Outage Behavior for Transmit Diversity OFDM-Based Systems

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Abstract – **The following transmit diversity schemes for OFDM-based systems are analyzed: Linear Delay Diversity (LDD), Cyclic Delay Diversity (CDD), Subcarrier Diversity (SD), and Time-Switched Transmit Diversity (TSTD). The overall system performance is investigated by computing the theoretic outage rates achievable over the composite radio channels induced by the different transmission strategies. In particular, the analysis points out that a convenient exploitation of the available diversity branches leads to lower outage probabilities and, consequently, to higher outage rates. Incidentally, the ergodic capacity limit for the considered diversity schemes remains equal to the single transmit antenna case. On the other hand, transmit diversity reduces the statistical dispersion of the mutual information about its mean. As a result, the optimal selection of the delay for both LDD and CDD is addressed. Nevertheless, in many cases of interest the failure probability is the same with either SD, LDD or CDD. It is also mentioned that SD and TSTD are equivalent if the propagation medium is sufficiently slow time-varying, while SD outperforms TSTD over highly time-selective radio channels.**

Keywords: Performance limits, outage rates, transmit diversity, OFDM, system design, wireless communications.

I. INTRODUCTION

The demand of reliable communications over wireless links requires the development of transmissions strategies able to fully exploit the diversity branches available in a radio system. Nevertheless, there exist propagation environments where the information can be reliably decoded at the cost of transmitting long codewords in order to capture the ergodic behavior of a channel [1]. Nowadays, that approach is unsuited to allowing high rate transmissions with delay and system complexity constraints. Therefore, wideband transmissions have been deeply studied with the twofold objective of increasing the effective data rates and reducing the latency of the information. In particular, a lot of attention is currently given to wideband systems employing OFDM (orthogonal frequency division multiplexing) as modulation format [2], [3]. OFDM is particularly suitable for wideband transmissions because it realizes a very efficient strategy to perfectly equalize the distortions introduced by a time-dispersive channel. However, as shown in this paper, if the propagation medium is not sufficiently frequency selective over the transmission bandwidth, codes with large constraint lengths, that work across both time and frequency domains, should be used to approach the theoretic system capacity limit, i.e., delay and complexity constraints may reduce the effective gain achievable by wideband transmissions. In those cases, it becomes of practical interest to introduce additional diversity branches into the system in order to increase the actual data rate by using simpler codes that guarantee low latencies.

As a matter of fact, several transmission diversity techniques have been proposed in the last decade [4]–[7] and applied to OFDM-based systems [8]–[10]. In particular, very often the study of these schemes is driven by the common sense and validated by simulation results. With that approach the technique named Cyclic Delay Diversity (CDD) seems to be the most promising one [2]. Indeed, the purpose of this paper is to re-examine several diversity strategies and give a comparison among them by studying the system from an information-theoretic perspective.

The following open-loop diversity techniques are considered: Linear Delay Diversity (LDD), CDD, Subcarrier Diversity (SD) [8], and Time Switched Transmit Diversity (TSTD) [5]. It is recalled that all these solutions have been proposed with the aim of leaving the receiver unchanged. In other words, these techniques employ multiple antennas at the transmitter side but, the received signal has the same structure as for a conventional single antenna scheme. This differs from the multiple-input single-output (MISO) [11] architecture where space-frequency coding can be performed at the cost of a more complex receiver, which has to detect several overlapped spatial bins.

Therefore, the performance limits are studied by analyzing the outage probability of the channels induced by the different transmission diversity schemes. The outage rate is the maximum achievable rate for which reliable communication is possible with a given outage probability. Indeed, the outage probability results to be equal to the frame error probability when capacity achieving codes are used. In particular, a second-order statistic description of the mutual information points clearly out the relevance of diversity on the system outage behavior.

With the proposed analysis the optimal system design parameters (in the outage rate maximization sense) for LDD, CDD and SD are found. It is shown that for many cases of practical interest these schemes are equivalent to each other. Hence, SD is so promising as CDD is. Moreover, if the channel coherence bandwidth is small enough, the performance of SD can be improved by applying a frequency hopping pattern. Finally, if the delay constraints are relaxed, TSTD results to be equivalent to SD for transmissions over slow time-varying channels. In general, SD with frequency hopping outperforms TSTD.

