporal resolution than conventional narrowband or wideband signals. Hence, channel fading cannot be assumed to be flat [8], and self-interference (SI) has to be taken into account [9]. To the best of our knowledge, this paper is the first to study the problem of radio resource allocation in a frequency-selective multipath environment using a game-theoretic approach. Previous work in this area has assumed flat fading [10], [11].

The remainder of this paper is organized as follows. The system model is given in Sect. II, whereas the proposed power

control game is described in Sect. III. In Sect. IV, we use the game-theoretic framework along with a large-system analysis to evaluate the performance of the system in terms of transmit powers and achieved utilities at the Nash equilibrium. The Pareto-optimal (cooperative) solution to the power control game is discussed in Sect. V, and its performance is compared with that of the noncooperative approach. Numerical results are discussed in Sect. VI, and finally some conclusions are drawn in Sect. VII.

II. SYSTEM MODEL

We consider a binary phase shift keying (BPSK) random TH-IR system using polarity code randomization, with K users in the network transmitting to a receiver at a common concentration point. The processing gain of the system is $N = N_f \cdot N_c$, where N_f is the number of pulses representing one data symbol, and N_c is the number of possible pulse positions in a frame [5]. The transmission is assumed to be over *frequency selective channels*, with the channel for user k modeled as a tapped delay line:

$$c_k(t) = \sum_{l=1}^L \alpha_l^{(k)} \delta(t - (l-1)T_c - \tau_k), \quad (1)$$

where T_c is the duration of the transmitted UWB pulse; L is the number of channel paths; $\alpha_k = [\alpha_1^{(k)}, \dots, \alpha_L^{(k)}]^T$ and τ_k are the fading coefficients and the delay of user k , respectively. Considering a chip-synchronous scenario, the symbols are misaligned by an integer multiple of T_c : $\tau_k = \Delta_k T_c$, for every k , where Δ_k is uniformly distributed in $\{0, 1, \dots, N-1\}$. In addition, we assume that the channel characteristics remain unchanged over a number of symbol intervals [9].

Due to high resolution of UWB signals, multipath channels can have hundreds of components, especially in indoor environments. To mitigate the effect of multipath fading as much as possible, we consider a base station where K Rake receivers [12] are used.¹ The Rake receiver for user k is in general composed of L coefficients, where the vector $\beta_k = \mathbf{G} \cdot \alpha_k = [\beta_1^{(k)}, \dots, \beta_L^{(k)}]^T$ represents the combining weights for user k , and the $L \times L$ matrix \mathbf{G} depends on the type of Rake receiver employed.

¹For ease of calculation, perfect channel estimation is considered throughout the paper.

The signal-to-interference-plus-noise ratio (SINR) of the k th user at the output of the Rake receiver can be well approximated (for large N_f , typically at least 5) by [9]

$$\gamma_k = \frac{h_k^{(\text{SP})} p_k}{h_k^{(\text{SI})} p_k + \sum_{\substack{j=1 \\ j \neq k}}^K h_{kj}^{(\text{MAI})} p_j + \sigma^2}, \quad (2)$$

where σ^2 is the variance of the additive white Gaussian noise (AWGN) at the receiver, and the gains are expressed by

$$h_k^{(\text{SP})} = \beta_k^H \cdot \alpha_k, \quad (3)$$

$$h_k^{(\text{SI})} = \frac{1}{N} \frac{\|\Phi \cdot (\mathbf{B}_k^H \cdot \alpha_k + \mathbf{A}_k^H \cdot \beta_k)\|^2}{\beta_k^H \cdot \alpha_k}, \quad (4)$$

$$h_{kj}^{(\text{MAI})} = \frac{1}{N} \frac{\|\mathbf{B}_k^H \cdot \alpha_j\|^2 + \|\mathbf{A}_j^H \cdot \beta_k\|^2 + |\beta_k^H \cdot \alpha_j|^2}{\beta_k^H \cdot \alpha_k}, \quad (5)$$

where the matrices

$$\mathbf{A}_k = \begin{pmatrix} \alpha_L^{(k)} & \cdots & \cdots & \alpha_2^{(k)} \\ 0 & \alpha_L^{(k)} & \cdots & \alpha_3^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \alpha_L^{(k)} \\ 0 & \cdots & \cdots & 0 \end{pmatrix}, \quad (6)$$

$$\mathbf{B}_k = \begin{pmatrix} \beta_L^{(k)} & \cdots & \cdots & \beta_2^{(k)} \\ 0 & \beta_L^{(k)} & \cdots & \beta_3^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \beta_L^{(k)} \\ 0 & \cdots & \cdots & 0 \end{pmatrix}, \quad (7)$$

$$\Phi = \text{diag} \{\phi_1, \dots, \phi_{L-1}\}, \quad \phi_l = \sqrt{\frac{\min\{L-l, N_c\}}{N_c}}, \quad (8)$$

have been introduced for convenience of notation.

III. THE NONCOOPERATIVE POWER CONTROL GAME

Suppose to apply noncooperative power control techniques to the network described above. Focusing on mobile terminals, where it is often more important to maximize the number of bits transmitted per Joule of energy consumed than to maximize throughput, an energy-efficient approach is considered.

It is possible to settle a noncooperative power control game (NPCG) in which each user seeks to maximize its own utility function. Let $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$ be the proposed game where $\mathcal{K} = \{1, \dots, K\}$ is the index set for the users; $P_k = [0, \bar{p}]$ is the strategy set, with \bar{p} denoting the maximum power constraint; and $u_k(\mathbf{p})$ is the payoff function for user k [3]:

$$u_k(\mathbf{p}) = \frac{D}{M} R \frac{f(\gamma_k)}{p_k}, \quad (9)$$

where $\mathbf{p} = [p_1, \dots, p_K]$ are the transmit powers; D is the number of information bits; M is the number of bits in a packet; R is the transmission rate; γ_k is the SINR for user k ; and $f(\gamma_k)$ is the efficiency function representing the packet

success rate (PSR), i.e., the probability that a packet is received without an error. In the following, we assume that $f(\gamma_k)$ is increasing, S-shaped, and continuously differentiable, with $f(0) = 0$, $f(+\infty) = 1$, $f'(0) = df(\gamma_k)/d\gamma_k|_{\gamma_k=0} = 0$. This assumption is valid in many practical systems.

Formally, the NPCG can be expressed as

$$\max_{p_k \in P_k} u_k(\mathbf{p}) = \max_{p_k \in P_k} u_k(p_k, \mathbf{p}_{-k}) \quad \text{for } k = 1, \dots, K, \quad (10)$$

where \mathbf{p}_{-k} denotes the vector of transmit powers of all terminals except user k . Using (9), (10) can be rewritten as

$$\max_{p_k \in P_k} \frac{f(\gamma_k(p_k, \mathbf{p}_{-k}))}{p_k} \quad \text{for } k = 1, \dots, K, \quad (11)$$

where we have explicitly shown that γ_k is a function of \mathbf{p} .

The solution that is most widely used for noncooperative game theoretic problems is the *Nash equilibrium* [1], which is a set of strategies such that no user can unilaterally improve its own utility. Formally, a power vector $\mathbf{p} = [p_1, \dots, p_K]$ is a Nash equilibrium of $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$ if, for every $k \in \mathcal{K}$, $u_k(p_k, \mathbf{p}_{-k}) \geq u_k(p'_k, \mathbf{p}_{-k})$ for all $p'_k \in P_k$.

Theorem 1: A Nash equilibrium exists and is unique in the NPCG $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$. Furthermore, the unconstrained maximization of the utility function occurs when each user k achieves an SINR γ_k^* that is a solution of

$$f'(\gamma_k^*) \gamma_k^* (1 - \gamma_k^*/\gamma_{0,k}) = f(\gamma_k^*), \quad (12)$$

where

$$\gamma_{0,k} = \frac{h_k^{(\text{SP})}}{h_k^{(\text{SI})}} = N \cdot \frac{(\beta_k^H \cdot \alpha_k)^2}{\|\Phi \cdot (\mathbf{B}_k^H \cdot \alpha_k + \mathbf{A}_k^H \cdot \beta_k)\|^2} \geq 1 \quad (13)$$

and $f'(\gamma_k^*) = df(\gamma_k)/d\gamma_k|_{\gamma_k=\gamma_k^*}$.

The Nash equilibrium can be seen from another point of view. The power level chosen by a *rational* self-optimizing user constitutes a *best response* to the powers chosen by other players. Formally, terminal k 's best response $r_k : P_{-k} \rightarrow P_k$ is the correspondence that assigns to each $\mathbf{p}_{-k} \in P_{-k}$ the set

$$r_k(\mathbf{p}_{-k}) = \{p_k \in P_k : u_k(p_k, \mathbf{p}_{-k}) \geq u_k(p'_k, \mathbf{p}_{-k})\} \quad \text{for all } p'_k \in P_k, \quad (14)$$

where P_{-k} is the strategy space of all users excluding user k .