This paper is organized as follows. In Sect. II the basic OFDM system model is described and its outage rates evaluated. That analysis is successively extended in Sect III to analyze LDD, CDD and SD. The delay-constraints are relaxed in Sect. IV where the TSTD scheme is also considered. Finally, some conclusions are drawn in Sect. V.[1]

[1]Notation: $\mathbb{E}[\cdot]$ represents the expectation. $\delta(\cdot)$ is the Dirac pulse. The operation "$a \bmod b$" returns the modulus of a/b. The function $\lceil a \rceil$ gives the nearest integer towards minus infinity of the real number a. The superscript ^{T} represents the transpose of a vector. A random variable x with complex circularly-symmetric Gaussian distribution, mean m and variance σ^2, is denoted as $x \sim CN(m, \sigma^2)$.

II. Outage Behavior: SISO OFDM

In this section, single-input single-output (SISO) OFDM-based transmissions are considered. The basic system and channel models are introduced in order to explain the information-theoretic approach developed throughout this paper. Indeed, the channel mutual information is a random variable with statistic description that, in many cases of interest, can be well-approximated as having Gaussian distribution [11], [12]. In particular, it results that while frequency diversity does not impact on the ergodic capacity (that is the mean mutual information), the length and shape of the channel power delay profile have an effect on the variance of the mutual information and so on the maximum achievable data rates. That aspect is relevant for the course of the investigation of the different transmission diversity techniques discussed in Sect. III.

A. System and channel model

It is considered a base-band representation of an OFDM modulator having M subcarriers that are equally spaced of $\Delta_f \triangleq 1/T$ being T the modulation rate. By following standard steps, an information data vector $\boldsymbol{A} \triangleq [A_0, A_1, \ldots, A_{M-1}]^T$ is OFDM modulated, parallel-to-serial converted, and cyclically extended by a prefix (CP) of $N_{cp} \leq M$ repeated samples. The transmission rate results $1/T_c = (M + N_{cp})/T$, where $T_c \triangleq 1/B$ is fixed by the available radio bandwidth B.

The modulated signal is transmitted over a multipath channel (WSS-US model) with discrete-time impulse response

$$h(\tau) = \sum_{p=0}^{N_p-1} h_p \, \delta(\tau - p\, T_c) \quad , \tag{1}$$

where the $N_p \leq M$ T_c-spaced channel taps h_p, $p = 0, 1, \ldots, N_p-1$, have attenuations modeled as zero-mean complex Gaussian random processes with variances $\sigma_p^2 = \mathbb{E}[|h_p^t|^2]$, $p = 0, 1, \ldots, N_p-1$, i.e., $h_p \sim CN(0, \sigma_p^2)$. The channel power delay profile (PDP) is normalized as $\sum_{p=0}^{N_p-1} \sigma_p^2 = 1$ to get equal average transmitter and receiver powers. In the first part of this paper the channel is assumed to be static over an OFDM symbol and to change independently on successive blocks (*block-fading assumption*). In Sect. IV, the study is briefly extended to take into account time-correlated channel models.

At the receiver side, the current channel state information is assumed to be perfectly known and also perfect time-frequency synchronization is performed. Hence, after cyclic prefix removal and OFDM demodulation the received data sample in correspondence to the m-th subcarrier is effected of neither intersymbol nor intercarrier interference and it reads

$$y_m = H_m \, A_m + n_m \quad , \tag{2}$$

where n_m represents the additive white noise component having complex circularly-symmetric Gaussian distribution with zero mean and power σ_n^2. The channel coefficient H_m is equal to the frequency responce of the channel evaluated at the frequency $f_m \triangleq m\,\Delta_f$, i.e., $H_m = \sum_{p=0}^{N_p-1} h_p \, e^{-j\,2\pi p \frac{m}{M}}$. The channel vector $[H_0, H_1, \ldots, H_{M-1}]$ is made up of jointly identically-distributed normal Gaussian random variables ($H_m \sim CN(0,1)$).

B. Mutual information: Second-order description

In this case, the ergodic capacity of the wideband discrete-time channel is maximized by selecting an OFDM symbol vector $\boldsymbol{A}$ with complex independent and identically distributed (i.i.d.) Gaussian entries [13]. The statistical power of each component under an average transmit power constraint P is then fixed to P/M [3], [11]–[13]. For a given channel realization, the mutual information is equal to

$$\begin{aligned} I &= \frac{1}{M} \sum_{m=0}^{M-1} \log_2\left(1 + \beta\, |H_m|^2\right) \\ &= \frac{1}{M} \sum_{m=0}^{M-1} I_m \qquad \text{[bit/s/Hz]} \; , \end{aligned} \tag{3}$$

where $\beta \triangleq (P/M)/\sigma_n^2$ and $I_m \triangleq \log_2\left(1 + \beta\,|H_m|^2\right)$ are the average signal-to-noise ratio (SNR) for each OFDM subcarrier at the receiver and the mutual information for the m-th OFDM tone, respectively. Without loss of generality, the spectral inefficiency due to the transmission of the cyclic prefix has been neglected.