The Nash equilibrium can be restated in a compact form: the power vector \mathbf{p} is a Nash equilibrium of the NPCG $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$ if and only if $p_k \in r_k(\mathbf{p}_{-k})$ for all $k \in \mathcal{K}$.

Prop. 1: Using (14), with a slight abuse of notation, user k 's best response to a given interference vector \mathbf{p}_{-k} is [3]

$$r_k(\mathbf{p}_{-k}) = \min(\bar{p}, p_k^*), \quad (15)$$

where

$$p_k^* = \arg \max_{p_k \in \mathbb{R}^+} u_k(p_k, \mathbf{p}_{-k}) = \frac{\gamma_k^* \left(\sum_{j \neq k} h_{kj}^{(\text{MAI})} p_j + \sigma^2 \right)}{h_k^{(\text{SP})} (1 - \gamma_k^*/\gamma_{0,k})} \quad (16)$$

is the unconstrained maximizer of the utility in (9). Furthermore, p_k^* is unique. The conclusion is that, at the Nash

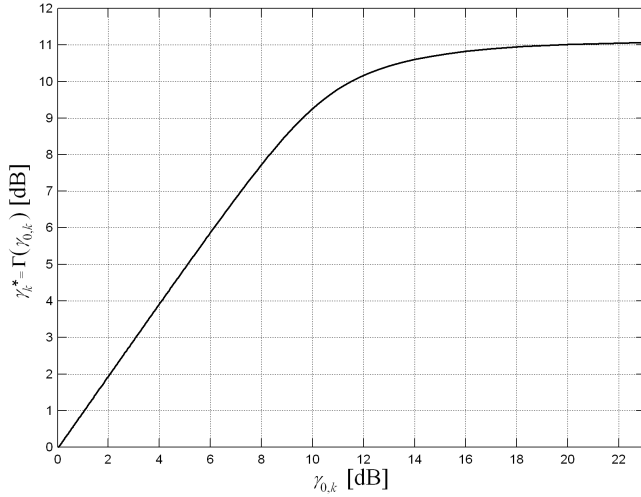


Fig. 1. Shape of γ_k^* as a function of $\gamma_{0,k}$ ($M = 100$).

equilibrium, a terminal either attains γ_k^* or it fails to do so and transmits at maximum power \bar{p} .

The proofs of Theorem 1 and Prop. 1 have been omitted because of space limitation. They can be found in [13].

IV. ANALYSIS OF THE NASH EQUILIBRIUM

In the previous section, it is seen that a Nash equilibrium for the NPCG exists and is unique. Note that, unlike the previous work in this area, γ_k^* is dependent on k , because of the SI in (2). Hence, each user attains a different SINR. More importantly, the only term dependent on k in (12) is $\gamma_{0,k}$, which is affected only by the channel of user k . This means that γ_k^* can be assumed constant when the channel remains unchanged, irrespectively of transmit powers \mathbf{p} and channel coefficients of the other users. For convenience of notation, we can express γ_k^* as a function of $\gamma_{0,k}$:

$$\gamma_k^* = \Gamma(\gamma_{0,k}). \quad (17)$$

Fig. 1 shows the shape of γ_k^* as a function of $\gamma_{0,k}$, where $f(\gamma_k) = (1 - e^{-\gamma_k/2})^M$, with $M = 100$. Even though γ_k^* is shown for values of $\gamma_{0,k}$ approaching 0 dB, it is worth emphasizing that $\gamma_{0,k} > 10$ dB in most practical situations.

Assumption 1: To simplify the analysis, let us assume the typical case of multiuser UWB systems, where $N \gg K$. In addition, \bar{p} is considered sufficiently large that $p_k < \bar{p}$ for those users who achieve γ_k^* . In particular, when $N \gg K$, at the Nash equilibrium the following property holds:

$$h_k^{(\text{SP})} p_k^* \simeq q > 0 \quad \forall k \in \mathcal{K}. \quad (18)$$

The heuristic derivation of (18) can be justified by SI reduction due to the hypothesis $N \gg K > 1$. Using (4), $\gamma_{0,k} \gg 1$ for all k . Hence, the noncooperative solution will be similar to that studied, e.g., in [4]. The validity of this assumption will be shown in Sect. VI through simulations.

Prop. 2: A necessary and sufficient condition for a desired SINR γ_k^* to be achievable is

$$\gamma_k^* \cdot (\gamma_{0,k}^{-1} + \zeta_k^{-1}) < 1 \quad \forall k \in \mathcal{K}, \quad (19)$$

where $\gamma_{0,k}$ is defined in (13), and $\zeta_k^{-1} = \sum_{j \neq k} h_{k,j}^{(\text{MAI})} / h_j^{(\text{SP})}$. When (19) holds, each user can reach the optimum SINR, and the minimum power solution to do so is

$$p_k^* = \frac{1}{h_k^{(\text{SP})}} \cdot \frac{\sigma^2 \gamma_k^*}{1 - \gamma_k^* \cdot (\gamma_{0,k}^{-1} + \zeta_k^{-1})}. \quad (20)$$

When (19) does not hold, the users cannot achieve γ_k^* simultaneously, and some of them would end up transmitting at the maximum power \bar{p} . The proof of Prop. 2 can be found in [13].

Based on Prop. 2, the amount of transmit power p_k^* required to achieve the target SINR γ_k^* will depend not only on the gain $h_k^{(\text{SP})}$, but also on the SI term $h_k^{(\text{SI})}$ (through $\gamma_{0,k}$) and the interferers $h_{k,j}^{(\text{MAI})}$ (through ζ_k). To derive some results for the transmit powers independent of SI and MAI terms, it is possible to resort to a large systems analysis. For convenience of notation, we introduce the following definitions, with $\text{Var}[\cdot]$ denoting the variance of a random variable:

- let \mathbf{D}_j^α be a diagonal matrix whose elements are

$$\{\mathbf{D}_j^\alpha\}_l = \sqrt{\text{Var}[\alpha_j^{(l)}]}; \quad (21)$$

- let \mathbf{D}_k^β be a diagonal matrix whose elements are

$$\{\mathbf{D}_k^\beta\}_l = \sqrt{\text{Var}[\beta_k^{(l)}]}; \quad (22)$$

- let \mathbf{C}_j^α be an $L \times (L-1)$ matrix whose elements are

$$\{\mathbf{C}_j^\alpha\}_{li} = \sqrt{\text{Var}[\{\mathbf{A}_j\}_{li}] / L}; \quad (23)$$

- let \mathbf{C}_j^β be an $L \times (L-1)$ matrix whose elements are

$$\{\mathbf{C}_k^\beta\}_{li} = \sqrt{\text{Var}[\{\mathbf{B}_k\}_{li}] / L}; \quad (24)$$

- let $\varphi(\cdot)$ be the matrix operator

$$\varphi(\cdot) = \lim_{L \rightarrow \infty} \frac{1}{L} \text{Tr}(\cdot), \quad (25)$$

where $\text{Tr}(\cdot)$ is the trace operator.

Theorem 2: Assume that $\alpha_k^{(l)}$ are zero-mean random variables independent across k and l , and \mathbf{G} is a deterministic diagonal matrix (thus implying that $\alpha_k^{(l)}$ and $\beta_j^{(m)}$ are dependent only when $j = k$ and $m = l$). In the asymptotic case where K and N_f are finite,² while $L, N_c \rightarrow \infty$, the terms ζ_k^{-1} and $\gamma_{0,k}^{-1}$ converge almost surely (a.s.) to

$$\zeta_k^{-1} \xrightarrow{\text{a.s.}} \frac{1}{N} \sum_{\substack{j=1 \\ j \neq k}}^K \frac{\varphi(\mathbf{D}_j^\alpha \mathbf{C}_k^\beta \mathbf{C}_k^{\beta H} \mathbf{D}_j^\alpha) + \varphi(\mathbf{D}_k^\beta \mathbf{C}_j^\alpha \mathbf{C}_j^{\alpha H} \mathbf{D}_k^\beta)}{\varphi(\mathbf{D}_j^\alpha \mathbf{D}_j^\beta) \cdot \varphi(\mathbf{D}_k^\alpha \mathbf{D}_k^\beta)}, \quad (26)$$

$$\gamma_{0,k}^{-1} \xrightarrow{\text{a.s.}} \frac{1}{N} \frac{\lim_{L \rightarrow \infty} \frac{1}{L^2} \sum_{i=1}^{L-1} \phi_i^2 \cdot \sum_{l=1}^i \theta_l^2 (l, L+l-i)}{\left(\varphi(\mathbf{D}_k^\alpha \mathbf{D}_k^\beta)\right)^2}, \quad (27)$$