Therefore, the mutual information (3) is a real random variable given by the sample average of the terms I_m. The random elements I_m are usually correlated since the channel frequency coefficients H_m are themselves usually correlated. In general, the complete statistical distribution of I cannot be found in closed-form but a second-order analysis is meaningful as shown in the sequel.[2]

1) Ergodic capacity: The ergodic capacity (also called average capacity or capacity in the Shannon sense) over an ergodic time-varying channel is indeed defined as the statistical average of the mutual information. To get close to that limit it is necessary to transmit long information codewords in order to reflect the ergodic behavior of the channel and to make the sample average (in time) of the mutual information to converge to the statistical mean [1].

Proposition 1: Let I be the mutual information of the SISO OFDM-based system, then from (3) it follows that the ergodic capacity is given by

$$\bar{I} \triangleq \mathbb{E}[I] = -\log_2(e)\, \mathrm{E}_i(-1/\beta)\, e^{1/\beta} \quad \text{[bit/s/Hz]} \; , \tag{4}$$

where $\mathrm{E}_i(x) = \int_{-\infty}^{x} \frac{e^t}{t}\, dt$, $x \in \mathbb{R}^+$, is the exponential integral function [16, Tab. 4.337(2)], [14].

Therefore, the ergodic capacity (4) is the same of transmissions over frequency-flat Rayleigh fading channels with SNR β. Tab. I reports some numerical values of the ergodic capacity for different transmission bandwidths and SNRs. It is worth noticing that the ergodic capacity is not effected by shape and length of the power delay profile (frequency selectivity) but it only depends on the SNR value. In particular for asymptotically high SNRs (4) simplifies as [15], [16]

$$\bar{I} \simeq \log_2(e) \Big[\log_2(1+\beta) - \gamma \Big] \quad , \quad \text{for } \beta \gg 1 \; ,$$

where γ stands for the Euler-Mascheroni constant ($\gamma = 0,57721566\ldots$). Hence, in the high SNR regime a loss of $\log_2(e)\,\gamma \simeq 0.8$ bit/s/Hz is found with respect to transmissions over AWGN channels.

2) Variance of the mutual information: The variance of the mutual information provides a measure of the statistic dispersion of I about its mean. For instance, the ergodic limit (4) is reached by averaging (3) over a sufficiently large set of independent channel realizations, i.e., by exploiting time diversity (Sect. IV). However, from (3) we deduce that frequency diversity can also help to reduce the variance of the mutual information by exploiting frequency selectivity due to multipath channels.

[2] In the rest of this paper, Propositions are reported without proofs but by discussing their meanings and technical implications. The interested readers are referred to [12] for further mathematical and technical details.

TABLE I

ERGODIC CAPACITY ([BIT/S])

β [dB] $\setminus$ B [Hz]	1	1.25M	10M	20M
5	1.7	2.1 M	17 M	34 M
10	2.9	3.6 M	29 M	58 M
15	4.3	5.4 M	43M	86 M
20	5.9	7.4 M	59 M	118 M
30	9.1	13.6 M	91 M	182 M

Proposition 2: Let $\rho_{m_1,m_2} \triangleq \text{cov}[\,|H_{m_1}|^2, |H_{m_2}|^2\,]$ be the covariance between the couple of frequency channel taps corresponding to the m_1-th and m_2-th OFDM subcarriers, then the variance of the mutual information $\sigma_I^2 = \text{var}[I] = \mathbb{E}[(I - \bar{I})^2]$ for a SISO OFDM system is given by (for any m)

$$\sigma_I^2 = \frac{\mathbb{E}[I_m^2]}{M} - \Big(\mathbb{E}[I_m]\Big)^2 + \frac{1}{M^2}\, 2 \sum_{m_1} \sum_{m_2 > m_1} \mathbb{E}[\, I_{m_1}\, I_{m_2}^*] \quad (5)$$

where

$$\mathbb{E}\,[\, I_{m_1}\, I_{m_2}^*] = $$

$$= \begin{cases} \log^2(e)\, \text{E}_i^2(-1/\beta)\, e^{2/\beta}\,, & \text{if } \rho_{m_1,m_2} = 0\,; \\ \int_0^{+\infty} \log_2^2 (1+\beta\, a)\; e^{-a}\; da\,, & \text{if } \rho_{m_1,m_2} = 1\,; \\ \iint_0^{\infty} \log_2 (1+\beta\, a) \log_2 (1+\beta\, b) \times & \\ \times \frac{e^{-\frac{a+b}{1-\rho_{m_1,m_2}}}}{1-\rho_{m_1,m_2}} \text{I}_0\!\left[\frac{2\sqrt{\rho_{m_1,m_2}\, a\, b}}{1-\rho_{m_1,m_2}}\right] da\, db\,, & \text{otherwise}\,, \end{cases} \quad (6)$$

and

$$\rho_{m_1,m_2} = \left| \sum_{p=0}^{N_p-1} \sigma_p^2\, e^{-j\, 2\,\pi p \frac{m_1 - m_2}{M}} \right|^2 \quad . \quad (7)$$

The function $\text{I}_0[\cdot]$ represents the zeroth-order modified Bessel function of the first kind.