²Considering regulations by the FCC [7], it is noteworthy that N_f could not be smaller than a certain threshold ($N_f \geq 5$).

where ϕ_i is defined as in (8) and

$$\theta_k(l, L+l-i) = \{\mathbf{D}_k^\alpha\}_l \{\mathbf{D}_k^\beta\}_{L+l-i} + \{\mathbf{D}_k^\beta\}_l \{\mathbf{D}_k^\alpha\}_{L+l-i}. \quad (28)$$

The proof of Theorem 2 can be found in [13].

It is worth emphasize that the results above can be applied to any kind of fading models, since only the second-order statistics are required. Furthermore, due to the symmetry of (26) and (27), it is easy to verify that the results are independent of large-scale fading models. Hence, Theorem 2 applies to any kind of channel, which may include both large- and small-scale statistics.

For ease of calculation, in the following we derive the asymptotic values when considering a flat Power Delay Profile (PDP) [14] for the channel coefficients. In addition, the variance of $\alpha_k^{(l)}$ is assumed dependent on the user k , but independent of l , i.e., $\text{Var}[\alpha_k^{(l)}] = \sigma_k^2$ for all l .

Prop. 3: Under the above mentioned hypotheses, when using an ARake, and thus $\mathbf{G} = \mathbf{I}$,

$$\zeta_k^{-1} \xrightarrow{a.s.} \frac{K-1}{N}, \quad (29)$$

$$\gamma_{0,k}^{-1} \xrightarrow{a.s.} \frac{\nu(\rho)}{N}, \quad (30)$$

where $\rho = N_c/L$, $0 < \rho < \infty$, and

$$\nu(\rho) = \begin{cases} \frac{2}{3}(3-3\rho+\rho^2), & \rho \leq 1, \\ 2/(3\rho), & \rho > 1. \end{cases} \quad (31)$$

Using definitions (8) and (21) – (25), and applying Theorem 2, after some algebraic manipulations, the proof is straightforward. As already noticed to justify Ass. 1, but also from (30), $\gamma_{0,k} \gg 1$ for all k . Thus, $\gamma_k^* = \Gamma(\gamma_{0,k})$ approaches $\bar{\gamma}^* = \Gamma(\infty)$, leading to a nearly SINR-balancing scenario.

An immediate consequence of Prop. 3 is the expression for transmit powers p_k^* and utilities u_k^* at the Nash equilibrium, which are independent of the channel realizations of the other users, and of the SI:³

$$p_k^* \simeq \frac{1}{h_k^{(\text{SP})}} \cdot \frac{\sigma^2 \bar{\gamma}^*}{1 - \bar{\gamma}^* \cdot [K-1 + \nu(\rho)]/N}, \quad (32)$$

$$u_k^* \simeq h_k^{(\text{SP})} \cdot \frac{D}{M} R_k \frac{f(\bar{\gamma}^*) (1 - \bar{\gamma}^* \cdot [K-1 + \nu(\rho)]/N)}{\sigma^2 \bar{\gamma}^*}, \quad (33)$$

where the condition (19) translates into

$$N_f \geq \left\lceil \bar{\gamma}^* \cdot \frac{K-1 + \nu(\rho)}{N_c} \right\rceil. \quad (34)$$

The validity of the claims above is verified in Sect. VI using simulations. To show that (29) – (30) represent good approximations not only for the model with flat PDP and equal variances, which has been used only for convenience of calculation, simulations are carried out using the exponential decaying PDP described in [15], which provides a more realistic channel model for the UWB scenario.

³Of course, the amount of p_k^* is dependent on the channel realization.

V. SOCIAL OPTIMUM

The solution to the power control game is said to be Pareto-optimal if there exists no other power allocation \mathbf{p} for which one or more users can improve their utilities without reducing the utility of any of the other users. It can be shown that the Nash equilibrium presented in the previous section is not Pareto-optimal. This means that it is possible to improve the utility of one or more users without harming other users. On the other hand, it can be shown that the solution to the following social problem gives the Pareto-optimal frontier [4]:

$$\mathbf{p}_{opt} = \arg \max_{\mathbf{p}} \sum_{k=1}^K \xi_k u_k(\mathbf{p}), \quad (35)$$

for $\xi_k \in \mathbb{R}^+$ (the set of positive real numbers). Pareto-optimal solutions are, in general, difficult to obtain. Here, we conjecture that the Pareto-optimal solution occurs when all users achieve the same SINRs, γ_{opt} . This approach is chosen not only because SINR balancing ensures fairness among users in terms of throughput and delay [4], but also because, for large systems, the Nash equilibrium is achieved when all SINRs are similar. We also consider the hypothesis $\xi_1 = \dots = \xi_K = 1$, suitable for a scenario without priority classes. Hence, the maximization (35) can be written as

$$\mathbf{p}_{opt} = \arg \max_{\mathbf{p}} f(\gamma) \sum_{k=1}^K \frac{1}{p_k}. \quad (36)$$

In a network where the hypotheses of Ass. 1 and Theorem 2 are fulfilled, and where ARake receivers are employed, at the Nash equilibrium all users achieve a certain output SINR γ with $h_k^{(\text{SP})} p_k \simeq q(\gamma)$, where

$$q(\gamma) = \frac{\sigma^2 \gamma}{1 - \gamma \cdot [K-1 + \nu(\rho)]/N}, \quad (37)$$

with $\rho = N_c/L$. Therefore, (36) can be expressed as

$$\gamma_{opt} \simeq \arg \max_{\gamma} \frac{f(\gamma)}{q(\gamma)} \sum_{k=1}^K h_k^{(\text{SP})}, \quad (38)$$

since there exists a one-to-one correspondence between γ and \mathbf{p} . It should be noted that, while the maximizations in (11) consider no cooperation among users, (36) assumes that users cooperate in choosing their transmit powers. That means that the relationship between the user's SINR and transmit power will be different from that in the noncooperative case.

Prop. 4: In a network where $L, N_c \rightarrow \infty$ and $N \gg K$, using ARake receivers, the Nash equilibrium approaches the Pareto-optimal solution. In addition, γ_{opt} is the solution of

$$f'(\gamma_{opt}) \gamma_{opt} \left[1 - \gamma_{opt} \cdot \frac{K-1 + \nu(\rho)}{N} \right] = f(\gamma_{opt}). \quad (39)$$

The proof can be found in [13]. The validity of the above claims is verified in Sect. VI using simulations.

TABLE I
RATIO σ_q^2/η_q^2 FOR DIFFERENT NETWORK PARAMETERS.

(N_c, N_f)	(L, K)			
	$(20,8)$	$(20,16)$	$(50,8)$	$(50,16)$
$(30,10)$	9.4E-4	3.2E-3	4.8E-4	1.7E-3
$(30,50)$	2.9E-5	6.4E-5	1.6E-5	3.4E-5
$(50,10)$	2.9E-4	6.8E-4	1.5E-4	3.7E-4
$(50,50)$	1.0E-5	2.2E-5	0.6E-5	1.2E-5
$(100,10)$	6.7E-5	1.5E-4	3.7E-5	7.8E-5
$(100,50)$	0.3E-5	0.6E-5	0.1E-5	0.3E-5

VI. NUMERICAL RESULTS

In this subsection, we present numerical results for the analysis presented in the previous sections. Simulations are performed using the algorithm described in detail in [13]. We assume that each packet contains 100 b of information and no overhead (i.e., $D = M = 100$). The transmission rate is $R = 100$ kb/s, the thermal noise power is $\sigma^2 = 5 \times 10^{-16}$ W, and the maximum transmit power is $\bar{p} = 1 \mu\text{W}$. We use the efficiency function $f(\gamma_k) = (1 - e^{-\gamma_k/2})^M$. Using $M = 100$, $\bar{\gamma}^* = \Gamma(\infty) = 11.1$ dB. To model the UWB scenario [8], channel gains are simulated following [15], where both small- and large-scale statistics are taken into account. The distance between the users and the base station is assumed to be uniformly distributed between 3 and 20 m.