Hence, the variance of the mutual information decreases with smaller correlation coefficients ρ_{m_1,m_2}. The minimum variance is for independent H_m, $m = 0, 1, \ldots, M-1$, that set $\rho_{m_1,m_2} = 0$ for $m_1 \neq m_2$ and simplify (5) into

$$\sigma_I^2 = \frac{1}{M} \left(\mathbb{E}[I_m^2] - (\mathbb{E}[I_m])^2\right) = \frac{1}{M}\, \text{var}[I_m] \quad . \quad (8)$$

Therefore, while the ergodic capacity is equal to the narrowband Rayleigh fading case, the variance is M times smaller. In particular, for independent I_m, in the limit of a number of subcarriers M going to infinity, the mutual information (3) converges almost surely to the ergodic capacity $\bar{I}$ for the strong law of large numbers.

3) Outage rates: The ergodic limit (4) cannot be reached in practical transmission scenarios where time-delay constraints are imposed in order to support Quality-of-Service, e.g., real-time software applications. In general, in a short decoding interval it is not possible "to collect enough time-diversity" but for wideband coherent transmissions frequency diversity can be exploited when available. A useful information-theoretic concept for the analysis of system performance is the channel outage rate (also called delay-constrained capacity) which is defined as the maximum achievable data rate that can be supported with a given outage (failure) probability.

Indeed, let R be a fixed normalized transmission rate, then the correspondent outage probability is equal to the probability that the mutual information I is lower than R

$$P_{out}(R) \triangleq P(I \leq R) \quad , \quad (9)$$

or, in other words, the probability that a channel is not able (in average) to support the rate R. Alternatively, we refer to $x\%$ *outage rate* as the maximum transmission rate that can reliably be supported with a failure probability equal to $x\%$

$$R(x\%) \triangleq \arg\max_R [P(I \leq R) = x] \quad . \quad (10)$$

In general, analytical computation of (10) requires the cumulative distribution function (cdf) of the mutual information. All the same, closed-form expressions are not of straightforward evaluation except for channels having one (flat Rayleigh) or two taps [14] and then in the most general case a simplified treatment is necessary to point out the role of the channel on the system outage behavior.

Nevertheless, we recall that the mutual information (3) is a sample average of correlated random variables. In the literature the general problem of identifying conditions under which such a kind of series converges to a Gaussian random variable has widely been studied by invoking the central limit theorem (CLT) and by stating several generalizations to the case of correlated (dependent) terms [17]. In particular, in [11] it is emphasized that the mutual information (3) converges almost surely to a Gaussian random variable whensoever the channel frequency taps H_m decorrelate sufficiently fast in the limit of a transmission bandwidth B going to infinity. For the purposes of this paper, we observe that for a frequency selective channel having coherence bandwidth B_{coh}, a frequency tap H_{m_1} is almost uncorrelated with any tap H_{m_2} with $m_2 \gtrless m_1 \pm \lceil B_{coh}/\Delta_f \rceil$, $m_1, m_2 \in \{0, 1, \ldots, M-1\}$. Therefore, the summation in (3) can be split into $M/\lceil B_{coh}/\Delta_f\rceil$ summations of about $M/\lceil B_{coh}/\Delta_f\rceil$ independent terms. Now, each sub-summation is known to fast approach (with 4 terms the approximation is just pretty tight) a Gaussian variable for the CLT and, being the sum of Gaussian variables still Gaussian, the mutual information is well-approximated as Gaussian distributed when $B/B_{coh} \gtrsim 4$.