Before showing the numerical results for both the noncooperative and the cooperative approaches, some simulations are provided to verify the validity of Ass. 1 introduced in Sect. IV. Table I reports the ratio σ_q^2/η_q^2 of the variance σ_q^2 to the squared mean value η_q^2 of the values $q = h_k^{(\text{SP})} p_k^*$, obtained averaging 10 000 realizations of channel coefficients for different values of network parameters using ARake receivers. We can see that, when the processing gain is much greater than the number of users, $\sigma_q^2/\eta_q^2 \ll 1$. Hence, (18) can be used to carry out the theoretical analysis of the Nash equilibrium.

Fig. 2 shows the utilities achieved at the Nash equilibrium as functions of the channel gains $h_k = \|\alpha_k\|^2$. These results have been obtained using random channel realizations for $K = 16$ users. The number of possible pulse positions is $N_c = 100$, while the number of paths is $L = 60$, in order to satisfy the large system assumption with $\nu(\rho) = 0.4$. The number of frame is $N_f = 10$, thus leading to a processing gain $N = 1000 \gg K$. The line represents the theoretical values of (33) when using an ARake, whereas the square markers correspond to the simulations. We can see that the simulations match closely with the theoretical results.

Fig. 3 shows the probability $P_o = \Pr\{\max_k p_k = \bar{p} = 1 \mu\text{W}\}$ of having at least one user transmitting at the maximum power, as a function of the number of frames N_f . We consider 10 000 realizations of the channel gains, using a network with ARake receivers at the base station, $K = 32$ users, $N_c = 50$, and $L = 100$ (thus $\rho = 0.5$ and $\nu(\rho) \simeq 1.17$). From (34), the minimum value of N_f that allows all K users to achieve the optimum SINRs is $N_f = \lceil 8.33 \rceil = 9$. The simulations thus

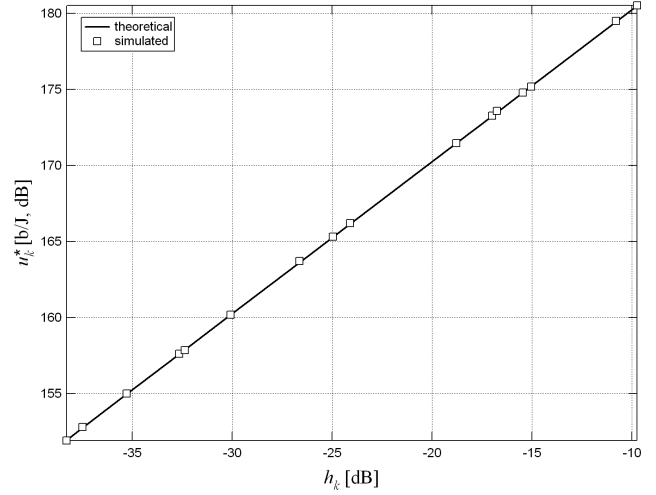


Fig. 2. Achieved utility versus channel gain at the Nash equilibrium for the ARake receiver.

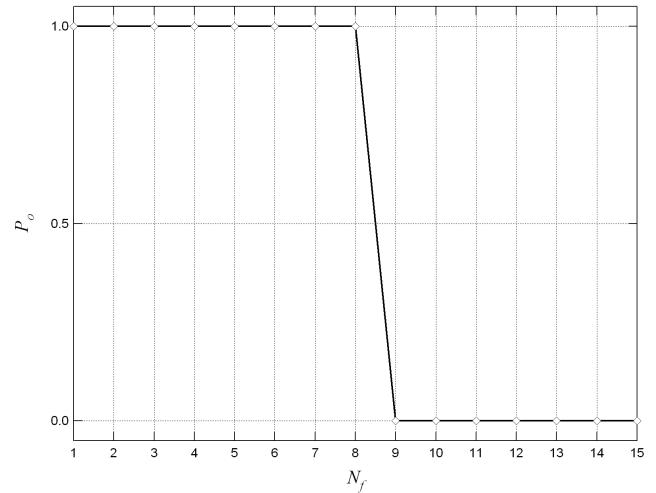


Fig. 3. Probability of having at least one user transmitting at maximum power versus number of frames.

agree with the analytical results of Sect. IV.

We now analyze the performance of the system when using a Pareto-optimal solution instead of the Nash equilibrium. Fig. 4 shows the normalized utility u_k/h_k as a function of the load factor ρ . We consider a network with $K = 5$ users, $N_f = 20$ frames and ARake receivers at the base station. The lines represent theoretical values of Nash equilibrium (dotted line), using (33), and of the social optimum solution (solid line), using (33) again, but substituting $\bar{\gamma}^*$ with the numerical solution of (39), γ_{opt} . The markers correspond to the simulation results. In particular, the circles represent the averaged solution of the simulation algorithm, while the square markers show averaged numerical results (through a complete search) of the maximization (35), with $\xi_k = 1$. As stated in Sect. V, the difference between the noncooperative approach and the Pareto-optimal solution is not significant. Fig. 5 compares the target SINRs of the noncooperative solutions

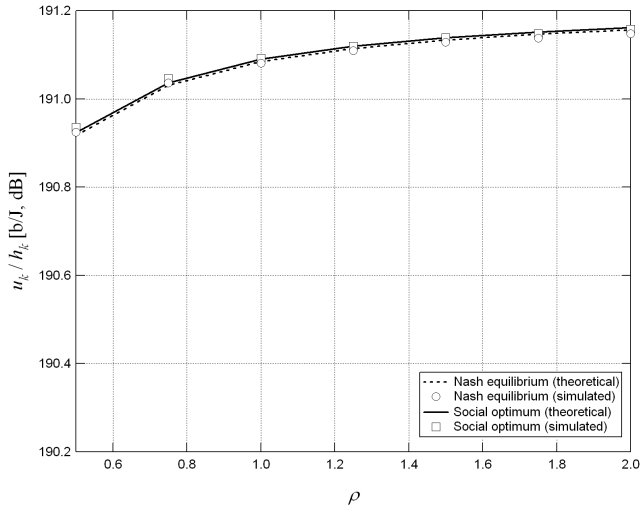


Fig. 4. Comparison of the normalized utility versus load factor for the noncooperative and Pareto-optimal solutions.

with the target SINRs of the Pareto-optimal solutions. As before, in both cases, the averaged target SINRs for the Nash equilibrium, γ , and the averaged target SINRs for the social optimum solution, γ_{opt} , are close to $\bar{\gamma}^*$, as shown in Sect. V.

VII. CONCLUSION

In this paper, we have used a game-theoretic approach to study power control for a wireless data network in frequency-selective environments, where the user terminals transmit IR-UWB signals and the common concentration point employs Rake receivers. A noncooperative game has been proposed in which users are allowed to choose their transmit powers according to a utility-maximizing criterion, where the utility function has been defined as the ratio of the overall throughput to the transmit power. For this utility function, we have shown that there exists a unique Nash equilibrium for the proposed game, but, due to the frequency selective multipath, this equilibrium is achieved at a different output SINR for each user, depending on the channel realization and the kind of Rake receiver used. Resorting to a large system analysis, we have obtained a general characterization for the terms due to multiple access interference and self-interference, suitable for any kind of channel model and for different types of Rake receiver. Furthermore, explicit expressions for the utilities achieved at the equilibrium have been derived for the case of an ARake receiver. It has also been shown that, under these hypotheses, the noncooperative solution leads to a nearly SINR-balancing scenario. In order to evaluate the efficiency of the Nash equilibrium, we have studied an optimum cooperative solution for the case of ARake receivers, where the network seeks to maximize the sum of the users' utilities. It has been shown that the difference in performance between Nash and cooperative solutions is not significant for typical values of network parameters.

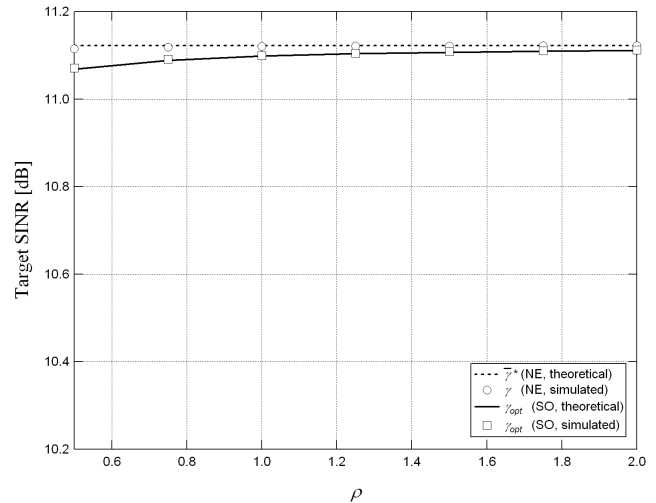


Fig. 5. Comparison of the target SINRs versus load factor for the noncooperative and Pareto-optimal solutions.

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