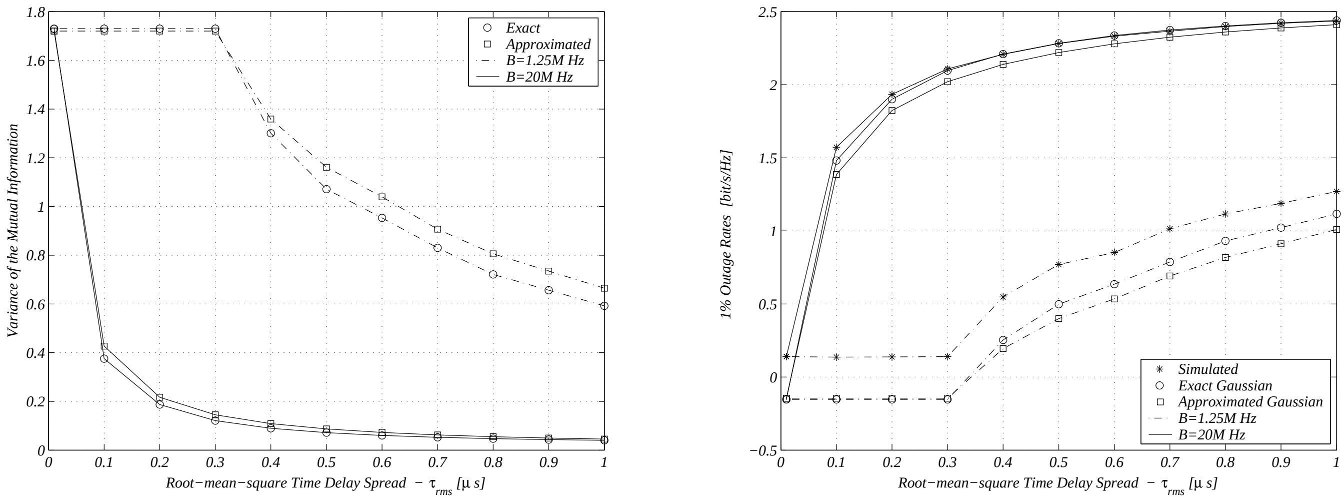
Therefore, in the first instance a Gaussian approximation for the mutual information is suitable in many cases of interests and then the $x\%$ outage rate $R(x\%)$ (10) can be approximated as $R(x\%) \approx \bar{I} + \Phi^{-1}(x)\, \sigma_I$, where $\Phi^{-1}(x) = -\sqrt{2}\, \text{erfc}^{-1}(2x)$ is the inverse standard normal cdf ($\Phi^{-1}(0.01) = -2.326$). All the same, a Gaussian distribution could not be tight over weakly frequency selective channels.

The outage rates are then higher for smaller values of the variance σ_I^2. From Proposition 2 it follows that channels having smaller correlation coefficients ρ_{m_1,m_2} should support higher outage rates. However, while (5) gives the exact value for σ_I^2 it does not point clearly out how β, shape and length of the channel power delay profile impact on ρ_{m_1,m_2}. In order to emphasize these aspects it is herein proposed to approximate σ_I^2 by using the so called "delta method" [18]. The delta method is also known as the Taylor series method because it gives an approximation of the moments of a function of a random variable by the expected value of a truncated Taylor series expansion of the function itself. In effective, the function $\log(1+\beta\, |H_m|^2)$ in (3) is in-series expanded about $\mathbb{E}[\,|H_m|^2] = 1$ and truncated to its second term giving the approximation

$$\sigma_I^2 \approx \tilde{\sigma}_I^2 \triangleq \sum_{m_1=0}^{M-1} \sum_{m_2=0}^{M-1} \frac{\partial I}{\partial |H_{m_1}|^2}\; \frac{\partial I}{\partial |H_{m_2}|^2}\; \text{cov}[\,|H_{m_1}|^2,\, |H_{m_2}|^2]$$

$$= \kappa^2 \frac{1}{M^2} \sum_{m_1, m_2 = 0}^{M-1} \rho_{m_1,m_2} \quad , \quad (11)$$

where the derivatives have been evaluated at $\mathbb{E}[\,|H_m|^2] = 1$ and it has been defined $\kappa \triangleq \log(e) \frac{\beta}{1+\beta}$.



(a) Variance of the mutual information

(b) Outage rates

Fig. 1. Variance of the mutual information and 1% outage rates for SISO OFDM.

Proposition 3: The variance of the mutual information I for a SISO OFDM-based system is approximated by

$$\tilde{\sigma}_I^2 = \kappa^2 \left(\sum_{p=0}^{N_p-1} \sigma_p^4 \right) \quad . \qquad (12)$$

Therefore, from Proposition 1 and approximation (12) we deduce the following.

- By increasing the SNR β, the ergodic capacity increases but the variance of the mutual information increases as well. In particular for asymptotically high SNRs ($\beta \gg 1$) the $x\%$ outage rate approximately reads

$$R(x\%) \simeq \log_2(e) \left(\log_2(1+\beta) - \gamma + \Phi^{-1}(x) \sqrt{\sum_{p=0}^{N_p-1} \sigma_p^4} \right)$$

  Hence, the effect of frequency diversity is clearly crucial in the high SNR regime.
- The variance of the mutual information scales with the sum of the square power delay profile. Hence, compared to the flat Rayleigh fading case ($N_p = 1$, $\sigma_0^2 = 1$), the effect of frequency diversity (multipath) is remarkable even with a few channel taps, e.g, for $N_p = 2$, $\sigma_p^2 = 1/2$, $p = 0, 1$, $\tilde{\sigma}_I^2$ is 1/2 of the Rayleigh fading case.
- Due to the channel power normalization $\sum_{p=0}^{N_p-1} \sigma_p^2 = 1$, it follows that longer power delay profiles reduce the variance of the mutual information. Among channels having the same number of taps N_p, the best one has uniform power delay profile.

In the sequel some numerical results are reported to validate the previous analysis. The radio channel has an exponentially decaying power delay profile truncated to $N_p = \min(\max(1, \lceil 5B\,\tau_{rms} \rceil), M)$ taps, where τ_{rms} is the root-mean-square time delay spread and $M = 64$ is the number of OFDM subcarriers. The SNR is fixed to $\beta = 10\,\text{dB}$. In Fig. 1(a) the approximation (12) is compared with the exact variance (5) for increasing τ_{rms} and two different transmission bandwidths B. As expected the variance decreases with the increasing of the time dispersion, or equivalently, with smaller coherence bandwidths B_{coh}. The approximation is pretty close to the real value. In Fig. 1(b) the correspondent 1% outage rates are reported by comparing their simulated values with the Gaussian assumption for both exact and approximated variances. A Gaussian approximation is very tight if the variance of the mutual information is small enough, i.e., over sufficiently frequency selective channels. In fact, in this case $B/B_{coh} \simeq B\,\tau_{rms}$ (> 4 for $B = 20\,\text{MHz}$ and $\tau_{rms} > 0.2\,\mu\text{s}$).

III. Outage Behavior: Transmit Diversity Schemes

In this section, we shall analyze the following transmit diversity techniques: spatial repetition (SR), LDD, CDD, and SD. In particular, it is shown that SR does not lead to any advantage with respect to a conventional SISO architecture. For LDD and CDD the optimal system design parameters are found and their effects are discussed. Finally, it is shown that LDD and CDD are equivalent to SD when the delays for LDD/CDD and subcarrier loading pattern for SD are properly selected.

In the sequel, we let N_t be the number of transmit antennas while the receiver is equipped with only one receive element. That represents the case of downlink transmissions where a base station (access point) transmits to a mobile terminal. The N_t radio links are assumed to be spatially uncorrelated (see [12] for the correlated case) and all with the same power delay profile.

A. Spatial repetition (SR)

In the case of spatial repetition depicted in Fig. 2(a), the same information is transmitted by all the N_t transmit antennas. In particular, each antenna has an average transmit power P/N_t.

Proposition 4: The mutual information for the spatial repetition scheme has the same second-order statistic description of the conventional SISO case.

Consequently, we conclude that there is neither gain nor loss in applying SR with respect to the standard SISO model considered in Sect. II. It is recalled that for transmissions over single-carrier SISO AWGN channels, repetition codes have not coding gain and, in addition, they leads to a spectral efficiency loss.

B. Linear delay diversity (LDD)

In Sect. II it has been established that longer power delay profiles reduce the outage probability. Indeed, LDD exploits multiple transmit antennas to make appear the channel power

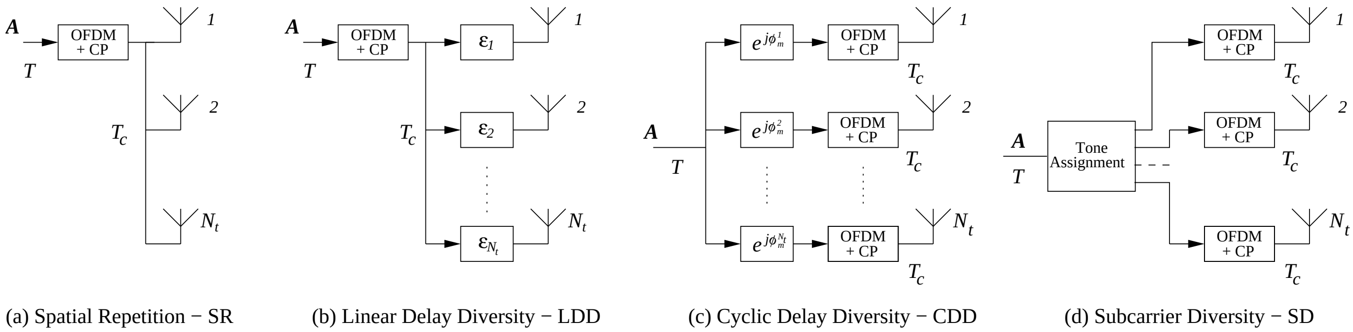


Fig. 2. Block models of SR, LDD, CDD and SD schemes.

delay profile longer. The basic idea of LDD is to introduce a linear delay on each spatial branch before data transmission, Fig. 2(b). In that way the overall system looks like a conventional SISO architecture transmitting over longer channels. That scheme has frequently been studied in the literature (see for instance [8] and [6]). In this paper, the aim is to understand system performance by analyzing information-theoretic limits.

Proposition 5: The ergodic capacity limit for LDD transmissions is independent of the applied delays and then equal to the conventional SISO case. On the contrary, the approximated variance of the mutual information for LDD becomes

$$\tilde{\sigma}^2_{I,\,LDD} = \kappa^2 \sum_{k=0}^{+\infty} \Big| \frac{1}{N_t} \sum_{t=1}^{N_t} \sum_{p=o}^{N_p-1} \sigma_p^2 \, \delta(k - \varepsilon_t - p) \Big|^2 , \quad (13)$$

where ε_t is the integer delay (in T_c-spaced samples) applied to the t-th transmit antenna, $t = 1, 2, \ldots, N_t$.

Therefore, the optimal delays that minimize (13) are

$$\varepsilon_t \geq N_p\,(t-1) \quad , \quad t = 1, 2, \ldots, N_t - 1 \, , \quad (14)$$

that give

$$\min_{\varepsilon_t} \tilde{\sigma}^2_{I,\,LDD} = \frac{1}{N_t}\,\tilde{\sigma}^2_I \quad . \quad (15)$$

Hence, with an optimal delay selection the variance of the mutual information with LDD is N_t times smaller that for the conventional SISO case. However, it should be noted that the channel length N_p is not known at the transmitter side. Besides, in order to retain orthogonality among the OFDM subcarriers a longer cyclic prefix has to be appended to the modulated signal and, consequently, an additional spectral inefficiency has to be introduced. By the way, that drawback is completely overcome by the CDD scheme.

C. Cyclic delay diversity (CDD)

For CDD transmissions a cyclic delay is applied instead of a linear delay on each transmit branch before cyclic prefix insertion. That operation is equivalent to introduce a phase rotation on the information symbols before OFDM modulation [8], Fig. 2(c). CDD does not require a longer cyclic prefix being phase rotation performed before OFDM modulation.

Proposition 6: The ergodic capacity limit for CDD transmissions is independent of the applied delays and then equal to the conventional SISO case. On the contrary, the approximated variance of the mutual information for CDD becomes

$$\tilde{\sigma}^2_{I,\,CDD} \triangleq \kappa^2 \sum_{k=0}^{+\infty} \Big| \frac{1}{N_t} \sum_{t=1}^{N_t} \sum_{p=o}^{N_p-1} \sigma_p^2 \, \delta\Big(k - ((\varepsilon_t + p) \bmod M)\Big) \Big|^2 \quad (16)$$

where ε_t is the integer cyclic delay (in T_c-spaced samples) applied to the t-th transmit antenna, $t = 1, 2, \ldots, N_t$. Equivalently, a phase rotation $e^{j\,\phi^t_m}$, $\phi^m_t = -2\,\pi\,\varepsilon_t\,m/M$ is applied to the m-th information symbol loaded on the m-th subcarrier at the t-th transmit antenna. CDD is equivalent to LDD for equal delays ε_t.

For transmissions over channels having decaying intensity profiles an effective delay selection, which is optimal if $N_p \leq M/N_t$, is given by

$$\varepsilon_t = \frac{M}{N_t}\,(t-1) \quad . \quad (17)$$

Indeed, with the delays (17), the channel frequency response H_m for the m-th subcarrier turns to be uncorrelated with $H_{m+(i+kN_t)}$ $i = 1, 2, \ldots, N_t - 1\, , \, k \in \mathbb{Z}\,.$

D. Subcarrier diversity (SD)

In SD schemes the information symbol A_m is loaded on the m-th subcarrier of one of the OFDM modulators corresponding to the transmit branches, Fig. 2(d).

Proposition 7: The optimal pattern allocation for SD transmissions is given by interleaving the information symbols A_m, $m = 0, 1, \ldots, M-1$ among the N_t modulators, i.e., to a t-th antenna we load the OFDM tones having indexes: $\{t-1, N_t + (t-1), 2N_t + (t-1), \ldots\}$ up to assign all the M data symbols.

With an interleaved allocation, SD is equivalent to either LDD or CDD with delays (17).

Therefore, the previous analysis has shown that for many practical cases, CDD, LDD and SD are equivalent to each other. We note the analogy between SD and OFDMA where to different users different subcarriers are allocated in an interleaved fashion.

In Fig. 3 it is plotted the 1% outage rates for SD transmissions with a different number of transmit antennas. The channel is like for Fig. 1. The benefit due to SD is remarkable.

IV. Further results

As it has been mentioned above, if the delay-constraint is relaxed and a Gaussian codebook that span more OFDM symbols is used, then there is the chance for time diversity exploitation when transmissions occur over time-varying radio channels. In particular, by using similar steps as those in the previous sections it can be proved the following Proposition (see [12] for more details).

Proposition 8: Let L be the number of OFDM symbols spanned by a codeword, then for SD OFDM-based systems equipped with N_t transmit antennas the next remarks hold.

- If the coherence bandwidth of the channel verifies $B_{coh} < B/N_t$ and its coherence time is sufficiently small (as a rule of thumb the normalized Doppler spread f_D/Δ_f is no

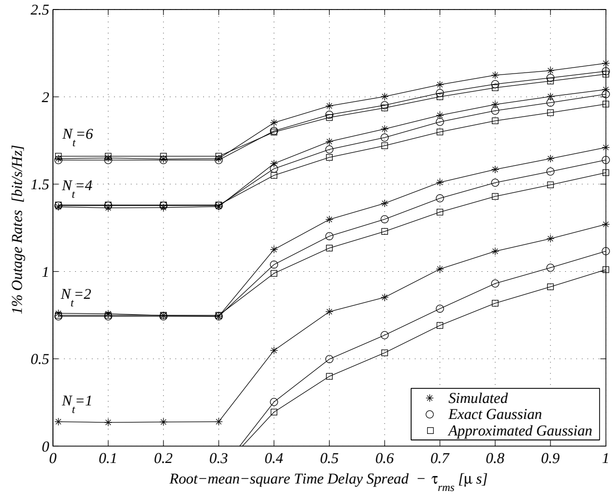


Fig. 3. Comparison between simulated 1% outage rates and Gaussian assumption with exact and approximated variances for subcarrier diversity (SD) transmissions. $B = 1.25\,\text{MHz}$.

greater than 0.1), a further outage probability reduction can be obtained by applying frequency hopping among the loaded OFDM subcarriers for each of the transmit antennas. For instance, with $L = N_t$ at time ℓ, $\ell = 0, 1, \ldots, L-1$, the symbol A_m is loaded on the antenna $((\ell+m) \bmod N_t)+1$. In that way, the available frequency diversity is completely exploited.

- If either $B_{coh} > B/N_t$ or the channel is fast time-varying, then there is no use applying frequency hopping.
- If the channel is almost static over N_t OFDM symbols, then SD (with or without frequency hopping) is almost equivalent to TSTD. In TSTD systems a whole OFDM symbol is transmitted by one antenna, where successive symbols are transmitted by circularly selecting one the N_t transmit elements [5].

V. Conclusions

In this paper, transmit diversity methods for OFDM-based systems were analyzed and compared by investigating the outage capacity limits of the radio channels induced by the different transmissions strategies.

It was shown that while ergodic capacity is independent of the specific diversity technique, the theoretic outage rates are sensitive to the number of diversity degrees exploitable by a system. Consequently, it was addressed the optimal design for LDD and CDD transmissions by noticing that they are equivalent to a SD scheme. That result is useful from both theoretic and practical point of views. In fact, most of the work presented in the literature has been focused on CDD transmissions but little has been done for SD schemes. All the same, many of the studies developed for CDD, i.e., channel estimation, code design, etc., can be straightforwardly applied to SD.

The analytical treatment proposed in this paper allows to look at different transmission strategies under a unified framework that point clearly out the actual gain of the analyzed systems.

The main results presented in this paper can be summarized as:

i) Optimized LDD, CDD and SD are equivalent to each other in many cases of interest.
ii) Over highly frequency selective, but slow time-varying channels, frequency hopping can improve the performance of SD.
iii) Over slow time-varying channels SD is almost equivalent to TSTD.

The above results are guidelines for the selection of the most suitable diversity strategy for a given transmission scenario.

Finally, we observe that a further performance improvement can be obtained by developing space-time-frequency coding techniques that lead to an increment on the ergodic limit at the cost of additional computational complexity.

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